with $1<m<2$, that is like $1/c^m$ so that the ratio of negative to positive magnetoresistance changes like c^{m+1}.

[10]In the case of CuMn small constant deviations occurred when plotting the negative magnetoresistance on a H/T plot; this was attributed to a systematic error made when assuming that the maximum value measured already corresponded to the low-temperature constant value. Accordingly, a single correction of a few percent increase was made. Some systematic positive and negative trend was observed in the variation of magnetoresistance with angle as the temperature was lowered which was not attributed to the normal positive part.

[11]Measurements on more concentrated alloys (280 and 2000 ppm) when plotted in the same way displayed a striking change in the position of the maximum of resistivity when an external field is applied. In a very rough way the temperature of the maximum of resistivity T_{max} varied like $T_0+(g\mu_B H_{ext}/k)$, where T_0 is the temperature of the maximum in zero external field, $g=2$, μ_B is the Bohr magneton, and k the Boltzman constant.

[12]J. A. Careaga, B. Dreyfus, R. Tournier, and L. Weil, in Proceedings of the Tenth International Conference on Low Temperature Physics, Moscow, 1966 (to be published).

[13]M. T. Beal-Monod and R. A. Weiner (to be published). In this paper a second-order perturbation calculation of the magnetoresistance is presented.

[14]C. M. Hurd, J. Phys. Chem. Solids 28, 1345 (1967).

ANALYSIS OF PERIODIC SCHOTTKY DEVIATIONS FROM IRIDIUM FILAMENTS

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The periodic deviation from the Schottky effect for thermionic emission has been measured for iridium polycrystalline wire over a temperature range from 1500 to 2000°K. A least-squares computer program was used to fit the experimental data with the periodic F_2 term predicted by the Miller-Good theory and deduce a complex reflection coefficient for the surface potential. However, the results also indicate that the amplitude of the experimental deviation increases slightly faster with field than is predicted by theory.

Periodic deviations from the Schottky curve in thermionic emission have been well established experimentally for tungsten,[1-3] tantalum,[1,4] molybdenum,[5,6] and rhenium.[7]

It is generally accepted that the deviations are due to a wave-type reflection of electrons at two relatively distinct points as they pass through the surface.[8] The first point, at the surface itself, is characterized by a complex reflection coefficient μ. The second point is at the motive maximum created by the superposition of applied field and the classical image potential, and has a reflection coefficient usually denoted by λ. Several theoretical calculations[9-12] have been made to evaluate the main periodic term in the deviations. Most of these calculations arrive at an expression containing μ and $\arg\mu$ (the amplitude and phase, respectively, of the reflection coefficient for the surface region) as parameters, and the assumption is that these quantities are field independent.

The differences between the various derived expressions are small, and for convenience the authors have used the main periodic term (usually denoted by F_2) derived by Miller and Good[9] for comparison with experiment. This term is given by

$$F_2=\frac{|\mu|f(\xi)}{T}\sin[R(\xi)+\arg\mu], \tag{1}$$

where

$$f(\xi)=1.3\times10^{-3}[C(\xi)]^{1/2}\xi^{7/4}, \tag{1a}$$

$$C(\xi)=1.007(1-0.079\ln\xi), \tag{1b}$$

$$R(\xi)=\frac{357.1}{\xi^{1/2}}+\tfrac{1}{2}\left[\tan^{-1}C+C\ln\frac{4+4C^2}{1+4C^2}\right]. \tag{1c}$$

T is the temperature in °K, and ξ is the square root of the field. The term in brackets on the right-hand side of Eq. (1c) is a slowly varying function of field and is usually approximated by a constant average value, but here all data were analyzed using the exact expression.

The thermionic emission from polycrystalline iridium wire 5.0×10^{-3} cm diam was measured as a function of applied field from 10^4 V/cm to 6.5×10^5 V/cm, and over a temperature range from 1500 to 2000°K. Measurements were made on two different wires both from the same stock.

The data from the emission measurements were analyzed on a digital computer using a least-squares program which fitted the experimental points with an assumed theoretical curve. Ideally one tries to fit the data with a theoretical curve of the form

$$\log_{10}J = P + Q\xi + F_2. \quad (2)$$

Expanding F_2 gives

$$\log_{10}J = P + Q\xi + [f(\xi)/T][M\cos R(\xi) + N\sin R(\xi)],$$

where J is current density and ξ, $f(\xi)$, T, and $R(\xi)$ are the same as defined in Eq. (1), and P, Q, M, and N are constants. The first two terms on the right-hand side of Eq. (2) represent the Schottky straight-line increase in $\log_{10}J$ with the square root of the field due to lowering of the mirror-image barrier. The coefficients P and Q as given by the least-squares program using Eq. (2) determine the best straight line from which the experimental periodic deviations are separated.

