potential measurements had an impedance of 500 MΩ and gave an approximately linear frequency response up to ion cyclotron frequency. Density measurements were based on probe theory by J. G. Laframboise [University of Toronto Report No. UTIAS 100, 1966 (unpublished)]. This theory is in approximate agreement with microwave and spectroscopic measurements and (at $\approx 10^{11}$ cm^{-3}) gives density values lower by a factor of $\frac{1}{3}$ than those from previous calculations, reviewed by F. F. Chen in Plasma Diagnostic Techniques, edited by R. H. Huddlestone and S. L. Leonard (Academic Press, Inc., New York, 1965).

[11]L. Spitzer, Physics of Fully Ionized Gases (Interscience Publishers, Inc., New York, 1962), 2nd ed.

[12]The use of Fick's law as the basis for the concept of diffusion holds when the pertinent plasma parameters can be averaged over distances small compared with the density scale length, e.g., $\lambda_\perp \ll n_0/\nabla n_0$. [F. Boeschoten, J. Nucl. Energy 6, 339 (1964).]

[13]S. von Goeler and R. W. Motley, Phys. Fluids 10, 1367 (1967). We note that a comparison of measured input flux and computed classical losses in the stable regime indicates an additional loss whose origin is not identified. A similar loss has been found in other Q devices. {S. von Goeler and R. W. Motley, in Proceedings of Conference on the Physics of Quiescent Plasmas, Frascati, Italy, 1967 [Laboratori Gas Ionizzati (Association Euratom-Comitato Nazionale per l'Energia Nucleare), 1967]; and private communications with F. F. Chen and G. Grieger.}

[14]To derive Eq. (3), the continuity equation is integrated over the length of the plasma column. Thus, F represents the radial flux averaged over z. Since the wave amplitude is maximum at the midplane and nearly zero at the end plates, the contribution to F due to the wave is approximately one-half of the wave flux at the midplane, Fig. 2(a).

[15]L. D. Landau and E. M. Lifshitz, Fluid Mechanics (Pergamon Press, London, 1959), p. 104; B. B. Kadomtsev, Plasma Turbulence (Academic Press, Inc., New York, 1965), p. 46.

[16]The enhanced loss thus may become comparable to the Bohm value before the wave amplitude reaches 100% (assuming ψ to be unchanged). We note that J. B. Taylor [Phys. Rev. Letters 6, 262 (1961)] showed from stochastic considerations that the maximum attainable transverse diffusion can exceed the Bohm value by a factor of 8 for $T_e = T_i$. A specific mechanism for Bohm diffusion based on nonlinear interaction of inertial drift modes, and resulting in relative amplitude fluctuations of the order of 100%, has recently been reported by B. Coppi, Princeton Plasma Physics Laboratory, Princeton University Report No. MATT-545, 1967 (unpublished).

MAGNETIC FIELD DEPENDENCE OF THE KONDO RESISTIVITY MINIMUM IN CuFe AND CuMn ALLOYS*

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We report here some quantitative measurements of resistivity, magnetoresistance, and magnetization made in dilute CuFe and CuMn alloys. Our main conclusions are these:

(a) It is possible to determine and separate out the (normal) positive magnetoresistance from the negative magnetoresistance of the impurities.

(b) In both dilute alloys the negative magnetoresistance varies like H^n with n about 1.7-1.8 for values of the magnetic field H of a few kilogauss.

(c) Whereas for CuMn a field of 20 kG will completely alter the logarithmic temperature dependence of the resistivity (namely creating a maximum of the resistivity at around 4°K), the same field will only lower the resistivity of CuFe by 0.3% at 1.4°K.

(d) At any temperature and magnetic field, the amplitude of the negative magnetoresistance varies like the square of the magnetization of the impurities. This relation is sufficient to account for the behavior of these two alloys and allows us to infer the s-d exchange constant.

Previous studies have been done by numerous workers on more concentrated alloys[1,2] or in a limited range of low temperatures[3] where these effects are readily observed but difficult to analyze quantitatively because of the presence of unknown internal fields arising from the magnetic interactions between impurities in concentrated alloys, and the presence of the large positive magnetoresistance[4,5] in dilute alloys. The correlation between negative magnetoresistance and magnetization was qualitatively demonstrated by Schmitt and Jacobs[2] on very concentrated alloys exhibiting hysteresis effects and was inferred theoretically by Yosida.[6]

