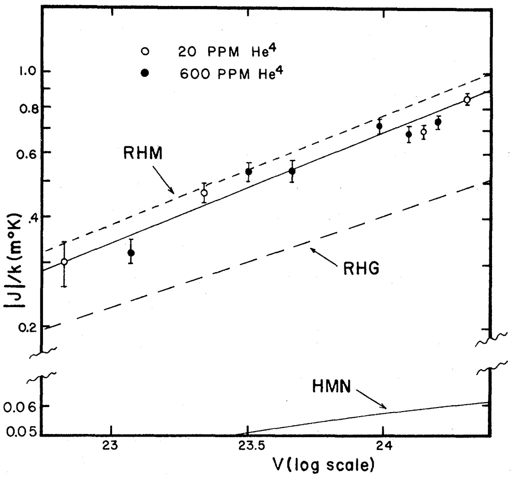

FIG. 3. The exchange integral versus molar volume. Circles, this work; RHM, Richardson, Hunt, and Meyer (Ref. 8); RHG, Richards, Hatton, and Giffard (Ref. 7); HMN, Hetherington, Mullin, and Nosanow (Ref. 3). Note that Refs. 3, 7, and 8 define J equal to twice the conventional J used here.

will show conclusively the sign of J as well as its magnitude.

We would like to thank J. W. Philp and Dr. R. Gonano for assistance in calibration of the germanium resistance thermometer, and the latter for helpful discussions. Also, we would like to thank Dr. L. Goldstein for a preprint and communications concerning this work.

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†Now at National Bureau of Standards, Boulder, Colorado.

[1]I. Pomeranchuk, Zh. Eksperim. i Teor. Fiz. 20, 919 (1950).

[2]N. Bernardes and H. Primakoff, Phys. Rev. 119, 968 (1960).

[3]L. H. Nosanow and W. J. Mullin, Phys. Rev. Letters 14, 133 (1965); J. H. Hetherington, W. J. Mullin, and L. H. Nosanow, Phys. Rev. 154, 175 (1967).

[4]A. L. Thomson, H. Meyer, and P. N. Dheer, Phys. Rev. 132, 1455 (1963).

[5]E. D. Adams, H. Meyer, and W. M. Fairbank, in Helium Three, edited by J. G. Daunt (Ohio State University Press, Columbus, Ohio, 1960), p. 57.

[6]D. O. Edwards, A. S. McWilliams, and J. G. Daunt, Phys. Rev. Letters 9, 195 (1962); Phys. Letters 1, 218 (1962).

[7]M. G. Richards, J. Hatton, and R. P. Giffard, Phys. Rev. 139, A91 (1965).

[8]R. C. Richardson, E. Hunt, and H. Meyer, Phys. Rev. 138, A1326 (1965).

[9]For a summary of the nmr work with complete references see J. Wilks, Liquid and Solid Helium (Oxford University Press, Oxford, England 1967).

[10]G. S. Rushbrooke and P. J. Wood, Mol. Phys. 1, 257 (1958).

[11]L. Goldstein, Phys. Rev. 159, 120 (1967); Ann. Phys. (N.Y.) 8, 390 (1959).

[12]R. C. Richardson, thesis, Duke University, 1965 (unpublished).

[13]Goldstein (Ref. 11) has used a value of J appropriate to $V = 24.8$ cm^3/mole, the molar volume at the melting-pressure minimum, rather than 24.1 cm^3/mole, the molar volume at melting near $T = 0$°K.

[14]G. C. Straty and E. D. Adams, Phys. Rev. 150, 123 (1966).

[15]A detailed study of pressure and volume changes associated with the phase separation is underway.

[16]R. A. Scribner, M. F. Panczyk, and E. D. Adams, to be published.

