[11]These may be taken as $|\vec{s}|$, $|\vec{p}_0|$, $\theta_{s,\,p_0}$ (the angle between $\vec{s}$ and $\vec{p}_0$), $|\vec{p}_1-\vec{p}_{-1}|$, and $\theta_{p_1-p_{-1},s}$ (the angle between $\vec{s}$ and $\vec{p}_1-\vec{p}_{-1}$).

[12]See H. Pilkuhn and B. E. Y. Svensson, Nuovo Cimento 38, 518 (1965), who include references to earlier work. The effects of absorption on correlation moments are discussed by B. E. Y. Svensson, Nuovo Cimento 39, 667 (1965); J. T. Donohue, thesis, University of Illinois, 1967 (unpublished); and J. D. Jackson et al., Phys. Rev. 139, B428 (1965).

$\pi\pi$ PHASE-SHIFT ANALYSIS FROM 600 TO 1000 MeV*

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A method[1] has been proposed (referred to hereafter as I) for extracting the $\pi\pi$ elastic scattering phase shifts from data on $\pi N\to\pi\pi N$. It is shown in I that complete prior knowledge of the helicity amplitudes is not necessary in the analysis. Although some of these become additional free parameters in fitting the data, a large number of constraints remain which test the validity of the model. We present here an analysis of this type for $\pi\pi$ effective mass $0.6<m_{\pi\pi}<1.0$ GeV and $\cos\theta_{\text{c.m.}}>0.9$ (nucleon momentum transfer $t\lesssim 0.175$ GeV^2), using a sample of data with beam momenta 2.1-3.2 GeV/c compiled from several laboratories[2]:

$$\pi^-+p\to\pi^-+\pi^++n \text{ (6740 events)}, \tag{1}$$

$$\pi^-+p\to\pi^-+\pi^0+p \text{ (3656 events)}, \tag{2}$$

where the numbers of events are those remaining after the $m_{\pi\pi}$ and $\theta_{\text{c.m.}}$ selection. The detailed analysis, described below, is concerned mainly with Reaction (1), with Reaction (2) used to obtain independent information on the $T=2$ s-wave interaction. Aside from demonstrating that the data satisfy well the tests suggested in I, the $T=0$ s-wave phase shift (δ_s^0) is shown to increase from ~60° to ~90° in the range $600<m_{\pi\pi}\lesssim 730$. For 730 MeV $<m_{\pi\pi}$, δ_s^0 most likely continues to increase, implying the existence of a $T=0$ scalar meson $\sigma(730)$.

We show in Fig. 1 the spherical harmonic moments $\langle Y_l^0\rangle$ of the π_{out}^- angular distribution in the $\pi\pi$ rest frame of Reaction (1) for $l\le 10$. As explained in I, the coordinate system used has its z axis along the direction of motion of the $\pi\pi$ system for reasons of simplifying the extraction of the helicity amplitudes in the subsequent analysis.[3] For both Reactions (1) and (2) (similar to Fig. 1, but not shown), small but significant (negative) moments exist for

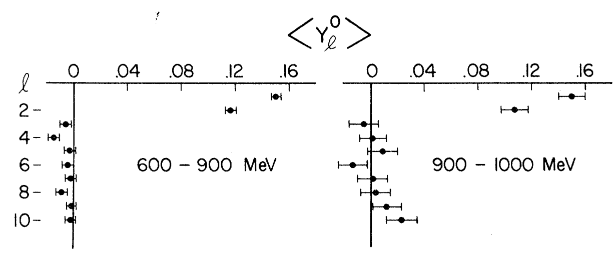

FIG. 1. Moments $\langle Y_l^0\rangle$ of the outgoing π^- in the $\pi\pi$ rest frame of $\pi^-p\to\pi^-\pi^+n$ with $\cos\theta_{\text{c.m.}}>0.9$. The polar axis is the helicity axis of the $\pi\pi$ system. The moments are separately given for $0.6<m_{\pi\pi}<0.9$ and $0.9<m_{\pi\pi}<1.0$ GeV.

l as high as 8. We take these to be due to πN^* background[4] but henceforth ignore their presence compared with the large $l=1,2$ moments. As in earlier analyses,[5] we assume that only s- and p-wave scattering need be considered for the $\pi\pi$ interaction in this region.