The CuMn samples were grown as single crystals in a graphite crucible heated in vacuum by induction.[7] The CuFe 110-ppm sam-

ple was from the same ingot where very low-temperature resistivity and initial susceptibility had already been measured.[8] The resistance of thin strips electrocut from the ingots was measured by a conventional four-electrode method. The relative accuracy was better than 10^{-4} and the absolute precision was 10^{-3}. The method for extracting the negative magnetoresistance term implies that the positive term remains constant at low temperature. This positive part is measured directly in the temperature range 15-20°K (where the phonon contribution to the scattering is negligible, and the negative term very small). This quantity is then systematically subtracted from all lower temperature measurements. That the normal, positive magnetoresistance is constant at low temperature was checked in two ways on CuFe: (1) On a 17-ppm alloy, it was observed that the measured magnetoresistance was constant within 1% between 20 and 1.4°K (although the resistance increased by 8%).[9] This is not in contradiction with the so-called Kohler's rule[3] provided that the magnetoresistance is linear with field, which was the case except at low fields. (2) The amplitude of the variation of magnetoresistance with magnet angle is temperature independent (for 800, 500, 110, and 17 ppm). This also proves that the negative part of the magnetoresistance is not itself dependent on the angle between the field and the current.[10]

The magnetization of samples of ~1 g cut from the same original ingots was measured using the Faraday method at fields up to 11 kG and temperatures down to 0.4°K. The sensitivity was about 10^{-9} emu/g but the precision was somewhat lower because of uncertainty in the sample holder correction.

CuMn results.—The resistivity of a 75-ppm CuMn alloy versus temperature at different fields is displayed in Fig. 1, once the positive magnetoresistance (18% of the resistance at 20 kG and 16.20°K) has been subtracted, as explained above. For this sample the deviation from logarithmic behavior in zero field is expected to occur at ≈0.25°K.[2]

The behavior of two more dilute samples (36 and 11 ppm) gave similar results. The negative magnetoresistance term ΔR which on Fig. 1 is the difference, at a given temperature, between the curve at zero field and the curve at a field H could be fitted with a unique function of H/T over the whole range of measurement at all fields between 2 and 20 kG and temperature between 1.40 and 10°K. This function varied like $(H/T)^n$, where $n = 1.7 \pm 0.1$ for $0.4 < H/T < 4$ kG/°K.[11]

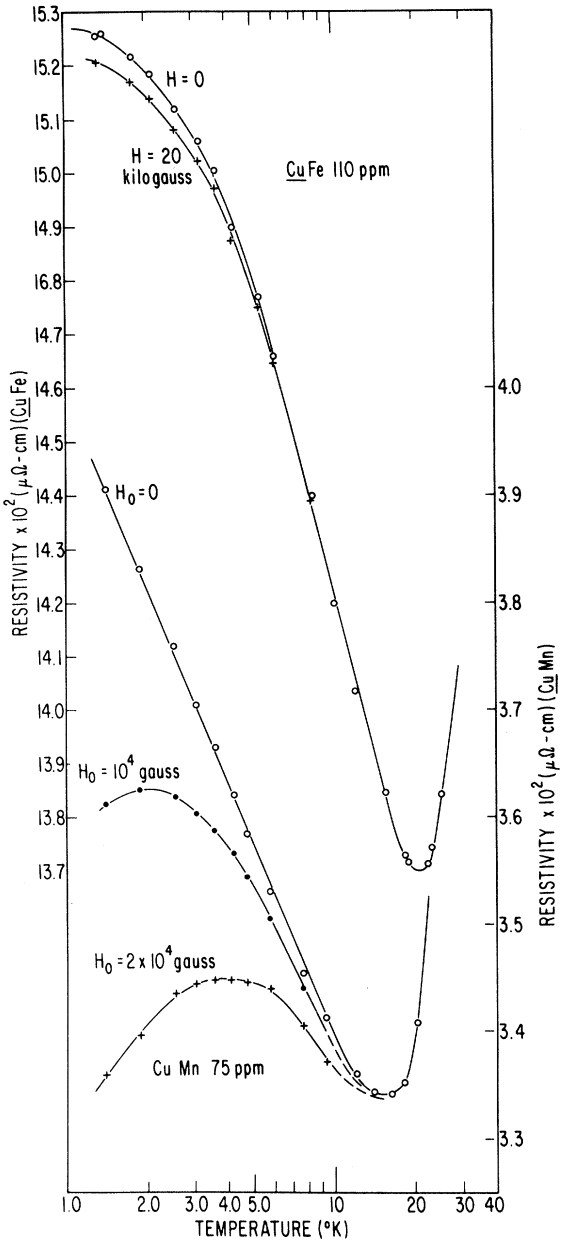

FIG. 1. Right scale and lower curve: resistivity of a 75-ppm-CuMn alloy as a function of temperature and magnetic field. The normal positive magnetoresistance has been subtracted everywhere. Left scale and upper curve: resistivity of a 110-ppm CuFe alloy as a function of temperature at zero magnetic field and 20 kG. The normal positive magnetoresistance has been subtracted.