THEORY OF FOUR-PLASMON PARAMETRIC EXCITATION AND COMPARISON WITH EXPERIMENT

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Recently, Stern and Tzoar[1] reported experiments in which a discharge plasma, "pumped" with microwaves at frequency ω_0, exhibited strongly enhanced incoherent signals at the frequencies Ω, $\omega_0-\Omega$, and $\omega_0+\Omega$. The frequency Ω corresponded to the lowest ion-acoustic-mode frequency which can propagate in the plasma. The enhancement occurred above a fairly well-defined threshold in the pump power. These observations were interpreted[1] as resulting from parametric excitation of modes at Ω and $\omega_0 \pm \Omega$ by the pump.

The purpose of this Letter is to present the results of the theory of four-mode parametric excitation in an infinite, homogeneous, collisionless plasma. Based on the reasonable extrapolation of these results to the experimental plasma of Stern and Tzoar (assuming that their measured and computed linear system parameters such as lifetimes, pump power, etc., are correct), we conclude that it is possible that an effect other than parametric excitation is responsible for their observations.

The present authors[2-4] and others[5] had previously given the theory for three-plasmon parametric coupling which is valid only[3,4] when the resonance width of the Langmuir waves (electron plasma waves), $\gamma_{\rm L}$, satisfies $\gamma_{\rm L} < \Omega$. This allows only the frequency pairs $(\Omega, \omega_0-\Omega)$ or $(\Omega, \omega_0+\Omega)$ to be on resonance. The three-mode theory cannot explain Stern and Tzoar's experiments because it fails to account for the fourth frequency observed. For the experimental conditions, $\gamma_{\rm L} \gg \Omega$; so all three frequencies $(\Omega, \omega_0 \pm \Omega)$ can be on resonance if $\omega_0 \simeq \omega_{\rm L}$.

The four-mode problem can be treated by the same methods as those used in Refs. 2-4 for the three-mode case. Since many steps are similar, we refer the reader to these papers for details. We make the usual parametric approximation of considering a monochromatic pump wave $\vec{E}_0 \sin(\omega_0 t - \vec{k}_0 \vec{x})$, where the amplitude $\vec{E}_0$ is taken to be a fixed parameter. The lowest order nonlinear charge density induced by the pump field is put into Poisson's equation for the components $U(k, \omega)$ of the scalar potential, which then becomes a set of four coupled linear equations which we write in matrix form:

$$\begin{bmatrix} (\vec{k}+\vec{k}_0)^2\tilde{\epsilon}(\omega+\omega_0) & -Q_1(+\omega_0+\omega, -\omega) & Q_2(\omega+\omega_0, \omega_0-\omega) & 0 \\ Q_1(\omega, -\omega-\omega_0) & k^2\tilde{\epsilon}(\omega) & -Q_1(\omega, \omega_0-\omega) & Q_2(\omega, 2\omega_0-\omega) \\ Q_2(\omega-\omega_0, -\omega-\omega_0) & +Q_1(\omega-\omega_0, -\omega) & (\vec{k}-\vec{k}_0)^2\tilde{\epsilon}(\omega-\omega_0) & -Q_1(\omega-\omega_0, 2\omega_0-\omega) \\ 0 & Q_2(-2\omega_0+\omega, -\omega) & Q_1(\omega-2\omega_0, \omega_0-\omega) & (\vec{k}-2\vec{k}_0)^2\tilde{\epsilon}(\omega-2\omega_0) \end{bmatrix} \begin{bmatrix} U(\omega+\omega_0) \\ U(\omega) \\ U(\omega-\omega_0) \\ U(\omega-2\omega_0) \end{bmatrix} = 4\pi \begin{bmatrix} \rho_s(\omega+\omega_0) \\ \rho_s(\omega) \\ \rho_s(\omega-\omega_0) \\ \rho_s(\omega-2\omega_0) \end{bmatrix}. \quad (1)$$