The moments $N\langle Y_1^m\rangle$ and $N\langle Y_2^m\rangle$ (N is the $\pi\pi$ effective-mass spectrum) evaluated[6] every 20 MeV for $600<m_{\pi\pi}<1000$ MeV are given in Fig. 2 for Reactions (1) and (2). As shown in Eqs. (3a)-(3f) of I, these quantities have a dependence on the effective $\pi\pi$-scattering amplitude functions which is determined only by l. Thus, $N\langle Y_1^m\rangle\sim\{\,\}\operatorname{Re}(A_{\pi\pi}^{S}A_{\pi\pi}^{P*})$ and $N\langle Y_2^m\rangle\sim\{\,\}|A_{\pi\pi}^{P}|^2$, where the brackets $\{\,\}$ denote functions of the helicity-amplitude vectors (defined in I) $\vec{p}_1$, $\vec{p}_0$, $\vec{p}_{-1}$, and $\vec{s}$. To the extent that these bracket quantities can be considered independent of $m_{\pi\pi}$, the data in Fig. 2 directly display the $m_{\pi\pi}$ dependence of the scattering-amplitude functions shown; the more rapidly varying the phase shifts, the better this approximation. Dirict tests of the fundamental factorization and reality assumptions of the formalism in I are that $N\langle Y_1^0\rangle$ and $N\langle \operatorname{Re}Y_1^1\rangle$ have the same $m_{\pi\pi}$ dependence and that $N\langle Y_2^0\rangle\sim N\langle\operatorname{Re}Y_2^1\rangle\sim N\langle\operatorname{Re}Y_2^2\rangle\sim$ (p-wave Breit-Wigner). Applying