The magnetic measurements on the 75-ppm

sample yielded magnetization curves $M(H/T)$ which are also only a function of H/T (except below 1°K). However, this function could not be fitted with a Brillouin function better than 10-20% even at low values of the magnetic field. These measurements closely reproduce the behavior found by Careaga.[12] The correlation between the magnetization $M(H/T)$ and the (negative) magnetoresistance $\Delta R(H/T)$ is expressed in Fig. 2 where the parameter H/T is a hidden common variable. Three things must be noted:

(a) The relationship is the same in the low-field region or in the high-field region where M reaches half its saturation value of $4.7\mu_B$.

(b) The slope of $\log\Delta R$ vs $\log M$ is not 2 but rather 1.93 ± 0.05. We believe this to be ascribable to the presence of small internal fields.

(c) The coefficient of proportionality between ΔR and M^2 is derived[13] in the limit of $g\mu_B H/kT < 1$ and first-order perturbation as

$$\Delta R = R(H,T) - R(0,T) = \frac{3\pi}{2\epsilon_F}\frac{m}{e^2\hbar} c V_0 J^2 M^2 \left[1 + \left(\frac{\mu_B}{\mu_{eff}}\right)^2\right], \qquad (1)$$

where e and m are the charge and mass of the electron, c the atomic concentration of magnetic impurities, ϵ_F the Fermi energy, V_0 the atomic volume, J the s-d exchange constant, M the magnetization of the magnetic impurities in μ_B per atom, and μ_{eff} the effective moment defined by

$$\chi = N\mu_{eff}^2/3kT,$$

where χ is the measured susceptibility, T the temperature, and N the number of impurities. We get from Eq. (1)

$$|J(\text{CuMn})| = 0.40 \pm 0.1 \text{ eV};$$

the uncertainty comes from the exponent of 1.93 rather than 2 in the $\Delta R \sim M^n$ relationships.

CuFe results.—Figure 1 shows also the resistivity variation of a 110-ppm CuFe alloy between 1.4 and 30°K in zero magnetic field and in 20 kG (with the positive magnetoresistance removed, as explained). The strong curvature in the 2°K region is due to the approach to the limiting value of resistivity as shown by Daybell and Steyert.[8] The negative magnetoresistance term follows H^n where n is about 1.75 ± 0.2 but is definitely not a function of H/T; indeed, for H/T constant, the negative magnetoresistance decreases by a factor of 5 when the temperature changes from 6 to 1.3°K. Likewise, the magnetization cannot be fitted to a Brillouin function of H/T; instead,[8,14] the susceptibility fits quite well the formula.

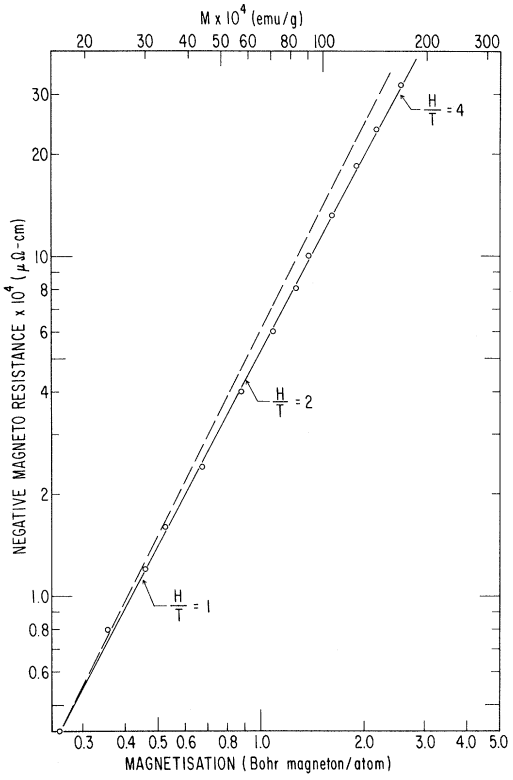

FIG. 2 Relation between negative magnetoresistance of the same 75-ppm CuMn and its magnetization ΔM. H/T is in kG/°K. The dashed line corresponds to a slope of 2. The factor $1+(\mu_B/\mu_{eff})^2$ is a constant with H and T and does not change the relationship.