In most runs the deviations show a slight curvature in their baseline below $\xi = 200$ if they are separated from a single straight line. (Similar phenomena observed by others have often been attributed to patch effects.) However, the amount of scatter in the experimental points was extremely small; therefore, in order to display the deviations over the complete range of field, the low-field curvature was compensated by separating the data about a line that had a constant plus a monotonically decreasing slope. (This technique is somewhat analogous to the practice of using different straight-line segments as outlined in Ref. 5.) A set of deviations separated in this manner is shown in Fig. 1. Similar monotonically varying corrections were used to compensate for the low-field curvature for other temperatures and filaments.

In data that had several periods present, as is the case here, the low-field curvature compensation could not introduce periodicity that was not already present in the data. Furthermore, in the range of field for which quantitative data are presented here, the amplitudes and phases remained unaffected, to within approximately 5%, whether curvature compensation was used or not.

Experimental values of $|\mu|$ were calculated by equating the Miller-Good amplitude term to the amplitude (A_m) of the experimental deviations at the values of $\xi = \xi_m$ corresponding to the maxima and minima points, i.e.,

$$|\mu|_{\xi_m} = A_m T/f(\xi_m), \quad (3)$$

where $f(\xi_m)$ is the Miller-Good amplitude term [Eq. (1c)] evaluated at ξ_m. Figure 2 is a plot of $|\mu|_{355}$ as a function of temperature. It can be seen that within experimental error $|\mu|_{355}$ is independent of temperature and an average value of $\langle|\mu|_{355}\rangle = 0.31$ is found. However, average values obtained in this manner increased with ξ_m (although all were independent of temperature), e.g., $\langle|\mu|_{265}\rangle = 0.29$ and $\langle|\mu|_{510}\rangle = 0.33$. This is expected since, as indicated in Fig. 1, the amplitude of the Miller-Good curve that "best fits" the experimental points does not increase quite as fast as the experimental am-

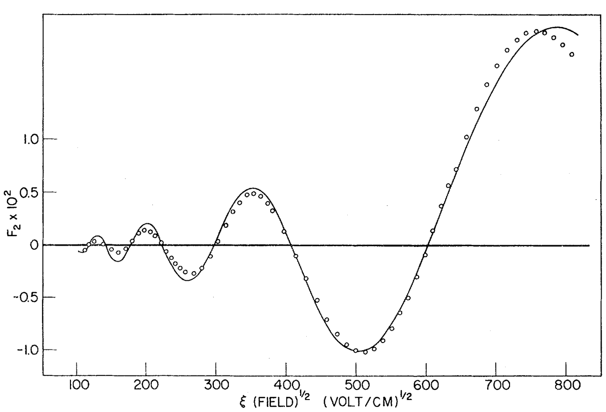

FIG. 1. Experimental periodic deviations for a 2-mil iridium filament at $T = 1865°K$. The solid curve is the Miller-Good F_2 term fitted to the experimental points by the least-squares program. The circles are the experimental deviations as separated using a curvature compensation proportional to $\xi^{3/2}$.

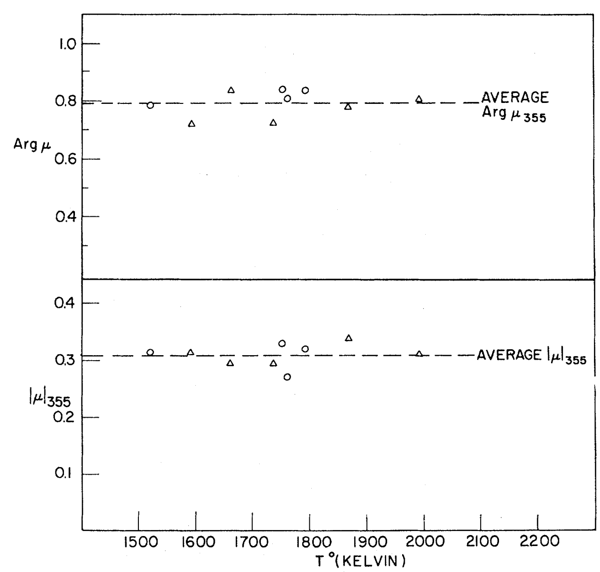

FIG. 2. Plot of $|\mu|$ and argμ vs T for the peak at $\xi \sim 355$. Triangles are filament No. 1. Circles are filament No. 2.