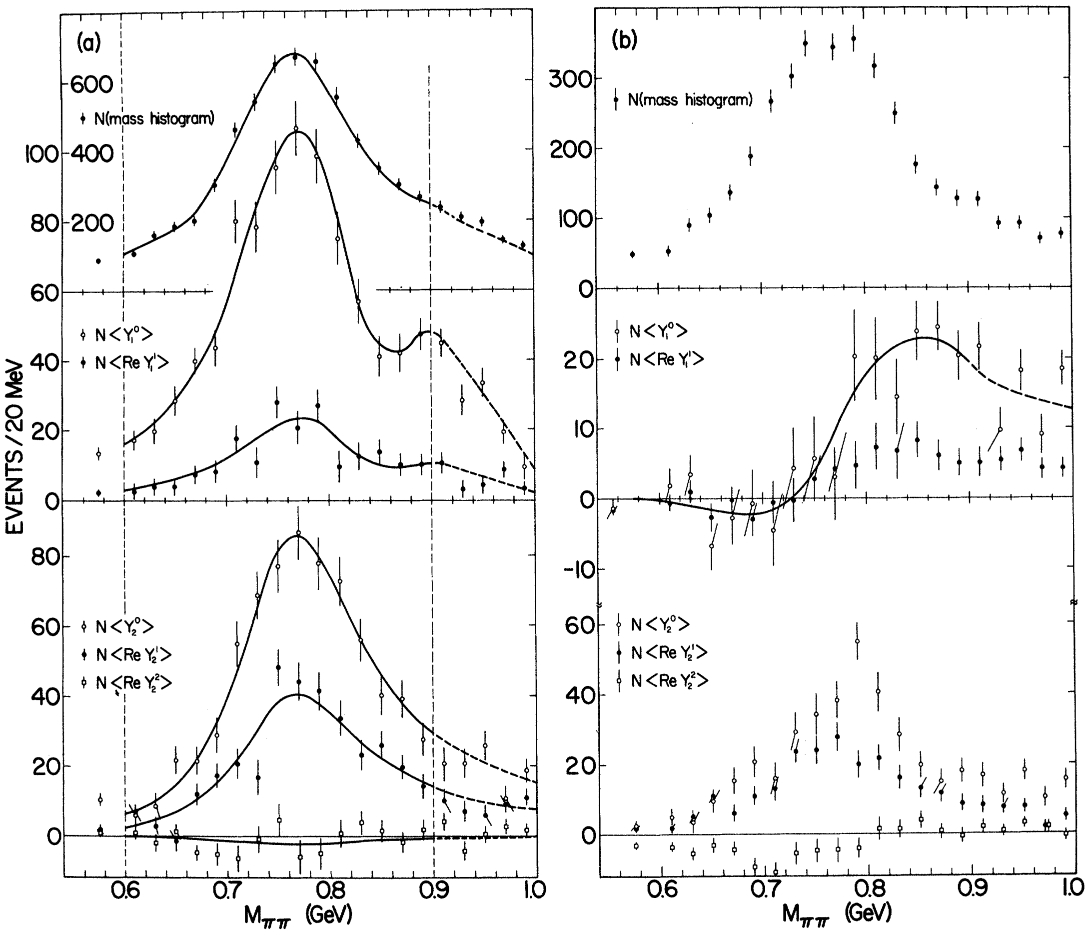

FIG. 2. Mass histograms (N) and moments $N\langle Y_l^m\rangle$ for (a) $\pi^-\pi^+n$ and (b) $\pi^-\pi^0p$ data with $\cos\theta_{\text{c.m.}} > 0.9$. Only the moments $N\langle Y_1^0\rangle$ for $\pi^-\pi^0p$ were used in the fits. The curves are calculated from the "Up-Up" solution; for the three moments $N\langle Y_2^m\rangle$, curves are those of a single Breit-Wigner function and differ only by multiplicative factors. The curves on N and $N\langle Y_1^m\rangle$ have been drawn smoothly to remove structure due to the fluctuations in δ_S seen in Fig. 3(d). Above 900 MeV the curves are drawn dashed to reflect the fact that the over-all fit is poor in this region.

these tests to the $\pi^+\pi^-n$ data in Fig. 2(a), we find acceptable χ^2 confidence levels of 55 and 7% for the $l=1$ and $l=2$ tests, respectively. For the $\pi^-\pi^0p$ data, we find 98 and <0.01%, respectively; excluding the $N\langle\text{Re}Y_2^2\rangle$ data from the $l=2$ test improves the fit to CL = 2%. This aspect of the $\pi^-\pi^0p$ data suggests the presence of two exchange components (π and ω) which contribute differently to the $m=0$ and $m=\pm1$ ρ helicity states.

For purposes of fitting Eqs. (3) of I to the data, the $\pi^+\pi^-n$ events were divided into three regions of $\cos\theta_{\text{c.m.}}$ containing approximately equal numbers of events (see Table II). There are $3\times20\times6=360$ independent $N\langle Y_l^m\rangle$ data points between 600 and 1000 MeV, in addition to the 20 moments[7] $N\langle Y_1^0\rangle$ for the $\pi^-\pi^0p$ events in Fig. 2(b). The $\pi^+\pi^-$ elastic-scattering amplitudes are assumed to have the form[8] $A_{\pi^+\pi^-}{}^S=\frac{1}{3}A(\delta_S{}^0)+\frac{1}{6}A(\delta_S{}^2)$ and $A_{\pi^+\pi^-}{}^P=\frac{1}{2}A(\delta_p)$, where $A(\delta_l{}^T)=\exp(i\delta_l{}^T)\sin\delta_l{}^T$. $\delta_S{}^0$, $\delta_S{}^2$, and δ_p were assumed independent of t (fits performed independently for each of the $\cos\theta_{\text{c.m.}}$ regions yielded phase shifts which were compatible with one another; thus the present data do not require a t dependence of these phase shifts). $\delta_S{}^0$ was assumed unknown at each of the 20 $m_{\pi\pi}$ intervals. $\delta_S{}^2$ was assumed unknown at seven different $m_{\pi\pi}$ values. δ_p was given by a p-wave Breit-Wigner amplitude[9] with $\cot\delta_p=[m_\rho{}^2-m_{\pi\pi}{}^2][1+(q/q_\rho)^2]/[2m_\rho\Gamma_\rho(q/q_\rho)^3]$, where q_ρ and q are the $\pi\pi$ c.m. decay momenta for $\pi\pi$ systems of mass m_ρ and $m_{\pi\pi}$, respectively, and m_ρ and Γ_ρ are variables in the fit. The six indepen-