$$\chi/N = \mu^2/3k(T+T')$$

in the range 1.3-20°K, where $T' = 9$°K and $\mu = 2.6\mu_B$. When the negative magnetoresistance ΔR is plotted versus the magnetization $\Delta M^2[1 + (\mu_B/\mu_{eff})^2]$, one gets a straight line as shown in Fig. 3. We must note that here too the exponent n in the ΔR-ΔM^n relation is a bit less than 2, namely, 1.90 ± 0.1. From Eq. (1) we can infer the s-d exchange constant:

$$|J(\text{CuFe})| = 0.91 \pm 0.2 \text{ eV}.$$

It is worth realizing that although Eq. (1) was obtained by first-order perturbation, it adequately correlates ΔR and ΔM below the critical temperature T_k (10-16°K) where these two quantities cannot be calculated by second-order

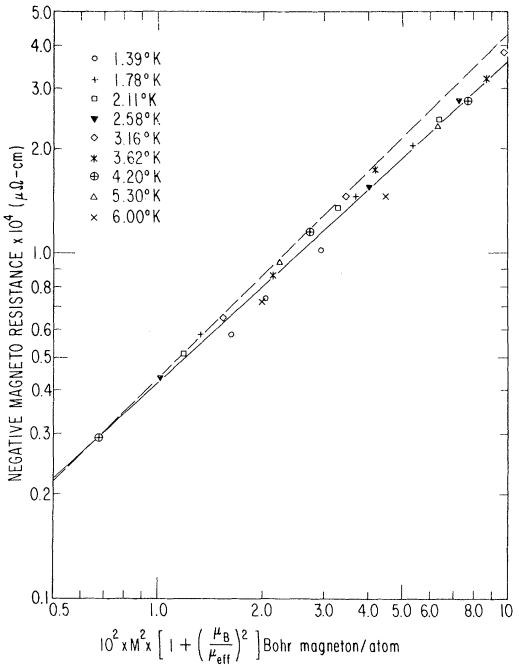

FIG. 3. Relation between the negative magnetoresistance of the same 110-ppm CuFe sample and the square of its magnetization. Here the factor $1+(\mu_B/\mu_{eff})^2$ is strongly varying with temperature. The dashed line is an arbitrary line of slope 1.

perturbation.[13]

Conclusions.—Inasmuch as the "normal" positive magnetoresistance can be measured and subtracted, it is possible to study quantitatively the negative contribution arising from dilute magnetic impurities in copper. From the relationship between negative magnetoresistance and the square of the magnetization, a value for the s-d exchange is inferred. The further implications are as follows:

(a) It might be possible to detect a change in the magnetization of magnetic impurities (due for instance to saturation with microwaves) by measuring the resistance change of the alloy.

(b) In the case of concentrated alloys showing deviations from dilute alloys behavior in resistivity, it is very tempting to interpret these deviations by the presence of internal magnetic fields. The resistivity difference between dilute and concentrated alloy (on appropriate scale) might lead to a measure of the local internal magnetization whereas a conventional magnetic measurement only gives the over-all magnetization of the sample.[2]

(c) Once the ΔR-ΔM relation is established, it is easier to follow the magnetization of isolated magnetic impurities by studying their negative magnetoresistance in regions of field or temperature where direct measurements are uncertain or awkward.

(d) If the relation between magnetization and negative magnetoresistance can be shown to hold far below T_k, it might serve to distinguish experimentally between impurities nonmagnetic by nature and those apparently nonmagnetic because the observation is made at too low a temperature. In the second case a small, temperature-independent, negative magnetoresistance would still be expected.

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†On leave from the Service de Physique des Solides et de Résonance Magnétique, Commissariat à l'Energie Atomique, Saclay, France.

[1]A. N. Gerritsen, Physica 19, 61 (1953).

[2]R. W. Schmitt and I. S. Jacobs, J. Phys. Chem. Solids 3, 324 (1957).

[3]Y. Muto, K. Noto, and F. T. Hedgcock, Can. J. Phys. 42, 15 (1964).

[4]N. E. Alekseevskii and I. P. Gaidukov, Zh. Eksperim. i Teor. Fiz. 31, 947 (1956) [translation: Soviet Phys.-JETP 4, 807 (1957)].

[5]Alekseevskii and Gaidukov studied carefully the resistivity minimum of AuFe and showed that a field of 8 kG was enough to make it disappear. They did not study the magnetization nor did they analyze quantitatively the negative magnetoresistance component.

