is an amplitude decay rate rather than a power decay rate as was assumed in Refs. 2 and 3.

[9]Expressed in terms of the variables $u=(\Delta_A-\Delta_S)/(\Delta_A+\Delta_S)\geqslant 0$ and $v=\gamma_L/(\Delta_A-\Delta_S)\geqslant 0$, $\Lambda^2/\Lambda_0{}^2\simeq\frac{1}{2}(u^{-1}+u)+u[v^2+(v^{-2}/16)(u^{-2}-1)]$, which has a minimum of unity for $u=1, v\ll 1$.

[10]See, for example, N. Bloembergen, Nonlinear Optics (W. A. Benjamin, Inc., New York, 1965), where a similar problem is discussed involving the Stokes and anti-Stokes modes in stimulated Raman scattering.

[11]The present theory does not take into account the heating of the plasma by the pump radiation. The heating rate is slow on the scale of times important for the parametric excitation process. Thus our "steady state" may be superimposed upon a secular change in the electron and ion temperatures.

MEASUREMENTS OF ENHANCED PLASMA LOSSES CAUSED BY COLLISIONAL DRIFT WAVES*

T. K. Chu, H. W. Hendel,† and P. A. Politzer
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey
(Received 26 July 1967)

We report measurements of enhanced plasma losses caused by collisional drift waves[1] in magnetically confined, thermally ionized alkali plasmas, based on (a) a density decrease in the central part and an increase in the outer part and outside of the plasma column, coincident with the abrupt onset of drift waves, and (b) correlation of density and plasma-potential oscillations. The enhanced loss, for a wave amplitude equal to 10% of the zero-order density, is measured to be an order of magnitude larger than the loss due to classical collisional diffusion.

Collisional drift waves can arise in a low-β plasma ($\beta\equiv 8\pi p/B^2$) in the absence of current as a result of the combined effects of transverse density gradient, ion inertia, and electron parallel thermal motion. Theory[2-4] indicates that these "universal" instabilities may cause transverse plasma loss above the lower limit set by "classical" binary-collision diffusion. However such a causal relation has not been conclusively demonstrated in prior experiments.[5-7] The difficulties are identification of the instability thought to cause the enhanced loss, separation of the loss due to the instability from other losses, and lack of a measurement of local plasma transport due to the instability.

In the present work, the instability is identified[8] as a density-gradient–driven collisional drift wave by measurements of (a) longitudinal and perpendicular phase velocities and their dependences on magnetic field and plasma temperature; (b) phase relation between density and potential oscillations; (c) radial, azimuthal, and longitudinal wavelengths; and (d) magnetic-field, plasma-temperature, ion-mass, and density dependence of mode transitions. The wave is localized in a region where $\nabla n_0/n_0 \gg \nabla T/T$, i.e., where temperature-gradient–driven drift waves and wave excitation due to large radial electric fields are excluded.

The principal results of this work are observations of a density reduction (up to 30%) in the central part and a density increase (up to 6%) in the outer part of the plasma column, coincident with the abrupt onset of the $m=1$ azimuthal mode of the collisional drift wave. This abrupt onset facilitates separation of wave-enhanced losses from other losses. Onset, from $n_1/n_0\approx 0$ to ~0.1, takes place when the magnetic field is increased by approximately 1% beyond theshold, with all other parameters constant. The abrupt destabilization to large amplitude is in agreement with the strong B dependence of the growth rate near onset, calculated from linearized theory.[1] Measurements of the correlation between the coherent density and potential oscillations over the radial extent of the plasma enable us to calculate local enhanced fluxes caused by the drift wave and a local "diffusion coefficient" throughout the plasma column. Finally, utilizing the continuity equation together with the feature that the self-excited wave can be "turned on and off" abruptly by variation of B, we can compare the enhanced-loss state with the stable state in an otherwise unchanged plasma (including the boundary conditions at the sheath), and show that the results of our measurements are consistent, i.e., the local loss flux obtained from wave-parameter measurements accounts for the measured change in plasma density.

The experimental work was done on the Princeton Q-1 device.[9] The plasma consists of cesium or potassium ions produced by surface ionization of atomic beams incident on hot tungsten plates ($T_{\text{plate}} \approx 2800°\text{K} \approx T_{\text{plasma}}$) located at both ends of the plasma column, and of thermionic electrons emitted from the same plates. The fully ionized plasma ($\beta \leqslant 10^{-6} < m_e/M_i$) is 3 cm in diameter and 128 cm long. Electric fields are not applied to the plasma column, and the thermoelectric voltage between ionizer (end) plates is maintained below 5 mV by temperature-balancing the plates. Both the nearly sinusoidal ion-density and plasma-potential oscillations are detected with Langmuir probes. The phase angle between density and potential oscillations is measured by two probes[10]: One determines the ion density oscillation as a reference and the other displays, successively, ion density and potential waves.

