and Radicati[12] from SU(6) and by Lee[13] from the current algebra.

In order to compare with the results of f obtained from the Cabbibo theory and from the weak hyperon-decay experiments, it is necessary to divide both sides of (6), respectively, by the square of the sum of masses of the baryons subscripted to the coupling constants, as the comparison can be made through the generalized Goldberger-Treiman relation and the current algebra.[14] Including this mass factor, the modified F-D mixing parameter obtained from (6) is given by

$$f' = 0.33 \pm 0.06. \quad (9)$$

This value agrees very well with 0.37, obtained by Willis et al.,[15] and 0.33 ± 0.02, by Brene et al.[16]

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STRONG DECAYS OF HIGHER SPIN BARYON RESONANCES IN O(4, 2) THEORY*

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The predictions of the relativistic O(4, 2) theory have been calculated for the decays of all the higher spin baryon resonances, giving results in good agreement with the experimental data. The SU(3) has been included in the rest frame implying a definite symmetry breaking for the vertex functions.

Recently, the relativistic framework of the dynamical group O(4, 2) has been successfully applied to the mass spectra and form factors of hadrons[1] and to the strong decays of meson resonances.[2] The strong decays of baryon resonances had been earlier calculated in the framework of the group O(3, 1) and it was found[3] that the simple representation of O(3, 1) (characterized by the vanishing of one of the Casimir operators, $\nu = 0$) could not explain the observed partial decay widths and that one had to go to a different representation characterized by $\nu \cong 3.5$ to describe the observed rates. This introduces a new unwanted parameter ν into the theory and, moreover, the form factors in this new representation have now an oscillating form for large t. This fact plus the complexity of the observed spectrum of baryons and the behavior of the form factor were the reasons for going to the larger dynamical group O(4, 2),[4] which now seems to account very well for most of the properties of hadrons.

The purpose of this note is to present the results of calculations on the strong decays of baryons in the framework of O(4, 2), and we will show that the simplest physical representation of this group, supplemented with the SU(3) internal symmetry in the rest frame, accounts for all the partial widths of the baryon decays up to $J=19/2$. The only parameter is a single over-all normalization constant in the amplitude.

We confine ourselves to the decays

$$B_j^* \to B_{\frac{1}{2}} + P, \qquad (1)$$

where B_j^*, $j^P = \frac{3}{2}^+, \frac{7}{2}^+, 11/2^+, 15/2^+, \cdots$, are $I=\frac{3}{2}$ baryon resonances and P the pseudoscalar mesons. In the rest frame the baryons are assigned to a doubled representation of O(4, 2) with $j^P = \frac{1}{2}^\pm, \frac{3}{2}^\pm, \cdots, (n-\frac{1}{2})^\pm, \cdots$. We also include, in the rest frame, the direct product of O(4, 2) with the internal symmetry group SU(3), so that a definite (minimal) symmetry-breaking effect is incorporated in any arbitrary frame via the relativistic boosts.

As in the previous work, the states are labeled by $|\alpha;a\rangle \equiv |n, j^P, j_z; (N) I, I_z, Y\rangle$, where n is the principal quantum number in O(4, 2) (i.e., eigenvalue of L_{56}), N is the dimension (better, the Casimir operators) of the SU(3) representation, and other labels have their usual meaning. The relativistic wave functions of a particle of momentum $p_\mu = (m\cosh\xi, \hat{\xi} m \sinh\xi)$ are obtained from the above states by the representations of the pure Lorentz transformations:

$$|\alpha;a;p\rangle \equiv e^{i\vec{\xi}\cdot\vec{M}}|\alpha;a\rangle, \quad \tanh\xi = p/E. \qquad (2)$$

The O(4, 2) formalism admits a mixing effect in the sense that the "physical" states for which the interactions have the simple transformation property of the group generators are given by

$$|\bar{\alpha};a;p\rangle = (\mathfrak{N})^{-1}\exp(i\vec{\xi}\cdot\vec{M})\exp(i\theta_\alpha T)|\alpha;a\rangle, \qquad (3)$$

where T is the rotational scalar operator L_{45} in the O(4, 2) algebra, θ_α are the parameters that are determined from current conservation and mass spectrum,[1] and $\mathfrak{N}$ is a normalization factor. The significance of the requirement (3) has been discussed previously.[1] As before, the O(4, 2) generators $L_{ab} = -L_{ba}$ $(a, b = 1, 2 \cdots, 6)$ have the following physical meaning: L_{ij} $(i, j = 1, 2, 3)$ the angular momentum operators; L_{i4} the analog of the Lenz vector; $M = L_{i5}$ the generators of the pure Lorentz transformations (boosts); $T = L_{45}$; $S = L_{46}$, a Lorentz scalar; and $L_{56} = n$, the principal quantum number.

