from Kay's[10] usage, thus the reference end in Eqs. (6), (8), and (9) is $+\infty$.] Note that $K(x, x)$ as determined by Eq. (12) is independent of values of $B(\xi)$ for $\xi < 2x$.

A number of results can be established by further elaboration of this method, which we mention without going into details.

When u represents a single soliton, there is perfect transmission $[b(k) \equiv 0]$ and exactly one discrete eigenvalue $\lambda_1 = \frac{1}{2}u_{\min}$. More generally, Kay and Moses[11] have given the general solution of Eq. (12) with $b(k) \equiv 0$ in closed form in terms of exponentials. This includes all cases where u decomposes exactly into solitons.

It is more difficult to find exact solutions when $b(k)$ does not vanish. The time dependence of $b(k)$ indicates a strong phase mixing in the integral of Eq. (13) as $t \to \infty$ for positive ξ. The behavior for negative ξ is more complicated since the integrand then has points of stationary phase. This is reflected (in computer studies) by the "tail" moving toward the left.

Since the c_n grow exponentially, as long as there is at least one of them $B(\xi)$ can be approximated by the summation when Eq. (12) is to be solved for $x > 0$ and $t \to \infty$. The solution then reduces to that found by Kay and Moses[11] described above. Thus the magnitude, velocity, and position of each soliton can be found in the limit of large time. Furthermore, the solitons for large negative time can be found from the usual version[10] of Eq. (12) where the reference end is $-\infty$.

A fuller treatment together with other applications and generalizations will be published subsequently.

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FREQUENCY BROADENING IN LIQUIDS BY A SHORT LIGHT PULSE*

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Several authors have reported the observations of frequency broadening in filaments which were produced by the self-focusing of a Q-switched laser in liquids.[1-3] This broadening has been attributed to the generation of new frequency components through an intensity-dependent refractive index and stimulated Rayleigh scattering.[1-4] Theories of the frequency broadening in an optical pulse by an intensity-dependent refractive index have been given in connection with the pulse distortion.[5,6] But comparison of experiment with theory has been difficult, because the broadening is usually irregular and the observation of the spectrum in filaments is obscured by the strong background. We report here the observation of the frequency broadening in a filament with short duration time, under such experimental conditions that the intensity of the stimulated Raman emission in the filament is much less than that of the laser. The structure of the frequency spectrum shows a pattern which can be explained by phase modulation through the intensity-dependent refractive index.

A Q-switched ruby laser beam with diameter 3 mm was used to produce filaments in a carbon disulfide liquid cell of 7.5 cm length. The intensity distribution of the light at the end of the cell was projected on the entrance slit of a spectrometer using a 5.2-cm focal-length lens with magnification of 17. In order to eliminate the nonscattered part of the laser beam from that in filaments, a wire with diameter of 1 mm was placed at the focusing point of the lens perpendicularly to the slit of the spectrometer. The entrance slit was wide open (about 2 mm). The resolution of the spectrum was determined from the magnitude of the image of the filament. The laser beam was composed of four or five groups of axial modes, the separation of the groups being approximately 0.45 cm^{-1}. With this mode structure the intensity of the laser beam changes considerably within the time interval $1/\Delta\nu$ where $\Delta\nu$ is the total spectral width. Therefore at relatively low laser power the duration time of the filament is expected to be 10^{-11} sec or less. (In the present experiments the laser power was kept lower than 50% above threshold for observations of optical trapping.) The total energy of a filament estimated from the photographic emulsion sensitivity is less than 10^{-7} J. Since the threshold for optical trapping in carbon disulfide is approximately 2 to 20 kW,[7] this energy also leads to a duration time of the order of 10^{-11} sec.

Typical spectra of the individual filaments in carbon disulfide are shown in Fig. 1. A regular periodic structure is observed in the low-frequency side of the broadening in every filament, when the broadening is smaller than the Raman shift. The period in a filament increases linearly with the amount of the broadening from the center. The average period for most filaments is 10 ± 3 cm^{-1} irrespective of the total magnitude of the broadening. The ratio between the intensity maxima and minima is at least 10. Most of the energy (about 90%) lies in the low-frequency side. The overall intensity is fairly uniform throughout the low-frequency side with an intensity maximum at the low-frequency extremity. In most cases an intensity peak is also observed at the laser frequency itself. A periodic structure but with larger spacing is also observed in the high-frequency side. The structure is smeared out when the low-frequency broadening is relatively large.

