chon is a fixed pole.

We would like to thank Professor Geoffrey F. Chew and Professor Stanley Mandelstam for several stimulating discussions.

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PION ELECTROMAGNETIC MASS DIFFERENCE FOR PHYSICAL PIONS

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We compute the on-mass-shell electromagnetic mass difference $\mu_{\pi^+}-\mu_{\pi^0}$, in a model consistent with chiral-algebra constraints and partial conservation of axial-vector currents, and find it to be logarithmically divergent. Some implications of this result are discussed.

In a recent Letter, Das, Guralnik, Mathur, Low, and Young[1] have presented a current-algebra calculation of the electromagnetic mass difference between the charged and neutral pi mesons

$$\delta\mu = \mu_{\pi^+} - \mu_{\pi^0}.$$

They work with pions of zero four-momentum and obtain $\delta\mu$ for the unphysical case $\mu_\pi = 0$. In this note we show how to generalize their calculation to on-shell pions and give the corrections of order μ_π^2/m_ρ^2 to their result.

We begin with the expression[2]

$$\delta\mu^2 = \frac{e^2}{4\pi} 2E_\pi \operatorname{Re}\int \frac{d^4q}{q^2-i\epsilon}\left(g_{\mu\nu} - \lambda\frac{q_\mu q_\nu}{q^2}\right)\int d^4x\, e^{iqx}\{\langle\pi^+(k)|T[V_\mu^{\mathrm{em}}(x)V_\nu^{\mathrm{em}}(0)] - \rho_{\mu\nu}(x)|\pi^+(k)\rangle - (\pi^+\to\pi^0)\}, \quad (1)$$

where the quantity inside the curly bracket is the difference of two current correlation functions[3] which are, separately, covariant and gauge invariant, and λ is an arbitrary parameter. We reduce both of the pions as in Ref. 1 to obtain

$$\delta_\mu^{\ 2}=\frac{e^2}{4\pi}\left(\frac{1}{2\pi}\right)^3\int\frac{d^4q}{q^2-i\epsilon}\left(g_{\mu\nu}-\lambda\frac{q_\mu q_\nu}{q^2}\right)T(k,-k,q)_{\mu\nu}, \tag{2}$$

where we define

$$T(k_1,k_2,q)_{\mu\nu}^{\ \ abcd}=(k_1^{\ 2}-\mu_\pi^{\ 2})(k_2^{\ 2}-\mu_\pi^{\ 2})\int d^4xd^4yd^4z\,e^{ik_1x}e^{ik_2y}e^{iqz}\langle 0|T^*[\pi^a(x)\pi^b(y)V_\mu^{\ c}(z)V_\nu^{\ d}(0)]|0\rangle \tag{3}$$

and

$$T(k,-k,q)_{\mu\nu}=\tfrac{1}{2}T(k,-k,q)_{\mu\nu}^{\ \ 1133}+\tfrac{1}{2}T(k,-k,q)_{\mu\nu}^{\ \ 2233}+T(k,-k,q)_{\mu\nu}^{\ \ 3333}$$

corresponding to Eq. (1). Using the definition of partially conserved axial-vector current (PCAC),

$$\pi^a(x)=F_\pi^{\ -1}\mu^{-2}\partial_\mu A_\mu^{\ a}(x),$$

in Eq. (3) we have

$$T(k_1,k_2,q)_{\mu\nu}^{\ \ abcd}=\frac{(k_1^{\ 2}-\mu_\pi^{\ 2})(k_2^{\ 2}-\mu_\pi^{\ 2})}{F_\pi^{\ 2}\mu_\pi^{\ 4}}\int d^4xd^4yd^4z\,e^{ik_1x}e^{ik_2y}e^{iqz}$$
$$\times\langle 0|T^*[\partial_\mu A_\mu^{\ a}(x)\partial_\nu A_\nu^{\ b}(y)V_\mu^{\ c}(z)V_\nu^{\ d}(0)]|0\rangle. \tag{4}$$

At this point Das et al. perform a sequence of partial integrations to remove the derivatives acting on the axial currents, using the equal-time commutation relations of the local chiral SU(2) ⊗ SU(2) current algebra, and take the limit $k_\mu=0$. This enables them to reduce the right-hand side of Eq. (2) to an expression involving only the two-point functions of the vector and axial-vector currents studied by Weinberg.[4] The sum rules

$$\int\frac{\rho_V(m^2)}{m^2}dm^2=\int\frac{\rho_A(m^2)}{m^2}dm^2+F_\pi^{\ 2}, \tag{5a}$$

$$\int\rho_V(m^2)dm^2=\int\rho_A(m^2)dm^2, \tag{5b}$$

insure the finiteness of $\delta_\mu^{\ 2}$. In the ρ and A_1 dominance model where one assumes