The notation here is the same as in Ref. 4 (k dependence is suppressed whenever it can be inferred from the ω dependence). The charge-density source vector ρ_s may represent fluctuating components of the charge density or externally controlled test sources. If the nonlinear coupling is weak, we expect that the resonant frequencies of the parametrically coupled system will be close to, but not identical to, those of the uncoupled system. Thus for a homogeneous, isotropic plasma the interesting frequencies are ω near ω_p, the Langmuir (or electron plasma) frequency, and ω near $\omega_i = c_i k$, the frequency of ion acoustic waves. The pump acting once couples ω to $\omega-\omega_0$ and $\omega+\omega_0$ through coefficients $\mp Q_1(\omega; \pm\omega_0-\omega)$ and acting twice, to $2\omega_0 \mp \omega$ through $Q_2(\omega; \pm 2\omega_0-\omega)$. If ω is near ω_p, then ω_0 can be chosen so that $\omega_0-\omega$ is near ω_i and $2\omega_0-\omega$ is near ω_p provided that the width $\gamma_{\rm L}$ of the resonance at ω_p satisfies $\gamma_{\rm L} \gg \omega_i$. In this case $\omega_0+\omega \approx 2\omega_p$ cannot be near its linear resonance and it is easy to show that only the 3×3 matrix obtained by striking out the first row and first column need be included.[6] Note that $\omega_0+\omega$ and $2\omega_0-\omega$ can only be coupled to third order in the pump amplitude. We will work only to second order and neglect this coupling. The pump acting twice can couple ω to itself via the function $Q_2(\omega, -\omega)$. This is taken into account by defining a shifted dielectric function[4]

$$\tilde{\epsilon}(k, \omega) = \epsilon_L(k, \omega) - (2/k^2) Q_2(\omega, -\omega), \quad (2)$$

where $\epsilon_L(k,\omega)$ is the familiar linear longitudinal dielectric function. The free modes of the parametrically coupled system are obtained by setting the determinant of the matrix equal to zero.

The coefficients or nonlinear susceptibilities are calculated from the dynamical equations describing the particles (e.g., the Vlasov equation), by expanding the induced charge density in powers of the total self-consistent potential field. The details of the calculation of Q_1 are given in Ref. 4 for both transverse and longitudinal fields. The calculation of the Q_2 coefficients also follows from the same type of analysis as in Ref. 4. The resulting 3×3 determinantal equation for the eigenmodes of frequency ω near ω_p turns out to be $D(\omega,\vec{k}) = k_D^2 k_A^2 k_S^2 \bar{D}(\omega,\vec{k}) = 0$, where ck_D is the Debye frequency and where

$$\bar{D}(\omega,k) = \begin{vmatrix} \bar{\epsilon}(\omega_0-\Omega) & -\Lambda\mu_S & \Lambda^2\mu_S\mu_A \\ -\Lambda\mu_S & (k_i^2/k_D^2)\bar{\epsilon}(-\Omega) & -\Lambda\mu_A \\ \Lambda^2\mu_S\mu_A & -\mu_A\Lambda & \epsilon(-\omega_0-\Omega) \end{vmatrix}. \quad (3)$$

We have defined $(\Omega,\vec{k}_i) = (\omega_0-\omega, \vec{k}_0-\vec{k})$ (the new acoustic eigenmode), $(\omega_0-\Omega, \vec{k}_S) = (\omega,\vec{k})$ (Stokes satellite), and $(\omega_0+\Omega,\vec{k}_A) = (2\omega_0-\omega, 2\vec{k}_0-\vec{k})$ (anti-Stokes satellite). The factors $\mu_{S,A}$ are cosines of the angle between the wave vectors $\vec{k}_{S,A}$ and the polarization $\hat{e}$:

$$\mu_{S,A} = (\hat{k}_{S,A}, \hat{e}_0). \quad (4)$$

In what follows we shall examine only the collinear case $|\mu_S| = |\mu_A| = 1$, since this is all that is pertinent to the experiment of Stern and Tzoar. Λ^2 is the perturbation parameter,

$$\Lambda^2 = E_0^2/8\pi n k_B T_e. \quad (5)$$

It is readily seen that if this dispersion relation has a root at $(\omega,k) = (\omega_0-\Omega, k_S)$, then it has a root at $(\tilde{\omega},\tilde{k}) = (\omega_0+\Omega^*, \vec{k}_A)$. Thus, roots near the Stokes and anti-Stokes frequencies are uniquely related and have the same imaginary part.[7]