With these definitions the vertex scalar transition probability amplitude between two states is given by

$$A = \langle \bar{n} j^\pm m; a|S|\bar{n}' j'^\pm m'; a'; p\rangle$$
$$= \sum_{n''} \langle n j^\pm m|L_{46}'|n'' j''^\pm m''\rangle f_{n''n}(\theta_n, \theta_{n'}, \xi), \qquad (4)$$

where

$$f_{n''n'}(\theta_n, \theta_{n'}, \xi) \equiv \langle n'' j'' m''|\exp(i\theta_n L_{45})\exp(-\xi L_{35})\exp(i\theta_n L_{45})|n' j' m'\rangle, \qquad (5)$$

and

$$L_{46}' = \exp(-i\theta_n L_{45}) L_{46} \exp(i\theta_n L_{45}) = L_{46}\cosh\theta_n + L_{56}\sinh\theta_n. \qquad (6)$$

The evaluation of $f_{n''n'}$ is very similar to a case done previously.[5] We first bring the operator in (5) into a O(2, 1)⊗O(2, 1) form generated by N_1^i, N_2^i $(i = 1, 2, 3)$, $[N_1, N_2] = 0$, where

$$L_{45} = (N_1^2 + N_2^2), \quad L_{46} = (N_1^1 + N_2^1),$$
$$L_{34} = (N_1^3 - N_2^3), \quad L_{35} = (N_1^1 - N_2^1).$$

Thus

$$f_{n''n'}=\langle n''j''m|\exp(i\theta_{n'n''}L_{45})\exp[-i\xi(\cosh\theta_{n'}L_{35}+\sinh\theta_{n'}L_{34})]|n'j'm'\rangle,$$

where

$$\theta_{n'n}=\theta_{n'}-\theta_{n''},\tag{7}$$

or, in terms of the Euler angles (α,β,γ), we have

$$f_{n''n'}=\langle n''j''m''|e^{-i\alpha L_{34}}e^{-i\beta(N_1^2+N_2^2)}e^{-\gamma L_{34}}|n'j'm'\rangle,\tag{8}$$

where

$$\cosh\tfrac{1}{2}\theta\cosh\tfrac{1}{2}\xi=\cos\tfrac{1}{2}(\alpha+\gamma)\cosh\tfrac{1}{2}\beta,\tag{9a}$$

$$\sinh\tfrac{1}{2}\theta\cosh\tfrac{1}{2}\xi=-\cos\tfrac{1}{2}(\alpha-\gamma)\sinh\tfrac{1}{2}\beta,\tag{9b}$$

$$\sinh\tfrac{1}{2}(\theta_{n''}+\theta_{n'})\sinh\tfrac{1}{2}\xi=\sin\tfrac{1}{2}(\alpha+\gamma)\cosh\tfrac{1}{2}\beta,\tag{9c}$$

$$\cosh\tfrac{1}{2}(\theta_{n''}+\theta_{n'})\sinh\tfrac{1}{2}\xi=-\sin\tfrac{1}{2}(\alpha-\gamma)\sinh\tfrac{1}{2}\beta.\tag{9d}$$

From the knowledge of ξ and the θ_n's, the angles α, β, and γ can be easily evaluated. We take the mixing angle as derived in a previous paper from current conservation and from a reasonable mass spectrum[1]: $\sinh\theta_n\cong 3/n$. This expression is approximate but good enough for the present case. Next we express the states $|n,j^{\pm}m\rangle$ in terms of the $|n_1,n_2,m\rangle$ basis. For this purpose we first combine $l=j-\frac{1}{2}$ with $S=\frac{1}{2}$ and obtain

$$|nj^{\pm}m\rangle=(-1)^{j-m-1}(2j+1)^{\frac{1}{2}}\begin{pmatrix}\frac{1}{2} & j-\frac{1}{2} & j\\ m' & m-m' & -m\end{pmatrix}[a_p^{\dagger}\pm i(-1)^l b_p^{\dagger}]|n,j-\tfrac{1}{2},m-m'\rangle,\tag{10}$$

