PLOT OF ALL HIGH-ENERGY p-p ELASTIC-SCATTERING DATA USING PARTICLE IDENTITY*

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During the past five years there has been an extensive experimental study of high-energy proton-proton elastic scattering at both small[1,2] and large angles.[2-5] These experiments have shown the differential cross section to decrease strongly as a function of both angle and energy, and to have at least two sharp breaks.[4,5]

There has also been an extensive phenomenological effort to find a single variable against which all cross sections could be plotted in an energy-independent way. The hope that such a variable might exist has been encouraged by the search for an optical-type model in which the interaction region does not change size with energy. The simple optical model proposed by Serber[6] suggested that $d\sigma/dt$ should be plotted against t. This plot turned out not to be energy independent when better data became available.[3] Next Krisch[7] and Narayan and Sarma[8] independently suggested plotting $d\sigma/dt$ against $P_\perp$, the transverse momentum. This gave a much better fit, but there was still some "shrinkage" or energy dependence remaining. Orear[9] then suggested plotting $sd\sigma/d\Omega$ against $P_\perp$ which gave a fair fit to large-angle data but a very poor fit to small-angle data. Krisch[10] suggested plotting $d\sigma/dt$ against $\beta^2P_\perp^2$,[11] where β is the center-of-mass velocity of the protons. This improved things considerably, giving a good fit over most of the angular range. However, the recent high-precision data of Allaby et al.[5] indicated that $d\sigma/dt$ was flatter near 90° than this model suggested. This flattening trend can be seen in the $\beta^2P_\perp^2$ plot in Ref. 4. Allaby et al.[5] then suggested plotting $d\sigma/dt$ against $s\sin\theta$. This plot looked good near 90° but was very poor at small angles.

We now propose that the flattening observed near 90° in the $\beta^2P_\perp^2$ plot is caused by the fact that the proton-proton system contains two identical particles. The cross sections from the two protons are equal at 90° and add together causing flattening in the angular distribution.

To study this effect we will study the cross section $d\sigma^\dagger/dt$, which we will call the "true" cross section for proton-proton elastic scattering when identity effects are subtracted out. The experimentally measurable cross section $d\sigma/dt$ is defined by

$$\text{No. Events} = N(N_0\rho t)(P^2/\pi)(d\sigma/dt)\Delta\Omega, \tag{1}$$

where N and $N_0\rho t$ are, respectively, the numbers of beam and target particles. These two cross sections are related by

$$d\sigma^\dagger/dt = I^{-1}(d\sigma/dt). \tag{2}$$

The function I contains the effects of the two protons being identical particles. We contend that this $d\sigma^\dagger/dt$ is the fundamental cross section which might be expected to behave in a simple and energy-independent way. Note that for other processes, such as π-p or n-p scattering, I is equal to 1 so that $d\sigma^\dagger/dt$ is identical to $d\sigma/dt$. Thus the plot of $d\sigma/dt$ against $\beta^2P_\perp^2$ for π-p scattering given in Ref. 9 is identical to the $d\sigma^\dagger/dt$ plot.

For proton-proton scattering the function I has two simple properties. It is equal to 2 at 90° and it is equal to 1 at small angles. We study its detailed form by noting that the experimental cross section has two terms for the two protons:

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma^\dagger}{dt}\right)_{\text{forward}} + \left(\frac{d\sigma^\dagger}{dt}\right)_{\text{backward}}. \tag{3}$$

Notice that we have added cross sections. The significance of this will be discussed later. Equation (3) can be rewritten as

$$\frac{d\sigma}{dt} = I\left(\frac{d\sigma^\dagger}{dt}\right)_{\text{f}}. \tag{4}$$

Then the function I is given by

$$I = 1 + \frac{(d\sigma^\dagger/dt)_{\text{b}}}{(d\sigma^\dagger/dt)_{\text{f}}}. \tag{5}$$

We now assume[7,10] that the cross section $d\sigma^\dagger/dt$ is given by the sum of three Gaussians in $\beta P_\perp$,

$$\frac{d\sigma^\dagger}{dt} = \sum_{i=1}^{3} A_i \exp(-a_i\beta^2P_\perp^2). \tag{6}$$