The dispersion relation (3) can be reduced to an algebraic equation if we assume that ω_0 can be chosen so that $\omega_0-\Omega$, Ω, and $\omega_0+\Omega$ all fall near a complex zero of $\bar{\epsilon}(\omega)$. Then $\bar{\epsilon}(\omega)$ is approximated by

$$\bar{\epsilon}(\omega) = \left.\frac{\partial\bar{\epsilon}(\omega)}{\partial\omega}\right|_{\omega=\omega_r} (\omega-\omega_r+i\gamma_r), \quad (6)$$

where $\mathrm{Re}\,\epsilon(\omega_r)=0$, and the index r can be S, A, or i, with $\omega_{S,A}^2 = \omega_p^2(1+3k_{S,A}^2/k_D^2)$ and $\omega_i = \omega_p(m_e/m_i)^{1/2}(k_i/k_D)$. The determinant can then be expanded out yielding a complex equation for the complex roots ω in terms of Λ^2 and other parameters. The threshold for simultaneous growth of the Stokes, anti-Stokes, and acoustic modes corresponds to pure real ω, and may be obtained from the following two consequences of the vanishing determinant in this case:

$$\frac{\Lambda^2}{\Lambda_0^2} = \left[1+\left(\frac{\Delta_i}{\gamma}\right)^2\right]\frac{\Delta_A-\Delta_S}{\Delta_A+\Delta_S}, \quad (7)$$

$$\frac{\Delta_i}{\gamma_i} = \frac{\gamma_L+\Delta_S\Delta_A/\gamma_L}{\Delta_A-\Delta_S} + \frac{2\gamma_i}{\omega_i}. \quad (8)$$

Here $\Lambda_0^2 = 4\gamma_i\gamma_L/\omega_i\omega_p$ is the three-mode threshold,[2,8] and $\Delta_i = \Omega-\omega_i$ and $\Delta_{S,A} = \omega_0 \mp \Omega - \omega_{S,A}$ are the shifts between old and new mode frequencies. The expansions (6) for $\bar{\epsilon}(\omega)$ are valid only provided that $|\Delta_i| \ll \omega_i$, and $|\Delta_S|$, $|\Delta_A| \ll \omega_p$. [The condition $|\Delta_i|/\omega_i \ll 1$ has further been used to simplify (8).] We have assumed that $\gamma_L(k_S) = \gamma_L(k_A) = \gamma_L \ll \omega_p$, which is equivalent to $k_S k_A \ll k_D$, so that collisional damping dominates. In what follows, $\gamma_i/\omega_i(\ll 1)$ is also taken independent of k_i. $\bar{\epsilon}(-\Omega)$ has been set equal to $\epsilon(-\Omega)$. (This assumes $\Lambda^2 k_i^2/k_0^2 \ll |\Delta_i|/\omega_i$.)

It is evident[9] from (7) and (8) that the lowest threshold value of Λ^2 is $\sim\Lambda_0^2$ and that this occurs for $\Delta_S = 0$, and $\omega_p \gg |\Delta_A| \gtrsim \gamma_L$. Physically, the first condition is the Stokes frequency-matching condition, and the second requires that the new anti-Stokes frequency be at least a linewidth removed from the old anti-Stokes frequency so that it is outside the old resonant region. The second condition is evidence of the system's attempt to suppress the anti-Stokes mode which, as is well known,[10] cannot be made to grow in the absence of the Stokes.