Measured quantities for a potassium plasma are given in Fig. 1 Figure 1(a) shows radial plasma density profiles (n_{0s}, n_{0w}), for stable and drift-wave regimes, taken adjacent to onset of the $m = 1$ mode. Measurements shown extend to $r = 15$ mm, equal to the radius of the end plate. An increase of density in the presence of drift waves was observed to $r = 25$ mm.

Figures 1(b) and 1(c) show the radial amplitude distribution of density (n_1) and potential (φ) oscillations, and the phase angle ψ by which n_1 leads φ.

The local radial diffusive flux due to electron-ion collisions F_{e-i} and the radial flux F_{wave} due to a drift wave of azimuthal mode number m can be calculated from the data shown in Fig. 1 according to the following relations (with standard notation[11]):

$$F_{e-i} = -n_0 \eta \nabla p / B^2, \qquad (1)$$

and

$$F_{\text{wave}} = \left(\frac{\omega}{2\pi}\right) \int_0^{t = 2\pi/\omega} dt \frac{n_1 E_\theta}{B} = \frac{m n_1 \varphi \sin\psi}{2 r B}. \qquad (2)$$

Results of the flux calculations are given in Fig. 2(a). Ion-ion diffusion is of the same order as electron-ion diffusion, but is not included in Fig. 2(a) because of the difficulty of its evaluation from measured density profiles.

Based on the local density-gradient scale length, a "diffusion coefficient" D_{wave} for the flux associated with the drift wave can be calculated. In Fig. 2(b) this diffusion coefficient is

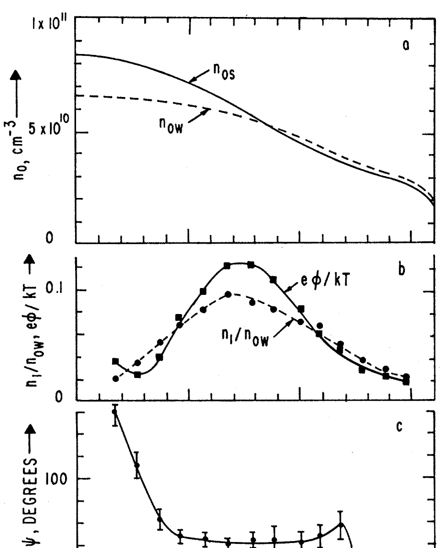

FIG. 1. Measured quantities versus radius for a potassium plasma at $T = 2760°$K. (a) Density profiles in the stable ($n_{0s}, B = 1964$ G) and drift-wave regimes ($n_{0w}, m = 1, B = 2050$ G). (b) Relative amplitude of density (n_1/n_0) and potential ($e\varphi/kT$) oscillation. (c) Phase angle ψ by which the density wave leads the potential wave.

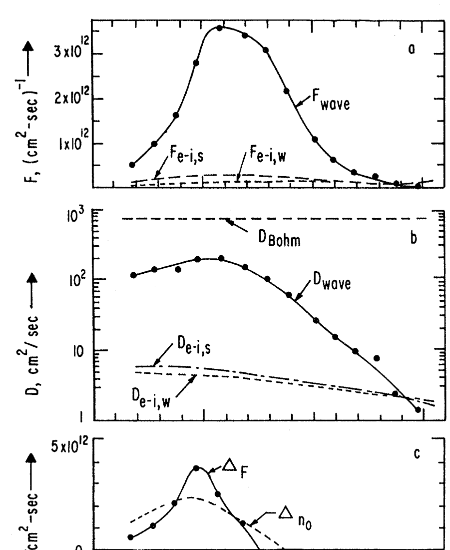

FIG. 2. Computed quantities versus radius for conditions of Fig. 1. (a) Radial particle flux due to wave (F_{wave}) and electron-ion collisions ($F_{e-i,s}, F_{e-i,w}$). (b) Comparison of diffusion coefficients. (c) Comparison of the change in divergence of radial flux, ΔF, with that in effective end-plate recombination loss, Δn_0.

compared with values of $D_{\text{classical}}$ and D_{Bohm}.[11] It should be pointed out, however, that the use of Fick's law as the basis for the concept of wave-enhanced diffusion may not be justified.[12] Therefore, greater significance must be attached to the wave-enhanced radial flux [Fig. 2(a)] than to the wave diffusion coefficient.