where

$$|nlm\rangle=(-1)^m(2l+1)^{\frac{1}{2}}\begin{pmatrix}\frac{1}{2}(n-1) & \frac{1}{2}(n-1) & l\\ \frac{1}{2}(m-n_1+n_2) & \frac{1}{2}(m+n_1-n_2) & -m\end{pmatrix}|n_1n_2m\rangle,\tag{11}$$

and

$$|n_1n_2m\rangle=[n_1!(n_1+|m|)!n_2!(n_2+|m|)!]^{\frac{1}{2}}\times a_1^{\dagger n_2+m}a_2^{\dagger n_1}b_1^{\dagger n_1+m}b_2^{\dagger n_2}|0\rangle,\quad m>0,$$

$$\times a_1^{\dagger n_2}a_2^{\dagger n_1-m}b_1^{\dagger n_1}b_2^{\dagger n_2-m}|0\rangle,\quad m<0.\tag{12}$$

With these formulas the amplitude (4) can be evaluated. We display in detail the evaluation of one amplitude, namely $\frac{3}{2}^+\to\frac{1}{2}+0^-$; the other cases can be carried out exactly in an analogous fashion.

The $|2,\frac{3}{2}^+,\frac{1}{2}\rangle$ and $|1,\frac{1}{2}^+,\frac{1}{2}\rangle$ states are given by

$$|2,\tfrac{3}{2}^+,\tfrac{1}{2}\rangle=(2\sqrt{3})^{-1}\{(a_1^{\dagger}-ib_1^{\dagger})(a_1^{\dagger}b_2^{\dagger}+a_2^{\dagger}b_1^{\dagger})|0\rangle+(a_2^{\dagger}-ib_2^{\dagger})a_1^{\dagger}b_1^{\dagger}|0\rangle\}$$

and

$$|1,\tfrac{1}{2}^+,\tfrac{1}{2}\rangle=(\sqrt{2})^{-1}(a_1^{\dagger}+ib_1^{\dagger})|0\rangle.\tag{13}$$

Thus

$$A=\frac{\frac{5}{2}\sinh\theta_{5/2}}{2\sqrt{3}}\langle(a_1+b_1)(a_1b_2+a_2b_1)+(a_2+ib_2)a_1b_1|e^{-i\alpha L_{34}}e^{-i\beta(N_1^2+N_2^2)}e^{-i\gamma L_{34}}|(\sqrt{2})^{-1}(a_1^{\dagger}+ib_1^{\dagger})\rangle$$

$$=(2\sqrt{6})^{-1}[\sqrt{2}e^{\frac{1}{2}i(3\alpha+\gamma)}\langle a_1^{\,2}b_2|e^{-i\beta N_2^2}|a_1^{\dagger}\rangle\langle 0|e^{-i\beta N_1^2}|0\rangle+2e^{-\frac{1}{2}i(\alpha-\gamma)}\langle a_1|e^{-i\beta N_2^2}|a_1^{\dagger}\rangle\langle a_2b_1|e^{-i\beta N_1^2}|0\rangle$$

$$-2e^{\frac{1}{2}i(\alpha-\gamma)}\langle b_1|e^{-i\beta N_1^2}|b_1^{\dagger}\rangle\langle a_1b_2|e^{-i\beta N_2^2}|0\rangle-\sqrt{2}e^{-\frac{1}{2}i(3\alpha+\gamma)}\langle a_2b_1^{\,2}|e^{-i\beta N_1^2}|b_1^{\dagger}\rangle\langle 0|e^{-i\beta N_2^2}|0\rangle],$$

or

$$A(\tfrac{3}{2}^+ \to \tfrac{1}{2}^+) = (3/2\sqrt{6})[\sqrt{2}e^{\frac{1}{2}i(3\alpha+\gamma)}V_{21}{}^{1}(-\beta)V_{\frac{1}{2}\frac{1}{2}}{}^{\frac{1}{2}}(\beta) + 2e^{\frac{1}{2}i(\alpha-\gamma)}V_{11}{}^{1}(-\beta)V_{\frac{3}{2}\frac{1}{2}}{}^{\frac{1}{2}}(\beta)$$
$$-2e^{\frac{1}{2}i(\alpha-\gamma)}V_{\frac{3}{2}\frac{1}{2}}{}^{\frac{1}{2}}(-\beta)V_{11}{}^{1}(\beta) - \sqrt{2}e^{\frac{1}{2}i(3\alpha+\gamma)}V_{\frac{1}{2}\frac{1}{2}}{}^{\frac{1}{2}}(-\beta)V_{21}{}^{1}(\beta)],$$