For a transverse pump, $\omega_0 = (\omega_p^2 + c^2k_0^2)^{1/2} \simeq (\omega_p^2 + 3v_e^2 k_{S,A}^2)^{1/2}$; so $k_0 \ll k_S, k_A$, and ω_S and ω_A are essentially equal. This constrains $\Delta_A - \Delta_S \simeq 2\omega_i$. If we demand that $\Delta_S = 0$, it follows that Δ_A is of order ω_i, and $\Lambda^2 = \Lambda_0^2$, provided that $\omega_i \gtrsim \gamma_L$ (three-mode parametric threshold condition). However, if $\omega_i \ll \gamma_L$ (four-mode condition), $\Delta_S = 0$ implies that $\Delta_A \sim \omega_i$ is too small to suppress the anti-Stokes line and the threshold will be far above Λ_0^2. The minimum threshold subject to the constraints $\Delta_A - \Delta_S$

$\simeq 2\omega_i$ and $\omega_i \ll \gamma_L$ occurs for $\Delta_S \simeq \Delta_A = \gamma_L/\sqrt{3}$ and is $\Lambda^2 = (4\sqrt{3}/9)(\gamma_L/\omega_i)\Lambda_0^2 \gg \Lambda_0^2$. Here, the Stokes line and the anti-Stokes line are both dragged to the edge of the old resonant region. A subsidiary condition for this threshold is that $\gamma_i\gamma_L \ll \omega_i^2$. This is required in order to have $|\Delta_i| \ll \omega_i$.

For a longitudinal pump, k_0 is of the same order as k_S and k_A. Therefore, $|\Delta_A - \Delta_S| \simeq |\omega_A - \omega_S|$ if $|\omega_A - \omega_S| \gg \omega_i$. Thus, the minimum threshold can be $\Lambda^2 \simeq \Lambda_0^2$, provided that $\Delta_S = 0$ and $\Delta_A \gtrsim \gamma_L$. This is automatically satisfied when $\omega_i \gtrsim \gamma_L$ (three-mode case) and can also be satisfied when $\omega_i \ll \gamma_L$ (four-mode case), provided that $|\Delta_A| = |\omega_A - \omega_S| \simeq \theta(\omega_p k_0^2/k_D^2) \gtrsim \gamma_L$. More precisely, the four-mode longitudinal pump threshold is $\Lambda^2 = \Lambda_0^2$, provided that $1 \gg k_0^2/k_D^2 \gg \gamma_L/12\omega_p$ and $\vec{k}_S = -\vec{k}_0$. These conditions are illustrated in Fig. 1, with $\omega_A - \omega_S \simeq \gamma_L$.

The experiment of Stern and Tzoar[1] utilizes the longitudinal Tonks-Dattner (TD) modes of an inhomogeneous cylindrical plasma for the pump as well as for the Stokes and anti-Stokes modes. Because of the discrete nature of these modes, the unperturbed frequencies correspond to the same (lowest) TD mode, i.e., $\omega_S = \omega_A$. Therefore the threshold properties should appear to be more closely related to the case considered above for a transverse pump in an infinite, homogeneous plasma. Compared either with our transverse or with our longitudinal pump results, their observed threshold for He or Hg plasmas with $\gamma_L \gg \omega_i$ occurs at anomalously low pump power—two to three orders of magnitude below the transverse threshold and one to two orders of magnitude below the minimum longitudinal threshold. The infinite, homogeneous plasma theory which we are using, of course, cannot be expected to be a very accurate model for the experimental plasma. For example, overlap integrals of the actual TD spatial modes should be used in the nonlinear susceptibilities instead of our plane wave modes. However, we believe it is unlikely that such corrections can substantially change this large discrepancy between theoretical and experimental thresholds.