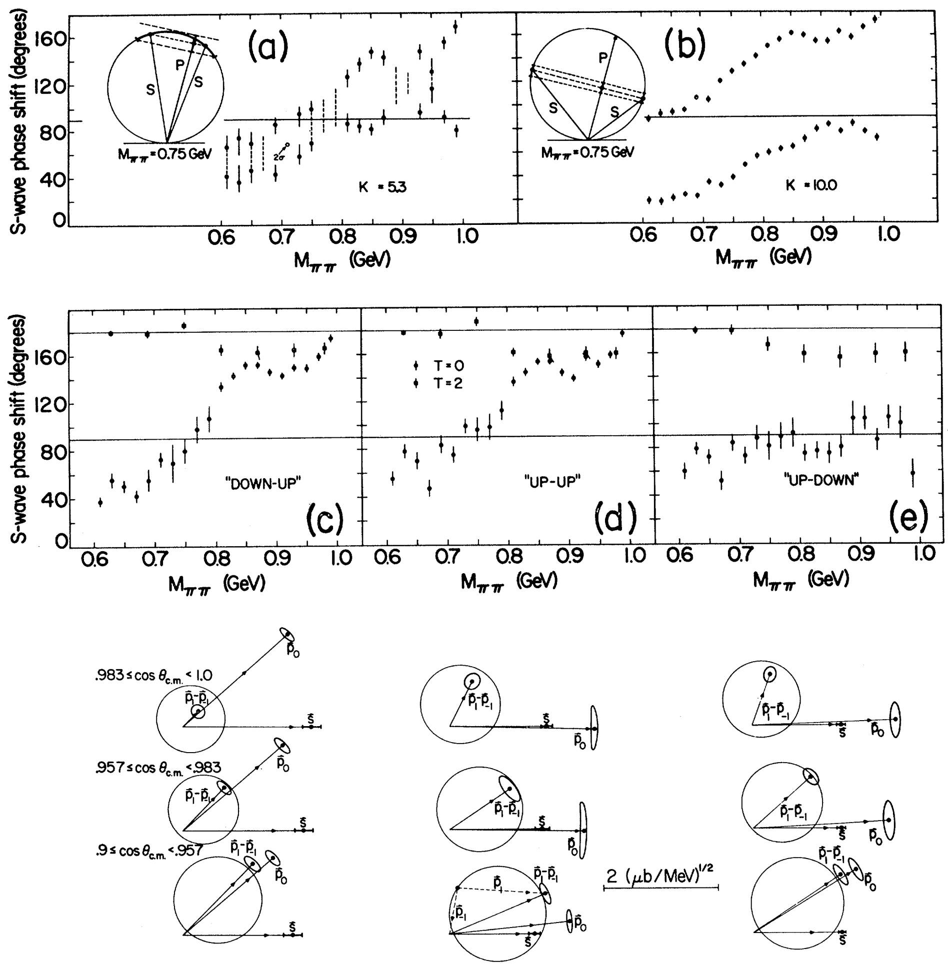

FIG. 3. (a), (b) Schematic diagrams showing how ambiguities in $\delta_S{}^0$ arise. δ_S is calculated from the moments $N\langle Y_1{}^0\rangle$ of Fig. 2(a) for the two indicated trial values of K in $N\langle Y_1{}^0\rangle = K\,\mathrm{Re}(A_{\pi\pi}{}^S A_{\pi\pi}{}^{P*})$. The unitarity circles show the projections of the complex S vectors on P at $m_{\pi\pi}=750$ MeV. For $K=5.3$ there exists poor separation between the two δ_S values at many $m_{\pi\pi}$ values, as demonstrated by the dotted lines. At 710 MeV the projection on P is 2σ outside the circle. (c)-(e) $\delta_S{}^0$, $\delta_S{}^2$, and the helicity-amplitude vectors for the solutions discussed in the text.

dent helicity-amplitude quantities discussed in I, namely $|\vec{s}|$, $|\vec{p}_0|$, $\theta_{\vec{p}_0,\vec{s}}$, $|\vec{p}_1-\vec{p}_{-1}|$, $\theta_{\vec{p}_1-\vec{p}_{-1},\vec{s}}$, and $(|\vec{p}_1|^2+|\vec{p}_{-1}|^2)$, were assumed unknown in each of the three $\cos\theta_{\mathrm{c.m.}}$ regions, yielding 18 additional free parameters. $\{\vec{s}\cdot\vec{p}_0\}_{\pi^-\pi^0 p}$ was also a free parameter.

To demonstrate the nature of the $\delta_S{}^0$ ambiguities expected in the fits, we show in Figs. 3(a) and 3(b) the values of $\delta_S{}^0$ (we ignore $\delta_S{}^2$ for this purpose) obtained from the $N\langle Y_1{}^0\rangle$ data of Fig. 2(a) for two assumed trial values of the multiplicative helicity amplitude factor (call it K) in Eq. (3b) of I. The value $K=5.3$ used in Fig. 3(a) is close to the preferred value in the