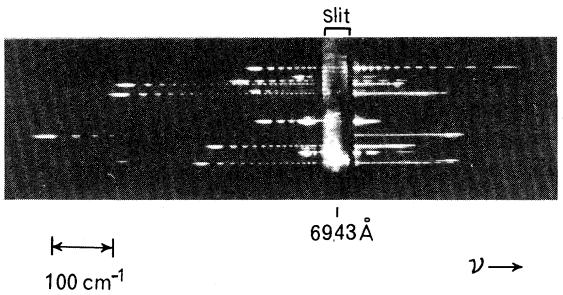

FIG. 1. Spectra of filaments in CS_2 excited with a single laser pulse. The vertical structure is produced by individual filaments occurring during the pulse. The frequency broadening (and periodic structure) is displayed in the horizontal direction and is evidently different for each filament. The image of the slit and the low-frequency side of the spectrum (at left) are attenuated by factors of 41 and 13, respectively, in comparison with the high-frequency side of the spectrum. (Some of the vertical fine structure within each filament is caused by diffraction by the masking wire placed at the focus of the objective lens.)

The regular periodic structure was usually observed only when the total spectral width of the laser radiation was fairly large. When filaments were produced by a laser with only two axial modes (separated by 0.8 cm^{-1}), the spectra of the filaments had a strong nonshifted center and were accompanied by a strong Raman Stokes spectrum. The structure in the spectrum was usually not as regular as observed in Fig. 1. When a single-mode laser was used, only a slight broadening was observed.

DeMartini et al.[6] have discussed pulse distortion by the intensity-dependent refractive index and have given examples of the numerical calculation of the frequency broadening where the distortion is fairly large. Since the amount of broadening in the present experiment is at most 5% of the laser frequency, we here treat the frequency broadening in the lowest order approximation neglecting the distortion of the intensity envelope. We will also discuss the mechanism of the periodic structure observed. The frequency broadening occurs through the phase modulation which is the result of the inhomogeneous phase velocity within a pulse. After a light pulse travels a distance L in the liquid, its phase is modulated by the intensity-dependent refractive index, as

$$\delta\varphi = (-k_0/n_0)L\delta n,$$

where k_0 is the wave vector of the initial laser beam in vacuum and δn is the change in the

refractive index given by

$$\delta n = \frac{n_2}{t}\int_{-\infty}^{t} dt' \langle E^2(t)\rangle e^{-(t-t')/\tau}.$$

In the above expression τ is the relaxation time and $n_2\langle E^2\rangle$ is the stationary value of the intensity-dependent refractive index. As an extreme case, if $\tau = 0$, the instantaneous frequency shift $d\delta\varphi/dt$ is proportional to $-d\langle E^2\rangle/dt$ and the frequency broadening occurs on either side of the center frequency. On the other hand, if τ is sufficiently large, $d\delta\varphi/dt$ is proportional to $-\langle E^2\rangle$ and the frequency broadening occurs only to the low-frequency side. The explanation of the periodic structure follows from a general consideration of a phase modulated wave. When the broadening is much larger than the inverse of the duration time of a pulse, the main contribution to the Fourier transform of $E(t)$, $\int E(t)\exp\{-i(\omega_0+\delta\omega)t\}dt$, at a particular angular frequency $\omega_0+\delta\omega$ comes from the integrals around the points at which the instantaneous frequency shift $d\delta\varphi/dt$ is equal to $\delta\omega$. For a pulse with a simple intensity envelope the condition $d\delta\varphi/dt = \delta\omega$ is satisfied at two points t_1 and t_2, for $\delta\omega < 0$. If we write values of integrals for these two parts as $I_1\exp(i\psi_1)$ and $I_2\exp(i\psi_2)$, respectively, the power spectrum is proportional to $I_1^2 + I_2^2 + 2I_1I_2\cos(\psi_2-\psi_1)$, where $\psi_2-\psi_1$ is expressed approximately by

$$\psi_2-\psi_1 \approx \{\delta\varphi(t_2)-\delta\varphi(t_1)\}-(t_2-t_1)\delta\omega.$$

The right-hand side of this expression is a steadily increasing function of $\delta\omega$ with positive second derivative. Therefore the interference term, $2I_1I_2\cos(\psi_2-\psi_1)$, produces a periodic structure in the spectrum with increasing spacing towards the low-frequency side. It can also be shown that the spectrum has its intensity maximum at the low-frequency extremity where the derivative of $d\delta\varphi/dt$ equals zero. If the pulse is symmetric, the minimum intensity in the periodic structure will be exactly zero. Although this is not the case for the pulses in Fig. 1, we may conclude that the pulse is not extremely asymmetric, because the observed ratio between the intensity maxima and minima is at least 10. The duration time of the filament is approximately the inverse of the average period in the low-frequency side, and is determined to be 6×10^{-12} sec. The high-frequency side can also be periodic, if τ is short enough compared with the duration time of the pulse.

In order to determine precisely the behavior of the broadening, more accurate measurements are necessary. Several factors, such as the temporal change in the intensity of the pulse and the deviation from the assumption of the simple relaxation time, which are not included in the above analysis, should be taken into account. A pulse with shorter duration time would be useful in order to study the dynamic behavior of the intensity-dependent refractive index in liquids.