An examination of the steady-state[11] fluctuation spectrum of the modes casts further light on discrepancies and suggests an alternative explanation. This spectrum is obtained by inverting the reduced (3×3) matrix version of Eq. (1), and using the fluctuation-dissipation theorem,

$$\lim_{V,T\to\infty} \frac{\langle |\rho_S(k,\omega)|^2\rangle}{VT} = \frac{\theta_e k^2}{2\pi\omega} \operatorname{Im}\epsilon_L(\vec{k},\omega),$$

to relate the fluctuating charge-density sources to the dielectric constant.[3,4] The fluctuation spectrum of the self-consistent field in the neighborhood of the Stokes line is then given by

$$\lim_{V,T\to\infty} \frac{\langle |U(\vec{k}_S,\omega_0-\Omega)|^2\rangle}{VT} = \frac{4\pi\theta}{\omega_p k_S^2} |\bar{D}(\vec{k}_S,\omega_0-\Omega)|^{-2} \left[\frac{\gamma_L}{\omega_p}\left|\frac{k_i^2}{k_D^2}\epsilon(-\Omega)\tilde{\epsilon}(-\omega_0-\Omega)-\Lambda^2\right|^2 + \frac{\Lambda^2\omega_p\gamma_i}{\omega_i^2}|\tilde{\epsilon}(-\omega_0-\Omega)-\Lambda^2|^2 + \frac{\Lambda^4\gamma_L}{\omega_p}\left|1-\frac{k_i^2}{k_D^2}\epsilon(-\Omega)\right|^2\right]. \quad (9)$$

Here it has been assumed that $\omega \equiv \omega_0 - \Omega$ is close to ω_p and that $|\Delta_i| \ll \omega_i$, and $|\Delta_S|, |\Delta_A| \ll \omega_p$. The spectrum near the anti-Stokes line is obtained by sending $\Omega \to -\Omega$ and interchanging $\vec{k}_S$ and $\vec{k}_A$.

With $\gamma_L \gg \omega_i$, and Λ^2 near threshold, the ratio of Stokes to anti-Stokes fluctuations is found to be of order unity for a transverse pump (or $\omega_S = \omega_A$), and is $\sim(k_A^2/k_S^2)[1+(\Delta_A^2/\gamma_L^2)] \gg 1$ for the optimum longitudinal pump conditions. In either case, it is the second term in the numerator of the right-hand side of (9) which dominates because of the large value of the ratio ω_p/ω_i. Stern and Tzoar observe the Stokes-to-anti-Stokes ratio to be unity. [This is not inconsistent with our observation that our transverse pump case ($\omega_S = \omega_A$) more closely parallels the excitation of TD modes.] This still leaves their observed "threshold" two to three orders of magnitude lower than the parametric threshold. Suppose we assume that Λ^2 is well below the parametric threshold Λ_0^2, and examine the fluctuations produced

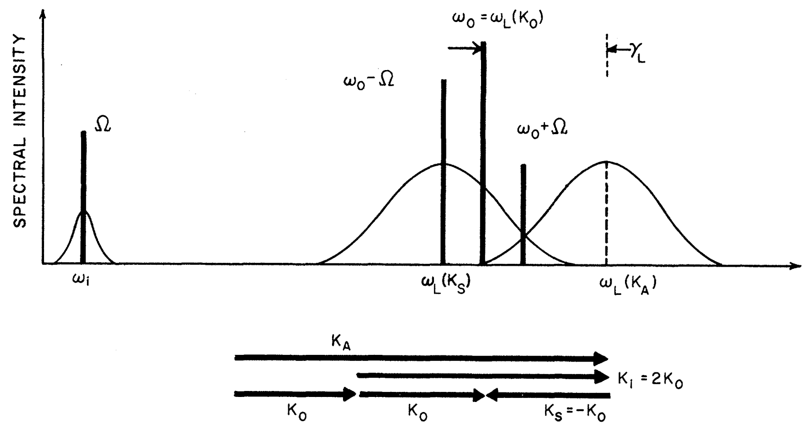

FIG. 1. Spectral intensity of self-consistent field fluctuations (proportional to density-density correlation function) pumped by a longitudinal plasma wave at ω_0. Bell-shaped curves indicate old eigenmodes (in the absence of the pump). Lines at $\omega_0 \pm \Omega$ and Ω indicate new eigenmodes of the coupled system. The condition on the momenta for minimum threshold is illustrated below.