The consistency of inferred $\langle n_1 E_\theta \rangle$ fluxes and zero-order density changes was examined by use of the continuity equation for ions. In the continuity equation for a Q device, the ion source term S is unaffected by the presence of a wave for an electron-rich plasma with a wave amplitude at the sheath much smaller than sheath voltage. The loss term in the stable regime is proportional to the density.[13] Thus, for steady state, the continuity equation is

$$r\partial F/\partial r + F + r(\alpha n_0 - S) = 0, \tag{3}$$

where F is the radial particle flux[14] and $\alpha = 310$ sec^{-1} for the experimental conditions of Fig. 1 as determined from measurements of input flux and density profile in the stable regime. The changes of flux divergence and loss term resulting from the presence of the drift wave can now be calculated:

$$r\partial(F_w - F_s)/\partial r + (F_\omega - F_s) + r\alpha(n_{0w} - n_{0s}) = 0, \tag{4}$$

where subscripts w and s denote drift wave and stable regimes, respectively, and where we have assumed that α is unaffected by the presence of the drift wave, in agreement with others.[13] The change in the loss term, $\Delta n_0 \equiv -r\alpha(n_{0w} - n_{0s})$, with wave onset is calculated and compared with the change in the divergence of radial flux, $\Delta F \equiv (1 + r\partial/\partial r)(F_w - F_s)$, in Fig. 2(c), which shows agreement. This agreement also indicates that the assumption regarding the loss term was valid.

The measured enhanced plasma loss due to the drift wave was found to vary as a function of magnetic field, density, temperature, and ion mass similar to the growth rate from linear theory. Although linearized calculations are inadequate to predict the large experimental saturation amplitudes and the n_1-φ phase angle, previous experiments[1] and theories[15] considering higher order terms indicate that the saturation amplitude is proportional to a power of the growth rate. Also, the present experiment shows that the phase angle ψ does not vary significantly with plasma parameters. Therefore, the enhanced plasma loss is expected to be proportional to a power of the growth rate, as shown by the experiment.

In conclusion, drift waves give rise to enhanced radial particle transport. This enhanced loss is an order of magnitude large than the loss due to classical collisional diffusion and smaller than Bohm diffusion for a wave amplitude of 10 %.[16] The magnitude of the particle loss is proportional to a power of the growth rate calculated from linear theory. This implies that this particular enhanced plasma loss due to (large-amplitude) collisional drift waves can be reduced by proper choice of plasma parameters as ascertained from the growth rate.

Additional results and data on the observed plasma losses related to higher modes will be reported elsewhere.

It is our pleasure to thank Dr. F. F. Chen, Dr. B. Coppi, Dr. C. R. Oberman, Dr. F. W. Perkins, and Dr. T. H. Stix for valuable discussions. We also thank L. Gereg and C. A. Johnson for help in the experimentation.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission, Contract No. AT(30-1)-1238.

†On leave from RCA Atomic Energy Division, Princeton, New Jersey.

[1]H. W. Hendel, B. Coppi, F. Perkins, and P. A. Politzer, Phys. Rev. Letters 18, 439 (1967).

[2]S. S. Moiseev and R. Z. Sagdeev, Zh. Eksperim. i Teor. Fiz. 44, 763 (1963) [translation: Soviet Phys. -JETP 17, 515 (1963)].

[3]A. A. Galeev, S. S. Moiseev, and R. Z. Sagdeev, J. Nucl. Energy 6, 645 (1964).

[4]F. F. Chen, Phys. Fluids 8, 912 (1965).

[5]N. S. Buchelnikova, Nucl. Fusion 6, 122 (1966); and in Proceedings of Conference on the Physics of Quiescent Plasmas, Frascati, Italy, 1967 [Laboratori Gas Ionizzati (Association Euratom-Comitato Nazionale per l'Energia Nucleare), 1967].

[6]B. J. Eastlund, K. Josephy, R. F. Leheny, and T. C. Marshall, Phys. Fluids 9, 2400 (1966).

[7]F. F. Chen and D. Mosher, Phys. Rev. Letters 18, 639 (1967).

[8]H. W. Hendel and P. A. Politzer, Princeton Plasma Physics Laboratory, Princeton University Report No. MATT-507, 1967 (unpublished); and in Proceedings of Conference on the Physics of Quiescent Plasmas, Frascati, Italy, 1967 [Laboratori Gas Ionizzati (Association Euratom-Comitato Nazionale per l'Energia Nucleare), 1967].

[9]N. Rynn, Rev. Sci. Instr. 35, 40 (1964).