actual fits. We refer to the set of larger δ_S values for all $m_{\pi\pi}$ as the "Up-Up" branch and the set of smaller δ_S values as the "Down-Down" branch; the proximity of the two branches for small K also yields in this case the cross-over branches "Up-Down" and "Down-Up" as possible solutions.

Starting values for the χ^2-minimization search program[10] were chosen to correspond to the possible classes of solutions illustrated in Figs. 3(a) and 3(b) for a large range of K. These choices excluded rapid changes of $\delta_S{}^0$ with $m_{\pi\pi}$ which would result from jumping back and forth between the branches. Three of the four possible branches are obtained as convergence points (the "Down-Down" solution is never found). The resulting fits are summarized in Table I and the phase shifts and helicity-amplitude

Table I. Phase-shift solutions.

		$m_{\pi\pi}$ dependence of $\lvert\vec{s}\rvert^2$, $\lvert\vec{p}_i\rvert^2$						
Solution	$m_{\pi\pi}$ range (GeV)	Form[a]	Change, 0.6-1.0 GeV (%)	χ^2	Constraints	Confidence level (%)	m_ρ (MeV)	Γ_ρ (MeV)
"Down-Up"	0.6 - 0.9	constant	...	240	244	56	767±3	152±7
	0.6 - 1.0	constant	...	378	332	4	767±3	150±7
		$1-0.2\Delta-0.1\Delta^2$	-8	378	330	3	769±5	149±8
"Up-Up"	0.6 - 0.9	constant[b]	...	232	244	70	767±2	149±5
	0.6 - 1.0	constant	...	372	332	6	766±2	153±5
		$1-0.3\Delta-\Delta^2$	-16	370	330	6	769±3	157±7
"Up-Down"	0.6 - 0.9	constant	...	258	244	26	762±2	139±6
	0.6 - 1.0	constant	...	441	332	<0.01	761±2	136±6
		$1-\Delta-2.6\Delta^2$	-47	389	330	1	771±4	153±7

[a] $\Delta = m_{\pi\pi} - 0.75$ GeV.