$$\rho_V(m^2)=g_\rho^{\ 2}\delta(m^2-m_\rho^{\ 2}), \tag{6a}$$

$$\rho_A(m^2)=g_A^{\ 2}\delta(m^2-m_{A_1}^{\ \ 2}), \tag{6b}$$

and taking in addition the Kawarabayashi-Suzuki-Fayyazuddin-Riazuddin[5] (KSFR) condition

$$g_\rho^{\ 2}m_\rho^{\ -2}=2F_\pi^{\ 2},$$

they obtain

$$\delta_\mu^{\ 2}=(3\alpha/4\pi)m_\rho^{\ 2}\times 2\ln 2. \tag{7}$$

Our calculation proceeds from Eqs. (2) and (4) directly. We have obtained expressions for $T(k_1, k_2,q)_{\mu\nu}^{abcd}$ which satisfy the constraints of the gauge conditions imposed by the chiral commutation relations and PCAC, in an approximation where the vertices and contact terms have the minimum allowable momentum dependence. The explicit details of this construction along with other applications will be presented elsewhere. Two of us (I.S.G. and H.J.S.) used a method which is a generalization of the three-point function calculation of Schnitzer and Weinberg[6] while the other two (B.W.L. and H.T.N.) used a phenomenological chiral Lagrangian technique.[7] In both cases, the result is identical on the mass shell.

Inserting our expression for $T(k, -k, q)_{\mu\nu}$ into Eq. (2) we obtain

$$\delta\mu^2 = \frac{e^2}{4\pi}\frac{m_\rho^{\,2}}{4\pi^2}\frac{1}{i}\int\frac{d^4q}{q^2}\frac{1}{(m_\rho^{\,2}-q^2)^2}\bigg\{-8+\frac{3}{2}\frac{\mu_\pi^{\,2}}{m_\rho^{\,2}}-\frac{1}{2}\bigg[\frac{1}{\mu_\pi^{\,2}-(q+k)^2}[(q^2+4\mu_\pi^{\,2}+4k\cdot q)-2m_\rho^{\,-2}(1+\delta)(\mu_\pi^{\,2}q^2-(k\cdot q)^2]$$
$$+\tfrac{1}{4}m_\rho^{\,-4}(1+\delta)^2q^2[\mu_\pi^{\,2}q^2-(k\cdot q)^2]+\frac{1}{m_{A_1}^{\,2}-(q+k)^2}(-8m_\rho^{\,2}+7\mu_\pi^{\,2}+14k\cdot q+q^2-6\delta k\cdot q+m_\rho^{\,-2}\{-6(k\cdot q)^2$$
$$-\tfrac{3}{2}\mu_\pi^{\,4}-6\mu_\pi^{\,2}k\cdot q+\delta[6(k\cdot q)^2+3\mu_\pi^{\,2}k\cdot q]-\delta^2[(k\cdot q)^2+\tfrac{1}{2}\mu_\pi^{\,2}q^2]+m_\rho^{\,-4}(1-\delta)^2q^2[\mu_\pi^{\,2}q^2-(k\cdot q)^2]\}+k\to -k\bigg]\bigg\}, \tag{8}$$

where δ is the anomalous magnetic moment of the A_1 as defined in Ref. 6. Evaluating Eq. (8) we find

$$\delta\mu^2 = \frac{3\alpha}{4\pi}m_\rho^{\,2}\bigg\{2\ln 2+\frac{\mu_\pi^{\,2}}{m_\rho^{\,2}}\bigg[-\frac{5}{2}+\ln\frac{m_\rho^{\,2}}{\mu_\pi^{\,2}}+\frac{19}{4}\ln 2+\frac{1}{8}\ln\frac{\Lambda^2}{m_\rho^{\,2}}+5\delta\left(\frac{1}{2}\ln 2-\frac{1}{4}\right)+\delta^2\left(-\frac{3}{4}+\frac{1}{4}\ln 2+\frac{1}{8}\ln\frac{\Lambda^2}{m_\rho^{\,2}}\right)\bigg]\bigg\}, \tag{9}$$

where Λ^2 is a cutoff. Our calculation, unlike that of Ref. 1, gives a logarithmically divergent result despite the use of the Weinberg sum rules, Eqs. (5). Numerically,

$$\delta_\mu^{\,2} \approx \frac{3\alpha}{4\pi}m_\rho^{\,2}2\ln 2\bigg\{1+\bigg[0.15+0.003(1+\delta^2)\ln\frac{\Lambda^2}{m_\rho^{\,2}}+0.01\delta(1-\delta)\bigg]\bigg\}, \tag{10}$$

where the zero-mass limit, Eq. (7) of Das et al., has been exhibited explicitly and the rest are the order-μ_π^2/m_ρ^2 corrections. Since according to Ref. 6 $|\delta|<1$, we have

$$\delta_\mu \approx 5\left[1+0.15+0.003(1+\delta^2)\ln\frac{\Lambda^2}{m_\rho^{\,2}}\right]\text{MeV},$$
$$\approx 6\ \text{MeV}.$$

