the ions of the plasma can however not be made since the nature of the background gas is not known. It was also found from these measurements by the use of a grid shutter and time-of-flight observations that most of the neutrals emitted are hydrogen atoms.

Since the aim of the experiment was to look for gross effects and scaling laws, no attempts have been made to investigate the nature and the type of the turbulence which causes the heating or the instability that limits containment. The possibility that counter-stream instabilities and the ion-sound instability are responsible for the turbulence and the heating has been pursued by a number of researchers.[4] The presence of energetic ions in this experiment and the scaling with voltage and density, as well as the radial expansion, may suggest that magnetohydradynamic instabilities cause the heating.

Thus, this experiment demonstrates that energy can be transferred from a condenser bank to a plasma by passage of current parallel to the confining field with efficiencies of 3-4%. Since classical Joule heating is completely negligible for the conditions of this experiment we have termed the heating mechanism "turbulent".

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EVIDENCE OF NUCLEAR SPIN ORDERING IN SOLID HELIUM-THREE*

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Measurements of the pressure versus temperature for constant-volume samples of solid He^3 to 20 mdeg K show the expected T^{-1} dependence due to spin ordering at low temperatures. A value for the exchange integral $|J|/k$ of about 0.7 mdeg K is obtained for a molar volume of 24 cm^3, with $\partial \ln|J|/\partial \ln V = 16.4$.

As absolute zero is approached, the properties of solid He^3, such as specific heat, susceptibility, expansion coefficient, and the melting curve, are largely determined by the ordering of the nuclear-spin system. In a rigid lattice the ordering would not take place until $T \sim 10^{-6}$ °K brought about by the dipole-dipole interaction.[1] It was pointed out by Bernardes and Primakoff[2] that, because of the large zero-point motion in solid He^3, there would be considerable overlap of the wave functions of neighboring atoms, resulting in an exchange interaction J which would cause ordering of the spins at a much higher temperature.

Theoretical calculations of Nosanow and others[3] have given $J/k \simeq -0.1$ mdeg K, indicating antiferromagnetic ordering. While susceptibility data[4] also indicate this type of ordering, attempts at direct experimental determination of J through susceptibility[4,5] and specific heat[6] have been unsatisfactory either because of He^4 impurity or because the measurements have not extended to low enough temperatures. Recent indirect determinations of J derived from nuclear-magnetic-resonance relaxation times give fairly consistent results,[7-9] although differing by as much as a factor of 5. In this paper we report the first direct determination of J through measurements of $(\partial P/\partial T)_V$ down to 20 mdeg K.

The exchange interaction between two nuclei is written in the usual way as

$$H=-2J\vec{I}_i\cdot\vec{I}_j, \qquad (1)$$

where I is the nuclear spin. [This is the conventional way of writing the Heisenberg Hamiltonian rather than that used in nmr work,[7-9] which omits the factor of 2 in Eq. (1).] For $kT \gg |J|$, there is an exchange contribution to the specific heat given by

$$C_{\mathrm{ex}}=\tfrac{2}{3}zI^2(I+1)^2(J/kT)^2R, \qquad (2)$$

where z is the number of nearest neighbors, and R is the gas constant.[10] With $I=\frac{1}{2}$ and $z=8$ in bcc He^3, this reduces to

$$C_{\mathrm{ex}}=3R(J/kT)^2. \qquad (3)$$

Using the Maxwell relation $(\partial P/\partial V)_T=(\partial P/\partial T)_V$, and the fact that C_{ex} depends only on the ratio J/kT, one obtains

$$\left(\frac{\partial P}{\partial T}\right)_{V,\mathrm{ex}}=-\frac{C_{\mathrm{ex}}}{V}\left(\frac{\partial\ln|J|}{\partial\ln V}\right)_T=-\frac{3R}{V}\left(\frac{J}{kT}\right)^2\left(\frac{\partial\ln|J|}{\partial\ln V}\right)_T. \qquad (4)$$

This can be written as

$$(\partial P/\partial T)_{V,\mathrm{ex}}=\gamma_{\mathrm{ex}}C_{\mathrm{ex}}/V, \qquad (5)$$

where

$$\gamma_{\mathrm{ex}}\equiv-(\partial\ln|J|/\partial\ln V)_T.$$

This is to be added to the usual lattice term

$$(\partial P/\partial T)_{V,L}=\gamma C_{V,L}/V, \qquad (6)$$

where γ is the Grüneisen constant. It has been pointed out[11,12] that with $|J|/k\sim 1$ mdeg K these two terms become equal in magnitude at $T\sim 0.20$°K. Thus the thermal expansion coefficient, $\alpha=\beta(\partial P/\partial T)_V$, where β is the compressibility, should become zero at $T\sim 0.20$°K, and at much lower temperatures it will be dominated by the exchange term. This provides a sensitive means of detecting exchange effects since they will be observable in the expansion coefficient at $T\simeq 200|J|/k$. With the value of J indicated above, Goldstein[11] has pointed out that the melting curve should have an inflection point at about 10.5 mdeg K, where the specific heats of the liquid and solid at melting become equal.[13]