Then except in the immediate region of one of the breaks the cross section can be rewritten as

$$d\sigma^\dagger/dt = A_i \exp(-a_i\beta^2P_\perp^2). \qquad (7)$$

We are now faced with the problem that for angles less than 90°, $(d\sigma^\dagger/dt)_\mathrm{b}$ must be smaller than $(d\sigma^\dagger/dt)_\mathrm{f}$, but both have the same $P_\perp^2$ and would thus be equal by Eq. (7). To fix this up we first note that

$$P_\perp^2 = P^2 - P_l^2, \qquad (8)$$

where P is the total momentum and P_l is the longitudinal momentum. Then we replace P_l^2 by $P_l|P_l|$, the product of P_l and the absolute value of P_l:

$$P_l^2 \to P_l|P_l|. \qquad (9)$$

Then we find that $(d\sigma^\dagger/dt)_\mathrm{f}$ and $(d\sigma^\dagger/dt)_\mathrm{b}$ behave properly:

$$(d\sigma^\dagger/dt)_\mathrm{f} = A\exp(-a\beta^2P^2)\exp(+a\beta^2P_l|P_l|),$$

$$(d\sigma^\dagger/dt)_\mathrm{b} = A\exp(-a\beta^2P^2)\exp(-a\beta^2P_l|P_l|). \qquad (10)$$

These functions have the desired property of continuing to drop after passing through 90°.

Now we notice that the ratio of the backward to the forward cross section is given by

$$\frac{(d\sigma^\dagger/dt)_\mathrm{b}}{(d\sigma^\dagger/dt)_\mathrm{f}} = \exp(-2a\beta^2P_l|P_l|). \qquad (11)$$

We substitute this ratio into Eq. (5) to obtain the form of I,

$$I = 1 + \exp(-2a\beta^2P_l^2). \qquad (12)$$

Note that we have now replaced $P_l|P_l|$ by P_l^2 since we are now referring to experimental cross sections where θ is always less than 90°. This form for I holds except in the immediate region of a break, where slightly more detail is needed to evaluate I.

Thus we have a prescription for evaluating I and hence obtaining values of $d\sigma^\dagger/dt$ from the experimental values of $d\sigma/dt$. The values of a_i used in this prescription were obtained from the data and are shown in Fig. 1. Using these a_i we have calculated $d\sigma^\dagger/dt$ for all existing high-energy proton-proton elastic-scattering measurements.[12] These cross sections are shown in Fig. 1 where $d\sigma^\dagger/dt$ is plotted against $\beta^2P_\perp^2$ on semilog paper.

The result is very impressive. There are several hundred points, many with errors of only a few percent. All are consistent[13] with a single smooth curve over a cross-section range of 10^{12},