[6]K. Yosida, Phys. Rev. 107, 396 (1957). This calculation is restricted to first-order perturbation.

[7]The copper was 99.999% pure from American Smelting and Refining Company. It contained a maximum of 2 ppm of magnetic impurities (Fe and Mn).

[8]M. D. Daybell and W. A. Steyert, Phys. Rev. Letters 18, 398 (1967).

[9]In this particular alloy the negative component was too small to be measured. It should be emphasized that the amplitude of the negative term is proportional to the concentration c of magnetic impurities, whereas the amplitude of the positive term varies like $(\omega_c\tau)^m$

with $1<m<2$, that is like $1/c^m$ so that the ratio of negative to positive magnetoresistance changes like c^{m+1}.

[10]In the case of CuMn small constant deviations occurred when plotting the negative magnetoresistance on a H/T plot; this was attributed to a systematic error made when assuming that the maximum value measured already corresponded to the low-temperature constant value. Accordingly, a single correction of a few percent increase was made. Some systematic positive and negative trend was observed in the variation of magnetoresistance with angle as the temperature was lowered which was not attributed to the normal positive part.

[11]Measurements on more concentrated alloys (280 and 2000 ppm) when plotted in the same way displayed a striking change in the position of the maximum of resistivity when an external field is applied. In a very rough way the temperature of the maximum of resistivity T_{max} varied like $T_0+(g\mu_B H_{ext}/k)$, where T_0 is the temperature of the maximum in zero external field, $g=2$, μ_B is the Bohr magneton, and k the Boltzman constant.

[12]J. A. Careaga, B. Dreyfus, R. Tournier, and L. Weil, in Proceedings of the Tenth International Conference on Low Temperature Physics, Moscow, 1966 (to be published).

[13]M. T. Beal-Monod and R. A. Weiner (to be published). In this paper a second-order perturbation calculation of the magnetoresistance is presented.

[14]C. M. Hurd, J. Phys. Chem. Solids 28, 1345 (1967).

ANALYSIS OF PERIODIC SCHOTTKY DEVIATIONS FROM IRIDIUM FILAMENTS

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The periodic deviation from the Schottky effect for thermionic emission has been measured for iridium polycrystalline wire over a temperature range from 1500 to 2000°K. A least-squares computer program was used to fit the experimental data with the periodic F_2 term predicted by the Miller-Good theory and deduce a complex reflection coefficient for the surface potential. However, the results also indicate that the amplitude of the experimental deviation increases slightly faster with field than is predicted by theory.

Periodic deviations from the Schottky curve in thermionic emission have been well established experimentally for tungsten,[1-3] tantalum,[1,4] molybdenum,[5,6] and rhenium.[7]

It is generally accepted that the deviations are due to a wave-type reflection of electrons at two relatively distinct points as they pass through the surface.[8] The first point, at the surface itself, is characterized by a complex reflection coefficient μ. The second point is at the motive maximum created by the superposition of applied field and the classical image potential, and has a reflection coefficient usually denoted by λ. Several theoretical calculations[9-12] have been made to evaluate the main periodic term in the deviations. Most of these calculations arrive at an expression containing μ and $\arg\mu$ (the amplitude and phase, respectively, of the reflection coefficient for the surface region) as parameters, and the assumption is that these quantities are field independent.

The differences between the various derived expressions are small, and for convenience the authors have used the main periodic term (usually denoted by F_2) derived by Miller and Good[9] for comparison with experiment. This term is given by

$$F_2=\frac{|\mu|f(\xi)}{T}\sin[R(\xi)+\arg\mu], \tag{1}$$

where

$$f(\xi)=1.3\times10^{-3}[C(\xi)]^{1/2}\xi^{7/4}, \tag{1a}$$

$$C(\xi)=1.007(1-0.079\ln\xi), \tag{1b}$$

$$R(\xi)=\frac{357.1}{\xi^{1/2}}+\tfrac{1}{2}\left[\tan^{-1}C+C\ln\frac{4+4C^2}{1+4C^2}\right]. \tag{1c}$$

T is the temperature in °K, and ξ is the square root of the field. The term in brackets on the right-hand side of Eq. (1c) is a slowly varying function of field and is usually approximated by a constant average value, but here all data were analyzed using the exact expression.

The thermionic emission from polycrystalline iridium wire 5.0×10^{-3} cm diam was measured as a function of applied field from 10^4 V/cm to 6.5×10^5 V/cm, and over a temperature range from 1500 to 2000°K. Measurements were made on two different wires both from the same stock.

The data from the emission measurements were analyzed on a digital computer using a least-squares program which fitted the exper-