It should be noted that among the liquids investigated (carbon disulfide, toluene, benzene, nitrobenzene, and mesitylene), the regular periodic patterns were observed in every filament in carbon disulfide only (also in some filaments in toluene). In all of these liquids, except carbon disulfide, a strong stimulated Raman emission was observed in each linear filament and the laser spectrum was either broad and irregular or very narrow. These observations can be explained if in carbon disulfide the laser pulse is not distorted appreciably by other nonlinear effects, such as stimulated Raman scattering, during the process of frequency broadening. In comparison with the other liquids, carbon disulfide has a large intensity-dependent refractive index (and the relaxation time for the relevant process is short). On the other hand, carbon disulfide has a relatively large gain for stimulated Raman scattering. However, this large gain is mainly due to the extremely narrow linewidth ($\sim$0.48 cm^{-1}).[8] When the stimulated Raman effect is excited by a pulse with a duration short compared with the relaxation time for the Raman process, the amplification is determined by the Raman cross section which is not so large in carbon disulfide compared with that in other liquids. Furthermore, since carbon disulfide has a large dispersion, the laser and Raman Stokes waves cannot long retain their favorable phase relation for maximum amplification of the Stokes wave. For these reasons, it is believed that distortion of the laser pulse is a minimum in carbon disulfide and results in the observed frequency broadening.

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TURBULENT HEATING OF PLASMA IN A MIRROR*

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A plasma has been heated substantially by passing a current through it. The plasma was injected axially into a mirror field, and the current was passed through the plasma parallel to the magnetic field. We obtained a heating efficiency, insensitive to variations of the discharge-circuit parameters as well as the density of the initial plasma, of 3-4%.

The possibility of rapidly heating a plasma to high temperatures by means of turbulence generated when current is passed through a plasma has excited wide interest because of its importance to controlled thermonuclear-fusion research. Publications of work done by Babykin et al.[1] reporting very high heating efficiency in such an experiment has aroused considerable debate because of their method of measuring plasma temperature and heating efficiency.

Hydrogen plasma injected axially into a magnetic-mirror field from two plasma guns has been heated by passing a current through the plasma. The current was passed axially through the plasma from two electrodes placed just outside the mirrors. This configuration is similar to the one used by Babykin et al.,[1] and the experiment is related to the one by Hamberger et al.[2] A schematic of the experiment is shown in Fig. 1. Under "standard" conditions

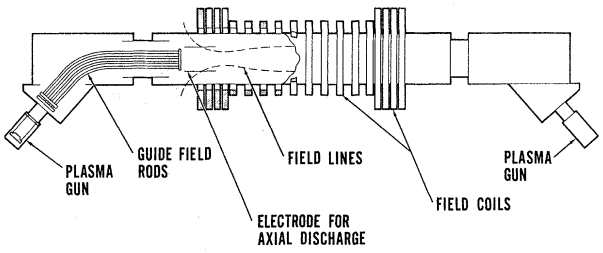
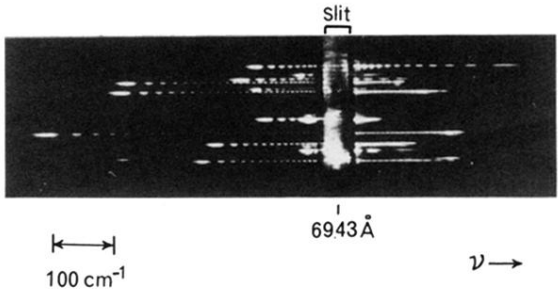


FIG. 1. Schematic of the experiment.

the field strength is 1600 G in the midplane of the mirror field, the mirror ratio is 2, and the distance between the mirrors is 180 cm. Electrostatic probe measurements show that the initial plasma has a maximum density of $\sim 5\times10^{13}$ cm^{-3} and a diameter of 2-3 cm, and that the electron temperature is of order 10 eV. From time-of-flight observations it is found that the mean ion directed energy is ~100 eV. The axial current results from the discharge of the two condensers in series. The circuit provides an open-circuit voltage of up to 80 kV between the electrodes and a ringing frequency of ~1 Mc/sec when shorted. The effective capacity of the circuit is 12 nF and thus the maximum current is ~5000 A.

By means of a compensated flux loop, with a diameter of 35 cm, the perpendicular energy of the plasma column was measured. The passage of the axial current through the plasma gave rise to a considerable increase of the kinetic energy of the plasma. Figure 2 shows the time dependence of the axial current and the perpendicular plasma energy measured by the flux loop. The efficiency of energy transfer to the plasma, in terms of the ratio between the perpendicular plasma energy and the energy initially stored in the condensers, has been found to be nearly constant. The perpendicular energy was measured not at the peak but at 7 μsec after the axial discharge starts. Fig-



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