We wish to make the following observations:

(1) The existence of a logarithmic divergence in an on-mass-shell calculation is hardly surprising since this is expected on general grounds as shown by Halpern and Segrè.[8] Our divergent term is two orders higher in g_ρ^{-1}, the effective $\rho\pi\pi$ coupling, than the (finite) zero-mass term, consistent with their result obtained using the algebra of fields.[9]

(2) The present theory, which is tailored to describe low-energy phenomena, has been unjustifiably extrapolated to a high-energy virtual process ($q^2\to\infty$). Thus, high-energy damping effects, which made it possible for Harari[10] to argue that the $\Delta I=2$ electromagnetic mass shifts should be dominated by low-lying excitations and therefore computible, are in fact lacking in our considerations. Within our model, if we require the $I=2$ (t-channel) Compton scattering amplitude to be superconvergent and relax the KSFR condition, we obtain the constraint, independent of δ,

$$g_\rho^{\,2}m_\rho^{\,-2} = -2F\pi^2,$$

which is precisely the condition that the coefficient of $\ln(\Lambda^2/m_\rho^2)$ vanish. This is related to the "fake" solutions of superconvergence relations found by Fubini.[11]

(3) Our numerical result is very insensitive to the value of the cutoff momentum Λ. Since present experimental data seem to indicate a much faster decrease of electromagnetic form factors than is implied by the ρ-dominance model, we may guess that Λ is relatively small, probably not much more than several ρ-meson masses. The cutoff term contributes ~0.07 MeV to $\delta\mu$ for $\Lambda=10m_\rho$ and ~0.14 MeV for $\Lambda=100m_\rho$.

(4) In general the σ term, coming from the commutator $[A_0^{\,a}(x), \partial_\mu A_\mu^{\,b}(0)]\delta(x_0)$, can add a correction of $O(\mu_\pi^2/m_\rho^2)$. Here we assume that this model-dependent commutator transforms as $I=0$ and hence does not affect our calculation.

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$A_1 \to \rho\pi$ AND $\rho \to 2\pi$ DECAYS*

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An analysis of the A_1 and the ρ decays is made using current algebra, partial conservation of axial-vector currents, and dispersion relations; the pions are treated as "soft" in the sense $K^2 \to 0$ instead of $K \to 0$ which is usually assumed.

The present paper is devoted to a study of the decays $\rho \to 2\pi$ and $A_1 \to \rho\pi$ using the techniques of current algebra and dispersion relations. It is well known that the usual treatment[1] of the $\rho \to 2\pi$ decays requires large extrapolation in the masses, so that it is surprising that the result comes out so close to the experimental value. Also in the calculation[2] of the $A_1 \to \rho\pi$ width one gets a gross overestimate of the decay width, if one uses current algebra and the soft-pion technique and assumes unsubtracted dispersion relations for the relevant form factors. We shall show how the ρ and the A_1 decays can be understood consistently in a single framework.[3]

On general grounds of invariance we define the matrix elements

$$\langle \pi^0(k) | V_{\mu_2}{}^1(0) | A_1{}^+(p)\rangle = \frac{i\epsilon_\nu{}^{(A)}(p)}{(4p_0k_0V^2)^{1/2}}[L_1(q^2)\delta_{\nu\mu} + L_2(q^2)k_\nu(p+k)_\mu + L_3(q^2)k_\nu(p-k)_\mu], \tag{1}$$

$$\langle \pi^0(k) | A_{\mu_2}{}^1(0) | \rho^+(p)\rangle = \frac{i\epsilon_\nu{}^{(\rho)}(p)}{(4p_0k_0V^2)^{1/2}}[K_1(q^2)\delta_{\nu\mu} + K_2(q^2)k_\nu(p+k)_\mu + K_3(q^2)k_\nu(p-k)_\mu]. \tag{2}$$

The conditions of conservation of vector currents on Eq. (1) and of partial conservation of axial-vector currents (PCAC) on Eq. (2) lead to the following constraints among the form factors:

$$L_1(q^2) + (M_A{}^2 - \mu^2)L_2(q^2) - q^2L_3(q^2) = 0, \tag{3}$$

$$K_1(q^2) + (M_\rho{}^2 - \mu^2)K_2(q^2) - q^2K_3(q^2) = 2\mu^2F_\pi G_{\rho\pi\pi}/(q^2+\mu^2), \tag{4}$$