at $\omega_0 \pm \omega_i$ due to ordinary mixing of the ion mode with the pump. The regenerative interaction of the modes is unimportant and $\bar{D} \simeq (k_i^2/k_D^2)\epsilon(-\omega_i)\epsilon(\omega_0-\omega_i)\epsilon(-\omega_0-\omega_i)$. The dominant part of the right-hand side of Eq. (9) is then proportional to $(\omega_p/\gamma_L)[1+(\omega_p/\omega_i)(\Lambda^2/\Lambda_0^2)]$, where the first term corresponds to the usual equilibrium fluctuations at $\omega_0-\omega_i$, and the second term corresponds to mixing. In the experiments of Stern and Tzoar, $(\omega_p/\omega_i)(\Lambda^2/\Lambda_0^2)$ ranges from 20 to 200 (for $\Lambda^2/\Lambda_0^2 \simeq 2\times10^{-2}$, the value at which the enhancement of the Stokes and anti-Stokes lines occurs). Moreover, this mixing leads to equally enhanced Stokes and anti-Stokes fluctuations. This effect alone cannot explain the observed simultaneous enhancement of fluctuations at the ion-acoustic frequency. However, the effect of the microwave "pump" may be to heat the electrons, lowering T_i/T_e and thereby enhancing the ion-acoustic mode. In the presence of a radial drift (not precluded by their experiments), this temperature change could drive the acoustic mode towards instability. Either mechanism decreases γ_i and hence Λ_0^2. Unless γ_i is experimentally determined, it is uncertain whether Stern and Tzoar have observed a parametric threshold or a combination of ion-acoustic mode enhancement and ordinary mixing.

Note added in proof.—Jackson[5] has considered the parametric coupling of $\omega_0+\Omega$ to $\omega_0-\Omega$. His theory is obtained from the present work by setting all Q_1's to zero in (1), retaining only the Q_2 coupling. It is easily shown that this leaves out the coupling, and therefore the enhancement, of the ion-acoustic mode and results in a different spectrum, threshold, and matching condition than those obtained in the present work. It is possible, in principle, for both types of resonances to occur for different pump frequencies. However, Jackson has shown that his type of resonance disappears if $\gamma_L/\omega_p > 10^{-4}$, which is the experimental case.

[1]R. A. Stern and N. Tzoar, Phys. Rev. Letters 17, 903 (1966).

[2]D. F. DuBois and M. V. Goldman, Phys. Rev. Letters 14, 544 (1965).

[3]M. Goldman, Ann. Phys. (N.Y.) 38, 95 (1966); in Proceedings of the Second Orsay Summer Institute of Plasma Physics—Nonlinear Effects in Plasmas (to be published).

[4]D. F. DuBois and M. V. Goldman, Phys. Rev. (to be published).

[5]E. A. Jackson, Phys. Rev. 153, 235 (1967); Y. C. Lee and C. H. Su, Phys. Rev. 152, 129 (1966).

[6]Similarly, if ω is near ω_i, ω_0 can be chosen so that $\omega+\omega_0$ is near ω_p and $\omega_0-\omega$ is near ω_p, again only if $\gamma_L \gg \omega_i$. In this case $2\omega_0-\omega$ is near $2\omega_p$, again off resonance, and only the 3×3 matrix obtained by striking out the fourth row and fourth column need be considered.

[7]If we examine the 3×3 determinantal equation valid for $\omega \simeq \omega_i, \vec{k} \simeq \vec{k}_i$, we find on setting $\omega = \Omega^*$ the same dispersion relation as (3), with $\omega = \omega_0 - \Omega, \vec{k} = \vec{k}_0 - \vec{k}_i$. Thus, a low-frequency root $\Omega \sim \omega_i$ and $k = k_i$ always occurs along with the Stokes and anti-Stokes roots $\omega_0 - \Omega$ and $\omega_0 + \Omega^*$ and has the same imaginary part as these roots.

[8]The factor 4 in the denominator arises because γ_r

is an amplitude decay rate rather than a power decay rate as was assumed in Refs. 2 and 3.