A schematic drawing of the pressure chamber and associated apparatus is shown in Fig. 1. Samples were formed at constant volume and annealed for 30 min at about 0.01°K below the melting temperature. Pressures on the solid samples were measured by means of a

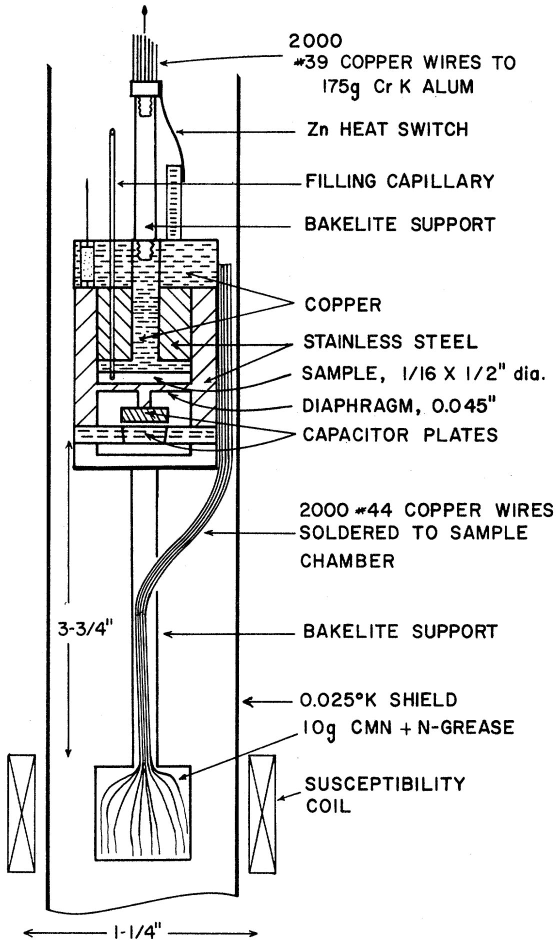

FIG. 1. Schematic drawing of the pressure chambers and the paramagnetic salt thermometer. The cooling salt and other details have been omitted for simplicity.

capacitive strain gauge similar to that described by Straty and Adams.[14] Changes in pressure of 3×10^{-5} atm could be detected. Temperatures to 20 mdeg K were obtained by demagnetization of 175 g of chromium potassium alum from about 0.3°K and 13 kG. The sample chamber was connected to the salt by a zinc heat switch, which permitted the sample temperature to be raised above that of the salt. The temperature of the sample was regulated electronically, and could be increased or decreased by adjustment of the regulating current. Heat leaks were very small so that a fixed sample temperature could be maintained for several hours. The susceptibility, measured ballistically, of 10 g of powdered cerium magnesium nitrate (CMN) served as a thermometer. The area of contact between the CMN and the wires embedded in it was 160 cm^2, giving a time constant of 5 min at 0.02°K. Germanium and carbon resistors were used as secondary thermometers above 0.04°K.

The nmr studies[7,8] have shown that the value of J depends strongly on the He^4 impurity. In this work two samples have been used with He^4 impurities of ~0.06 and 0.002%. These undergo phase separation[6] at $T\simeq0.1$ and 0.07°K, respectively. The phase separation in the 0.06% sample was observed through its effect on the pressure,[15] with the time constant for the separation varying from 1 to 30 min for molar volumes from 24.15 to 23.02 cm^3/mole. While the exchange contribution to the pressure is obscured in the vicinity of the phase separation for the 0.06% sample, the separation can be used to advantage to increase the sample purity. Below 0.05°K the He^4 impurity in the He^3-rich phase, which is almost the entire sample, is only about 0.2 ppm or less, and the effect of the phase separation on $(\partial P/\partial T)_V$ is negligible. Thus, between 0.05 and 0.02°K the values of $(\partial P/\partial T)_V$ are for very pure He^3 with the only contribution being the T^{-2} exchange term.

