DETERMINATION OF $N\Lambda K$ AND $N\Sigma K$ COUPLING CONSTANTS*

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Using KN forward-dispersion relations, both the $N\Lambda K$ and $N\Sigma K$ coupling constants have been determined[1]: $g_{p\Lambda K}^2 = 16.0 \pm 2.5$ and $g_{p\Sigma^0 K}^2 = 0.3 \pm 0.5$. The results are consistent with SU(3) invariance for baryon-baryon-pseudoscalar meson coupling constants, contrary to previous conclusions. The value of the F-D mixing parameter obtained from these coupling constants for strong interactions agrees well with the value obtained from the Cabbibo theory and weak hyperon-decay experiments.

During the last two years, there have been many determinations of $g_{N\Lambda K}^2$. Because of the lack of experimental information on the pole term from $Y_1^*(1385)$, it was only possible to set an upper limit on $g_{N\Sigma K}^2$. All previous determinations of the NYK coupling constants suffered from common systematic errors introduced by too simple and approximate treatment of the unphysical region below the KN threshold.[2] Thus, a good portion of this Letter will be used in explaining these systematic errors which were neglected previously. It has become possible to take care of the unphysical region satisfactorily only since the recent multichannel effective-range analysis of $\bar{K}N$ interactions.[3]

The dispersion relation used was originally derived by Mathews and Salam.[4] It is given for the $K^{\pm}p$ system in the form

$$\frac{1}{2M_K}[D_-(M_K)-D_+(M_K)]-\frac{1}{4\pi^2}\int_{M_K}^{\infty}\frac{\sigma_-(\omega')-\sigma_+(\omega')}{k'}d\omega'-\frac{1}{\pi}\int_{\omega_{\Lambda\pi}}^{M_K}\frac{A_-(\omega')}{k'^2}d\omega'=\frac{X(\Lambda)}{\omega_\Lambda^2-M_K^2}+\frac{X(\Sigma^0)}{\omega_\Sigma^2-M_K^2}, \quad (1)$$

where $D_\pm(\omega')$ and $A_\pm(\omega')$ are real and imaginary parts of $K^\pm$-p scattering amplitudes, ω' and k' are energy and momentum of K meson,

$$\omega_\Lambda = (M_\Lambda^2 - M_p^2 - M_K^2)/2M_p,$$

$$X(\Lambda) = [(M_\Lambda - M_p)^2 - M_K^2]g_{p\Lambda K}^2/4M_pM_\Lambda,$$

and similar expressions are for ω_Σ and $X(\Sigma^0)$ with Λ replaced by Σ^0. Also, the same dispersion relation holds for the $K^\pm n$ system except that the Λ-pole term is missing. All quantities in Eq. (1) are defined in the laboratory system.

In evaluating Eq. (1) numerically, it is convenient to consider three energy regions. The low-energy region extends from the $\Lambda\pi$ threshold to 0.73 BeV. The high-energy region covers from 0.73 to 20.0 BeV. The asymptotic region lies above 20.0 BeV.

In integrating over the high-energy region, very accurate measurements of the $K^\pm p$ and $K^\pm n$ total cross sections by Cool et al.[5] and Davies et al.[6] have been used in addition to the cross sections which were previously used and listed.[7] The asymptotic region has been integrated over using the Regge-pole parameters which were obtained by fitting the experimental cross sections available from 6 to 20 BeV/c by Phillips and Rarita.[8]

The imaginary part of the K^-p forward scattering amplitudes for s wave, calculated both from the effective-range analysis[3] and from the constant-scattering-length analysis,[9] are shown in Fig. 1. The two amplitudes agree very well for the physical region where fittings to the same experimental data have been made, but disagreement between two amplitudes comes in rapidly below the $\bar{K}N$ threshold. The location and the width of the $Y_0^*(1405)$ pole term from the effective-range parameters are, respectively, lower and considerably wider compared with the values from the constant scattering lengths.[10] Therefore, the value of the integral for the unphysical region using the effective-range parameters is much larger than the value of this integral previously obtained from the constant scattering length. This is the main reason for the value of the new $N\Lambda K$ coupling constant being much larger than the previous values. Also, in Fig. 1, the imaginary part of K^-p forward-scattering amplitude for the P_3 wave is shown. It is shown for the first time experimentally through the multichannel effective-range analysis that the

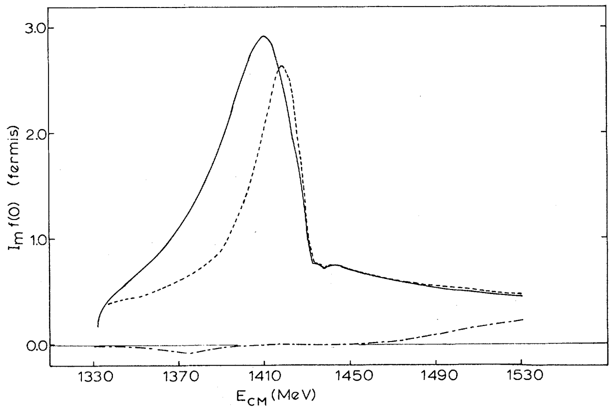

FIG. 1. The solid line is the s-wave imaginary part of the forward K^-p scattering amplitude calculated from the effective range parameters of Ref. 3. The dashed line is the same amplitude calculated from the constant-scattering-length parameters by Kim, Ref. 9. The dot-dashed line is 10 times the imaginary part of the P_3 K^-p amplitude calculated from the effective-range parameters of Ref. 3.

coupling of $Y_1^*(1385)$ to the $\bar{K}N$ channel is small.[3] This has made it possible for the first time to determine the $N\Sigma K$ coupling constant experimentally.