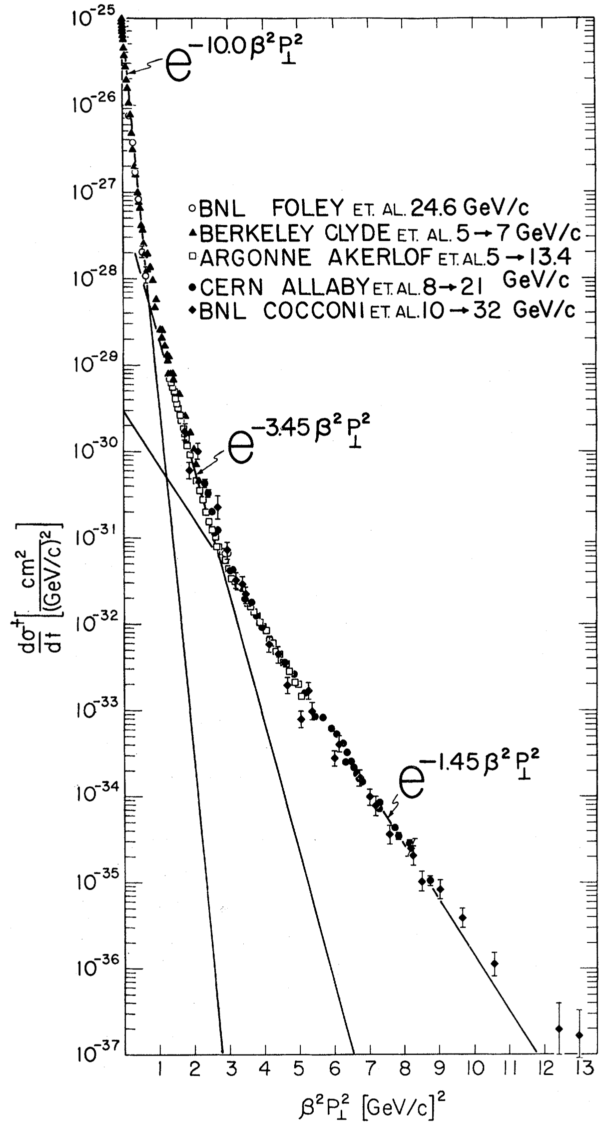

FIG. 1. Plot of $d\sigma^\dagger/dt$ vs $\beta^2P_\perp^2$ for all high-energy proton-proton elastic-scattering data (Refs. 1-5). Not all small-angle data (Refs. 1 and 2) are shown on this plot to avoid crowding. (All small-angle data are shown in detail in Fig. 2 of Ref. 10, where $d\sigma/dt$, which is identical to $d\sigma^\dagger/dt$ for small angles, is plotted against $\beta^2P_\perp^2$.) The lines drawn are straight-line fits to the data.

$$\frac{d\sigma^\dagger}{dt}\left[\frac{\mathrm{mb}}{(\mathrm{GeV}/c)^2}\right] = 90\exp(-10.0\beta^2P_\perp^2) + 0.74\exp(-3.45\beta^2P_\perp^2) + 0.0029\exp(-1.45\beta^2P_\perp^2). \qquad (13)$$

Thus it appears that there are three Gaussian regions in the proton-proton interaction.[14]

A comment should be made on the fact that we used I in relating $d\sigma^\dagger/dt$ to $d\sigma/dt$. This is equivalent to adding cross sections rather than amplitudes, and is necessary to obtain the dramatic fit in Fig. 1. The need to add cross sections in this way implies a relationship near 90° between the different spin amplitudes, which we will not study in detail.

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[11]$\beta^2 P_\perp{}^2$ should be the correct variable in a diffraction model in which a spherically symmetric interaction probability density is squashed down by a factor γ by the Lorentz transformation. See Ref. 10.

[12]At some low energies the nature of elastic scattering changes and is no longer diffractive. We have set a cutoff at 5 GeV/c and have only considered data above that momentum.

[13]We have calculated the χ^2 for all points in Refs. 3-5 except those in the immediate region of the break, where the interference must be considered. (See Ref. 14.) For these 93 points we got a $\chi^2 = 158$. For Refs. 3 and 4 we used the point-to-point errors quoted, and for Ref. 5 we added to the point-to-point errors 5% of the quoted 10% normalization error.

[14]As pointed out in Ref. 4, there appears to be some destructive interference at the second break.

ASYMPTOTIC SYMMETRY FOR VECTOR MESONS*

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Assuming the asymptotic validity of the nonet symmetry we calculate (1) electromagnetic decays of vector mesons, (2) the $\varphi \to K\bar{K}$ decay, and (3) the coupling constants for ω and φ to baryons.

Following the idea that symmetries may become exact asymptotically at high energies,[1] we discuss properties of vector mesons. For the relevant symmetry scheme we take the nonet model of Okubo,[2] which properly accounts for singlet-octet (ω-φ) mixing.[3]

To postulate asymptotic nonet symmetry we consider a set of nine vector currents $V_{\mu,\beta}{}^{\alpha}(x)$ to be associated with the nine vector mesons. μ is the four-vector index. The indices α and β each take on values 1, 2, and 3. Specifically, φ is coupled to $V_{\mu,}3^3$; ω to $(1/\sqrt{2})(V_{\mu,}1^1 + V_{\mu,}2^2)$; ρ^0 to $(1/\sqrt{2})(V_{\mu,}1^1 - V_{\mu.}2^2)$; and K^* to $V_{\mu,}3^1$. Thus the currents constitute the usual nonet tensor. Note that the upper and lower indices of this mixed tensor may be considered to refer to two commuting SU(3) algebras in a manner discussed in detail by Bose and Sudarshan.[2]

A convenient method of imposing asymptot-