where the V's are the universal functions occurring in all the calculations using representations of noncompact groups.[1-5] If we insert the values of V, we obtain, finally,

$$A = -\frac{3}{2\sqrt{6}}\left(e^{\frac{1}{2}i(3\alpha+\gamma)} + e^{-\frac{1}{2}i(3\alpha+\gamma)}\right)\frac{2\sinh\frac{1}{2}\beta}{\cosh^{4\frac{1}{2}}\beta}$$
$$-\left(e^{\frac{1}{2}i(\alpha-\gamma)} + e^{-\frac{1}{2}i(\alpha-\gamma)}\right)\frac{2\sinh\frac{1}{2}\beta}{\cosh^{4\frac{1}{2}}\beta},$$

i.e.,

$$A = -\frac{12}{\sqrt{6}}\sin\alpha\sin[\tfrac{1}{2}(\alpha-\gamma)]\frac{\sinh\frac{1}{2}\beta}{\cosh^{4\frac{1}{2}}\beta}. \qquad (14)$$

The calculated amplitude (14) is relativistically invariant; we square it and multiply with the invariant phase space to obtain the decay width. The decay of strange baryon resonances will be calculated by assuming the SU(3) Clebsch-Gordan coefficients in the rest frame. The universal formula obtained is in terms of the relativistic velocity parameter ξ. If we insert the experimental masses, we obtain automatically a symmetry breaking that is the minimal symmetry breaking in the formalism of dynamical groups. Thus the final formula

Table I. Comparison between calculation and experiment.

Decay	Γ (theory) (MeV)	Γ (expt) (MeV)
$\Delta_{3/2}\to N\pi$	120	120 ± 2
$\Sigma_{3/2}\to N\pi$	35.82	33.67 ± 2.73
$\Sigma_{3/2}\to \Sigma\pi$	1.2	3.33 ± 1.11
$\Xi_{3/2}\to \Xi\pi$	5.25	7.3 ± 1.7
$\Delta_{7/2}\to N\pi$	76.7	100.0
$\Delta_{7/2}\to \Sigma K$	1.74	seen
$\Sigma_{7/2}\to N\bar{K}$	40.9	25.6
$\Sigma_{7/2}\to \Sigma\pi$	10.1	seen
$\Sigma_{7/2}\to \Lambda\pi$	32.9	40.0
$\Delta_{11/2}\to N\pi$	45.0	~27.5
$\Delta_{11/2}\to \Sigma K$	5.23	?
$\Delta_{15/2}\to N\pi$	13.9	~10
$\Delta_{19/2}\to N\pi$	3.9	~ 3

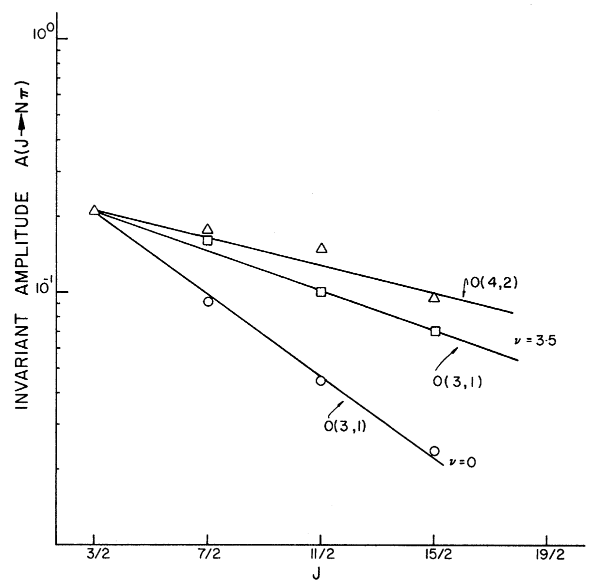

FIG. 1. Comparison of the invariant amplitudes in O(3, 1) theory ($\nu = 0$, most degenerate representation, and $\nu = 3, 5$) with that in O(4, 2) theory (most degenerate representation).

for the partial decay rates is

$$\Gamma = (2J+1)^{-1}(p_f/M_i)M_f(\text{C.G.})^2|A|^2, \qquad (15)$$

where M_i, M_f are the masses of the initial and baryon masses, J_i spin of the initial baryon, p_f the momentum of the final baryon, and (C.G.) the usual SU(3) Clebsch-Gordan coefficients.