plitudes.[13] This behavior was typical of all runs taken on both filaments. The experimental values of $\arg\mu$ obtained by matching the phase of the periodic term in Eq. (1) at the value of $\xi = \xi_m$ for which extrema occur in the experimental curve were determined from the equation[14]

$$\arg\mu = [\tfrac{1}{2}(2m+1)]\pi - [9.4\times 10^{-3}\xi_m^{\;1/2} + R(\xi_m)], \quad (4)$$

$$m = \text{integer}.$$

The phase obtained from Eq. (4) for fields below $\xi = 600$ was, within experimental error, independent of field and temperature. The values of $\arg\mu$ evaluated at $\xi_m = 355$ are plotted against temperature in Fig. 2. The plots for $\xi_m < 355$ had somewhat more scatter, but averaged to essentially the same value of $\arg\mu = 0.79 \pm 0.1$.

It can be seen from Fig. 1 that ξ_m for the experimental peak near $\xi = 750$ does not agree with the corresponding ξ_m for the peak in the theoretical curve. (This discrepancy was slightly larger if curvature compensation was not used.) The observed shift in phase of approximately -0.3 rad could be due to a field dependence of $\arg\mu$ which becomes significant at high fields, or the approximations made in arriving at $R(\xi)$ could be poor at high fields.

It is to be noted that the values of M and N in Eq. (2a), which are obtained from the least-squares fitting, can also be used to determine experimental values for $|\mu|$ and $\arg\mu$. However, because of the anomalous increase in the amplitudes, and the fact that the least-squares curve tended to fit better at higher fields, the values of $|\mu|$ and $\arg\mu$ obtained in this way were not the best averages for the entire curve.

It can be concluded that the periodic deviations from iridium are in approximate agreement with the Miller-Good theory, and that the experimental measurements interpreted in terms of this theory give an average amplitude of the reflection coefficient for the surface potential of iridium of $|\mu| = 0.31$. However, there is enough consistent evidence in the present measurements to indicate that the field dependence of the theoretical amplitude does not agree exactly with experiment.

Except for the unexplained phase shift above $\xi = 600$, the phase of the deviations is in good agreement with the phase term predicted by Miller and Good,[9] and gives an experimental value of $\arg\mu = 0.79$ that is constant over a relatively wide range of field. Because of the lack of agreement[15] between theory and experiment, especially with regard to $|\mu|$, it is not possible to obtain an unambiguous reflection coefficient from the amplitudes of the periodic deviations.

[1] R. L. E. Seifert and T. E. Phipps, Phys. Rev. 56, 652 (1939).

[2] D. Turnbull and T. E. Phipps, Phys. Rev. 80, 663 (1950).

[3] D. F. Stafford and A. H. Weber, J. Appl. Phys. 34, 2667 (1963).

[4] R. J. Munich, W. B. LaBerge, and E. A. Coomes, Phys. Rev. 80, 887 (1950).

[5] E. G. Brock, A. L. Houde, and E. A. Coomes, Phys. Rev. 89, 851 (1953).

[6] G. A. Haas and E. A. Coomes, Phys. Rev. 93, 1421 (1954).

[7] I. J. D'Haenens and E. A. Coomes, Phys. Rev. Letters 17, 516 (1966).

[8] C. Herring and M. H. Nichols, Rev. Mod. Phys. 21, 185 (1949).

[9] S. C. Miller, Jr., and R. H. Good, Phys. Rev. 92, 1367 (1953).

[10] Eugene Guth and Charles Mullin, Phys. Rev. 59, 575 (1941).

[11] P. H. Cutler and J. J. Gibbons, Phys. Rev. 111, 394 (1958).

[12] D. W. Juenker, G. S. Colladay, and E. A. Coomes, Phys. Rev. 90, 772 (1953).

[13] This same effect has also been found in some recent new deviations taken by the authors using tungsten and molybdenum filaments (investigation of this effect is still in progress).

[14] Equation (4) is determined by setting $(\partial F_2/\partial\xi)_{\xi=\xi_m} = 0$. The form of Eq. (4) is an approximation valid where $9.4\times10^{-3}\xi_m^{\;1/2} \ll 1$. The error in $\arg\mu$ due to the approximation is less than 1% for $\xi_m < 600$.

[15] The discrepancies observed in amplitude and phase could arise from many sources. However, in the calculation of the F_2 term Miller and Good (Ref. 9) indicated that some of their approximations caused the amplitude to be low by about 10%. It seems very likely that this could be a field-dependent error. Consequently, it is possible that a numerical calculation of F_2 using the λ term of Miller and Good (Ref. 1) would resolve the discrepancies observed here.