[9]Expressed in terms of the variables $u=(\Delta_A-\Delta_S)/(\Delta_A+\Delta_S)\geqslant 0$ and $v=\gamma_L/(\Delta_A-\Delta_S)\geqslant 0$, $\Lambda^2/\Lambda_0^2\simeq\frac{1}{2}(u^{-1}+u)+u[v^2+(v^{-2}/16)(u^{-2}-1)]$, which has a minimum of unity for $u=1, v\ll 1$.

[10]See, for example, N. Bloembergen, Nonlinear Optics (W. A. Benjamin, Inc., New York, 1965), where a similar problem is discussed involving the Stokes and anti-Stokes modes in stimulated Raman scattering.

[11]The present theory does not take into account the heating of the plasma by the pump radiation. The heating rate is slow on the scale of times important for the parametric excitation process. Thus our "steady state" may be superimposed upon a secular change in the electron and ion temperatures.

MEASUREMENTS OF ENHANCED PLASMA LOSSES CAUSED BY COLLISIONAL DRIFT WAVES*

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We report measurements of enhanced plasma losses caused by collisional drift waves[1] in magnetically confined, thermally ionized alkali plasmas, based on (a) a density decrease in the central part and an increase in the outer part and outside of the plasma column, coincident with the abrupt onset of drift waves, and (b) correlation of density and plasma-potential oscillations. The enhanced loss, for a wave amplitude equal to 10% of the zero-order density, is measured to be an order of magnitude larger than the loss due to classical collisional diffusion.

Collisional drift waves can arise in a low-β plasma ($\beta\equiv 8\pi p/B^2$) in the absence of current as a result of the combined effects of transverse density gradient, ion inertia, and electron parallel thermal motion. Theory[2-4] indicates that these "universal" instabilities may cause transverse plasma loss above the lower limit set by "classical" binary-collision diffusion. However such a causal relation has not been conclusively demonstrated in prior experiments.[5-7] The difficulties are identification of the instability thought to cause the enhanced loss, separation of the loss due to the instability from other losses, and lack of a measurement of local plasma transport due to the instability.

In the present work, the instability is identified[8] as a density-gradient–driven collisional drift wave by measurements of (a) longitudinal and perpendicular phase velocities and their dependences on magnetic field and plasma temperature; (b) phase relation between density and potential oscillations; (c) radial, azimuthal, and longitudinal wavelengths; and (d) magnetic-field, plasma-temperature, ion-mass, and density dependence of mode transitions. The wave is localized in a region where $\nabla n_0/n_0 \gg \nabla T/T$, i.e., where temperature-gradient–driven drift waves and wave excitation due to large radial electric fields are excluded.

The principal results of this work are observations of a density reduction (up to 30%) in the central part and a density increase (up to 6%) in the outer part of the plasma column, coincident with the abrupt onset of the $m=1$ azimuthal mode of the collisional drift wave. This abrupt onset facilitates separation of wave-enhanced losses from other losses. Onset, from $n_1/n_0\approx 0$ to ~0.1, takes place when the magnetic field is increased by approximately 1% beyond theshold, with all other parameters constant. The abrupt destabilization to large amplitude is in agreement with the strong B dependence of the growth rate near onset, calculated from linearized theory.[1] Measurements of the correlation between the coherent density and potential oscillations over the radial extent of the plasma enable us to calculate local enhanced fluxes caused by the drift wave and a local "diffusion coefficient" throughout the plasma column. Finally, utilizing the continuity equation together with the feature that the self-excited wave can be "turned on and off" abruptly by variation of B, we can compare the enhanced-loss state with the stable state in an otherwise unchanged plasma (including the boundary conditions at the sheath), and show that the results of our measurements are consistent, i.e., the local loss flux obtained from wave-parameter measurements accounts for the measured change in plasma density.

The experimental work was done on the Princeton Q-1 device.[9] The plasma consists of cesi-