[10]Phase angles introduced by the apparatus were measured to be less than 5° for the pertinent frequency range. The isolation transformer used in the floating

potential measurements had an impedance of 500 MΩ and gave an approximately linear frequency response up to ion cyclotron frequency. Density measurements were based on probe theory by J. G. Laframboise [University of Toronto Report No. UTIAS 100, 1966 (unpublished)]. This theory is in approximate agreement with microwave and spectroscopic measurements and (at $\approx 10^{11}$ cm^{-3}) gives density values lower by a factor of $\frac{1}{3}$ than those from previous calculations, reviewed by F. F. Chen in Plasma Diagnostic Techniques, edited by R. H. Huddlestone and S. L. Leonard (Academic Press, Inc., New York, 1965).

[11]L. Spitzer, Physics of Fully Ionized Gases (Interscience Publishers, Inc., New York, 1962), 2nd ed.

[12]The use of Fick's law as the basis for the concept of diffusion holds when the pertinent plasma parameters can be averaged over distances small compared with the density scale length, e.g., $\lambda_\perp \ll n_0/\nabla n_0$. [F. Boeschoten, J. Nucl. Energy 6, 339 (1964).]

[13]S. von Goeler and R. W. Motley, Phys. Fluids 10, 1367 (1967). We note that a comparison of measured input flux and computed classical losses in the stable regime indicates an additional loss whose origin is not identified. A similar loss has been found in other Q devices. {S. von Goeler and R. W. Motley, in Proceedings of Conference on the Physics of Quiescent Plasmas, Frascati, Italy, 1967 [Laboratori Gas Ionizzati (Association Euratom-Comitato Nazionale per l'Energia Nucleare), 1967]; and private communications with F. F. Chen and G. Grieger.}

[14]To derive Eq. (3), the continuity equation is integrated over the length of the plasma column. Thus, F represents the radial flux averaged over z. Since the wave amplitude is maximum at the midplane and nearly zero at the end plates, the contribution to F due to the wave is approximately one-half of the wave flux at the midplane, Fig. 2(a).

[15]L. D. Landau and E. M. Lifshitz, Fluid Mechanics (Pergamon Press, London, 1959), p. 104; B. B. Kadomtsev, Plasma Turbulence (Academic Press, Inc., New York, 1965), p. 46.

[16]The enhanced loss thus may become comparable to the Bohm value before the wave amplitude reaches 100% (assuming ψ to be unchanged). We note that J. B. Taylor [Phys. Rev. Letters 6, 262 (1961)] showed from stochastic considerations that the maximum attainable transverse diffusion can exceed the Bohm value by a factor of 8 for $T_e = T_i$. A specific mechanism for Bohm diffusion based on nonlinear interaction of inertial drift modes, and resulting in relative amplitude fluctuations of the order of 100%, has recently been reported by B. Coppi, Princeton Plasma Physics Laboratory, Princeton University Report No. MATT-545, 1967 (unpublished).

MAGNETIC FIELD DEPENDENCE OF THE KONDO RESISTIVITY MINIMUM IN CuFe AND CuMn ALLOYS*

P. Monod†

Department of Physics, University of California, San Diego, La Jolla, California

(Received 18 September 1967)

We report here some quantitative measurements of resistivity, magnetoresistance, and magnetization made in dilute CuFe and CuMn alloys. Our main conclusions are these:

(a) It is possible to determine and separate out the (normal) positive magnetoresistance from the negative magnetoresistance of the impurities.

(b) In both dilute alloys the negative magnetoresistance varies like H^n with n about 1.7-1.8 for values of the magnetic field H of a few kilogauss.

(c) Whereas for CuMn a field of 20 kG will completely alter the logarithmic temperature dependence of the resistivity (namely creating a maximum of the resistivity at around 4°K), the same field will only lower the resistivity of CuFe by 0.3% at 1.4°K.

(d) At any temperature and magnetic field, the amplitude of the negative magnetoresistance varies like the square of the magnetization of the impurities. This relation is sufficient to account for the behavior of these two alloys and allows us to infer the s-d exchange constant.

Previous studies have been done by numerous workers on more concentrated alloys[1,2] or in a limited range of low temperatures[3] where these effects are readily observed but difficult to analyze quantitatively because of the presence of unknown internal fields arising from the magnetic interactions between impurities in concentrated alloys, and the presence of the large positive magnetoresistance[4,5] in dilute alloys. The correlation between negative magnetoresistance and magnetization was qualitatively demonstrated by Schmitt and Jacobs[2] on very concentrated alloys exhibiting hysteresis effects and was inferred theoretically by Yosida.[6]

The CuMn samples were grown as single crystals in a graphite crucible heated in vacuum by induction.[7] The CuFe 110-ppm sam-