The present measurements are limited to large molar volumes by long equilibrium times for the phase separation in the 0.06% sample, and by pressure sensitivity for the 0.002% sample. Figure 2 is a plot of ΔP vs T^{-1} for various molar volumes, where ΔP is the increase in pressure relative to some arbitrary reference point. According to Eq. (4), the slopes of these curves are equal to $3\gamma_{\rm ex}RJ^2/k^2V$. Since this contains J and its logarithmic derivative, a self-consistent procedure is used to solve for J. Based on the values of J from nmr work,[7,8] an initial value of $\gamma_{\rm ex}$ is assumed, then Eq. (4) solved for $J(V)$ for each molar volume. A new, self-consistent, value of $\gamma_{\rm ex}$ is obtained from $J(V)$ and is used to compute the final $J(V)$. Over the limited range of molar volumes covered, $\gamma_{\rm ex}=-16.4$ is valid.

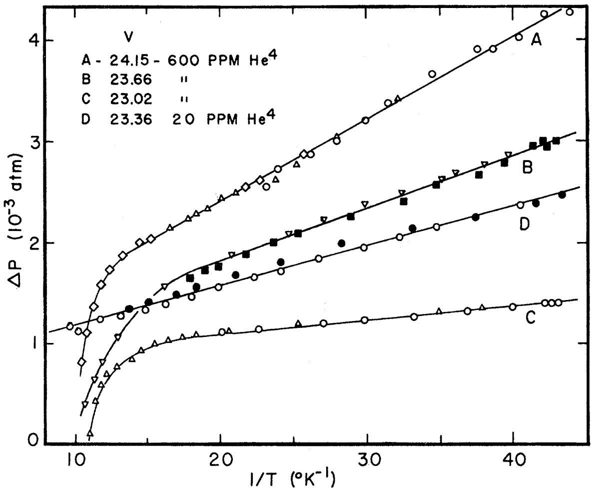

FIG. 2. The exchange contribution to the pressure, ΔP, versus T^{-1} for typical molar volumes. The various symbols indicate different runs on that sample, closed symbols indicate cooling, open warming. As explained in the text, the slopes determine the exchange integral J. The sharp decrease in ΔP near $T^{-1}=12$ for the 600-ppm samples is due to the onset of "mixing" (phase separation).

In Fig. 3 is shown J obtained by the above procedure. Also shown for comparison are the results derived from nmr work[7-9] and those calculated by Hetherington, Mullin, and Nosanow.[3] The good agreement between our values of J and those of Richardson, Hunt, and Meyer[8] is particularly gratifying considering the vast differences in the two methods.

In conclusion, we see that, for molar volumes near the melting curve at $T=0$°K, the thermal expansion has given $|J|/k=0.7$ mdeg K with $|J|$ decreasing approximately as $\partial\ln|J|/\partial\ln V=16.4$ as the volume is decreased. The inflection point in the melting curve would then be at 7 mdeg K. This is in agreement with measurements of the melting curve to 17 mdeg K which do not show an inflection point.[16] As usual, we plan to extend this work to lower temperatures and molar volumes. Also we plan to make susceptibility measurements which

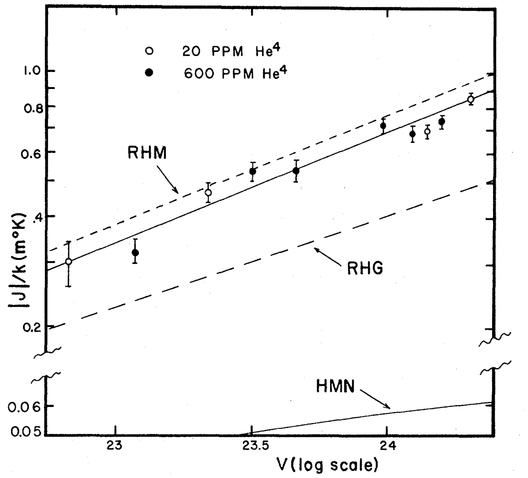

FIG. 3. The exchange integral versus molar volume. Circles, this work; RHM, Richardson, Hunt, and Meyer (Ref. 8); RHG, Richards, Hatton, and Giffard (Ref. 7); HMN, Hetherington, Mullin, and Nosanow (Ref. 3). Note that Refs. 3, 7, and 8 define J equal to twice the conventional J used here.

will show conclusively the sign of J as well as its magnitude.

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THEORY OF FOUR-PLASMON PARAMETRIC EXCITATION AND COMPARISON WITH EXPERIMENT

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Recently, Stern and Tzoar[1] reported experiments in which a discharge plasma, "pumped" with microwaves at frequency ω_0, exhibited strongly enhanced incoherent signals at the frequencies Ω, $\omega_0 - \Omega$, and $\omega_0 + \Omega$. The frequency Ω corresponded to the lowest ion-acoustic-mode frequency which can propagate in the plasma. The enhancement occurred above a fairly well-defined threshold in the pump power. These observations were interpreted[1] as resulting from parametric excitation of modes at Ω and $\omega_0 \pm \Omega$ by the pump.