Table I shows the comparison of the calculated and observed partial decay widths. In Fig. 1 we compare the O(4, 2) amplitude as a function of the spin with that of O(3, 1) and see that the single physical representation of O(4, 2) accounts automatically for the mixing effect ($\nu \neq 0$) in O(3, 1).

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CURRENT ALGEBRA AND VERTEX FUNCTIONS*

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A method is given for calculating vertex functions involving three currents obeying chiral $SU(2)\otimes SU(2)$ commutation relations, without using soft-pion approximations. The procedure assumes one-particle dominance of intermediate states. The Weinberg sum rule emerges as a consequence of the vanishing of q-number Schwinger terms. The charge radius of the pion is obtained and is in agreement with existing data.

Recently Schnitzer and Weinberg[1] have developed techniques for calculating T-products of three currents obeying chiral $SU(2)\otimes SU(2)$ commutation relations. These results are remarkable as they allow the calculation of such processes as $\rho\to\pi+\pi$ and $A_1\to\rho+\pi$ without resort to soft- or massless-pion approximations. Their method involves the use of the Ward identities for the vertex functions, combined with the assumption of single-particle dominance by π, A_1, and ρ mesons. We here present an alternative analysis also based on single-particle dominance, but more directly connected to the current algebras.[2] The two procedures give identical answers for the T-product of three currents but represent different techniques for extending these results to more complicated problems, such as four currents and higher groups.

We start by considering the T-product $\langle T[A_a{}^\alpha(x)A_b{}^\beta(y)V_c{}^\gamma(0)]\rangle$, where a, b, and c represent the SU(2) indices. This function may be expanded into its six time orderings of which a characteristic one is

$$\langle A_a{}^\alpha(x)V_c{}^\gamma(0)A_b{}^\beta(y)\rangle=\sum_{n,m}\langle 0|A_a{}^\alpha|n\rangle\langle n|V_c{}^\gamma|m\rangle\langle m|A_b{}^\beta|0\rangle. \tag{1}$$

Imposing single-particle dominance (i.e., saturating the right-hand side with π and A_1 states for this case) one encounters the matrix element $\langle 0|A_a{}^\alpha|\pi,qb\rangle$ which is proportional to $F_\pi q^\alpha$ [by partial conservation of axial-vector current (PCAC)[3]], and $\langle 0|A_a{}^\alpha|A_1,qb\rangle$ which is proportional to $g_A\epsilon^\alpha(q)$. Here ϵ^α is the polarization vector of the A_1, and g_A is the coupling strength of the axial current to the A_1 meson, defined by Weinberg.[4,3] (For other time orderings, there appears the factor $\langle 0|V_c{}^\gamma\times|\rho,qb\rangle$ which involves the coupling strength g_ρ of the vector current to the ρ meson.) For the one-meson–one-meson matrix elements, we use the single-particle–dominance hypothesis in the stronger form that the vector current couples to the mesons only through the ρ, and the axial current only through the π and A_1 particles. Thus, for example, the vertex $\langle\pi|V_c{}^\gamma|\pi\rangle$ has a ρ pole in the momentum transfer, and so one writes phenomenologically

$$(2\pi)^3(2\omega_q2\omega_p)^{\frac{1}{2}}\langle\pi qb|V_c{}^\gamma|\pi pa\rangle=i\epsilon_{abc}\Delta_\rho{}^{\gamma\lambda}(k)\Gamma_\lambda(q,p)=i\epsilon_{abc}\Delta_\rho{}^{\gamma\lambda}(k)[a_1+a_2k^2+\cdots](q+p)_\lambda. \tag{2}$$

Here $\Delta_\rho{}^{\mu\lambda}(k)$ (where $k\equiv q-p$) is the ρ propagator and Γ_λ is the π-π-ρ vertex function. If one is not considering processes with too high a momentum transfer, presumably only a few terms in the series expansion of Γ_λ need be kept. Similarly, expressions for other one-meson matrix elements can be written down, the axial currents possessing corresponding A_1 and π poles.