The results of calculation of the dispersion relations are shown in Table I. The resultant sum rules for both $K^\pm p$ and $K^\pm n$ systems are

$$34.5 \pm 6.2 = 2.12 g_{p\Lambda K}^2 + 1.83 g_{p\Sigma^0 K}^2, \qquad (2)$$

$$1.2 \pm 2.0 = 1.83 g_{n\Sigma^- K}^2, \qquad (3)$$

where $g_{n\Sigma^-K}^2 = 2g_{p\Sigma^0K}^2$.

Solving these two equations simultaneously, we determine both $g_{p\Lambda K}^2$ and $g_{p\Sigma^0 K}^2$ coupling constants:

$$g_{p\Lambda K}^2 = 16.0 \pm 2.5, \qquad (4)$$

$$g_{p\Sigma^0 K}^2 = 0.3 \pm 0.5, \qquad (5)$$

where error correlation has been taken care of in solving two equations. The value of $g_{p\Lambda K}^2$ given by (4) is 2-3 times larger than the previous determinations of this coupling constant. This large difference mainly comes from the correct treatment of the unphysical region and from the salient feature of cancellation of various terms in the dispersion relation. All the previous values of the $N\Lambda K$ coupling constant would agree with the present value within a large error limit, if they included the proper estimate of systematic errors in their statistical error quoted.

From pure SU(3) invariance, we obtain the following relations[11]:

$$g_{p\Lambda K}^2 = g_{NN\pi}^2 (1+2f)^2/3, \qquad (6)$$

$$g_{p\Sigma^0 K}^2 = g_{NN\pi}^2 (1-2f)^2, \qquad (7)$$

where $f = F/(F+D)$ and $g_{NN\pi}^2 = 14.5 \pm 0.4$. From (4) and (6), the value of the F-D mixing parameter f has been determined:

$$f = 0.41 \pm 0.07. \qquad (8)$$

Using this value of f in (7), the calculated value of $g_{p\Sigma^0K}^2 = 0.5$. This value agrees very well with the experimental value given by (5). From this agreement, it can be concluded that the baryon-baryon-pseudoscalar meson coupling constants are consistent with pure SU(3) invariance within the present statistics. It is also interesting to compare (8) with the theoretical value of 0.4 obtained both by Gürsey, Pais,

Table I. Details of the numerical evaluation of the dispersion relations for $K^\pm p$ and $K^\pm n$ systems in units of 10^{-7} MeV^{-2}. E.R. and C.S.L. refer to the effective-range and the constant-scattering-length parameters of Kim (Ref. 3) and J. J. DeSwart [Rev. Mod. Phys. 35, 916 (1963)], respectively.

Energy range (BeV)	$K^\pm p$ E.R.	$K^\pm p$ C.S.L.	$K^\pm n$ E.R.	$K^\pm n$ C.S.L.
K^- from $\omega_{\Lambda\pi}$ to 0.73	37.9 ± 5.9	18.0 ± 1.1	2.7 ± 1.7	12.6 ± 1.5
K^+ from m_K to 0.73	29.4 ± 1.3		16.4 ± 0.7	
$(\sigma_- - \sigma_+)$ integral from 0.73 to 20	−27.5 ± 0.5		−14.6 ± 0.8	
$(\sigma_- - \sigma_+)$ integral above 20	−5.3		−3.3	
Sum	34.5 ± 6.2	14.6 ± 2.0	1.2 ± 2.0	9.1 ± 2.0

and Radicati[12] from SU(6) and by Lee[13] from the current algebra.

In order to compare with the results of f obtained from the Cabbibo theory and from the weak hyperon-decay experiments, it is necessary to divide both sides of (6), respectively, by the square of the sum of masses of the baryons subscripted to the coupling constants, as the comparison can be made through the generalized Goldberger-Treiman relation and the current algebra.[14] Including this mass factor, the modified F-D mixing parameter obtained from (6) is given by

$$f' = 0.33 \pm 0.06. \quad (9)$$

This value agrees very well with 0.37, obtained by Willis et al.,[15] and 0.33 ± 0.02, by Brene et al.[16]

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STRONG DECAYS OF HIGHER SPIN BARYON RESONANCES IN O(4, 2) THEORY*

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The predictions of the relativistic O(4, 2) theory have been calculated for the decays of all the higher spin baryon resonances, giving results in good agreement with the experimental data. The SU(3) has been included in the rest frame implying a definite symmetry breaking for the vertex functions.

Recently, the relativistic framework of the dynamical group O(4, 2) has been successfully applied to the mass spectra and form factors of hadrons[1] and to the strong decays of meson resonances.[2] The strong decays of baryon resonances had been earlier calculated in the framework of the group O(3, 1) and it was found[3] that the simple representation of O(3, 1) (characterized by the vanishing of one of the Casimir operators, $\nu = 0$) could not explain the observed partial decay widths and that one had to go to a different representation characterized by $\nu \cong 3.5$ to describe the observed rates. This introduces a new unwanted parameter ν into the theory and, moreover, the form factors in this new representation have now an