[b] Accepting the validity of the "Up-Up" solution, the parameters of this fit should be the most reliable determination.

vectors for the best confidence limit fits for 600-1000 MeV shown in Figs. 3(c)-3(e) (confidence limits 4, 6, and 1%, respectively). It is important to note, however, that for all three solutions, neither the δ_S values nor the vectors $\vec{p}_i$, $\vec{s}$ obtained in the 600- to 900-MeV fits are significantly altered when the 900- to 1000-MeV data are added to the fit. However, the confidence limits decrease considerably, indicating that the formalism may be showing signs of breaking down when the fits are extended over the full 600- to 1000-MeV range. The "Down-Up" and "Up-Up" solutions are found to be extremely insensitive to the inclusion of the quadratic $m_{\pi\pi}$ dependence of $|\vec{s}|^2$ and $|\vec{p}_i|^2$ (shown in Table I) in the fit, whereas the confidence limit for the "Up-Down" solution improves from <0.01 to 1%; neither the phase-shifts nor the helicity-amplitude vectors change significantly in this process, however.

As shown in Figs. 3(c)-3(e), the three solutions have in common that $\delta_S^{\,0} \sim 90°$ in the region $m_{\pi\pi} \sim 750$ MeV, a result which is thus independent of the following discussion. The "Down-Up" and "Up-Up" solutions differ essentially in their $\delta_S^{\,0}$ values for $m_{\pi\pi} < 750$ MeV and in their helicity amplitudes. The smaller average contribution to the mass spectrum of the "Down-Up" solution in this $m_{\pi\pi}$ range causes $|\vec{s}|$ to be larger, resulting in $\theta_{p_0,s} \sim 45°$ in order to retain approximately the same value for $\vec{p}_0 \cdot \vec{s}$. Since the absorption model predicts the ratio of nucleon helicity-flip/nonflip amplitudes to be the same for production of a zero-helicity $\pi\pi$ system of any l wave,[11] the "Up-Up" solution is preferred over the "Down-Up" solution on this basis.

The "Up-Up" and "Up-Down" solutions are nearly identical for $m_{\pi\pi} \lesssim 790$ MeV. Above this energy $\delta_S^{\,0}$ increases fairly rapidly for the former solution while for the latter solution it remains near 90° up to 1000 MeV. Since for $\delta_S^{\,0}$ slowly varying, the results of a fit are most sensitive to false assumptions concerning the $m_{\pi\pi}$ dependence of $|\vec{p}_i|^2$ and $|\vec{s}|^2$, it may be unreliable to rule out the "Up-Down" solution either on the basis of the large $\theta_{p_0,s}$ in Fig. 3(e) or because of the seemingly excessive fall-off of the $m_{\pi\pi}$ dependence of $|\vec{p}_i|^2$ and $|\vec{s}|^2$ shown in Table I for this solution. However, a relatively constant s-wave $\pi\pi$ cross section from 600 to 1000 MeV is not compatible with the experimental results of Corbett et al.,[12] Strugalski et al.,[13] and Wahlig et al.,[14] who present $\pi^0\pi^0$ mass spectra all of which show a significant drop-off in this region (although Wahlig et al. point out that uncertain background contributions may be in part responsible for their results).

For the preferred "Up-Up" solution, $\delta_S^{\,0}$ passes through 90° at $m_{\pi\pi} \sim 730$ MeV implying the existence of a scalar meson $\sigma(730)$. The "Up-Up" solution for $600 < m_{\pi\pi} < 900$ MeV is not well fitted by a Breit-Wigner distribution, however, but requires $\cot\delta_S^0$ to be a more complex function of $m_{\pi\pi}$ with $d\cot\delta/dm_{\pi\pi}$ increasing as δ_S^0 passes through 90° (the slope at the 90° point corresponds to $\Gamma \sim 150$ MeV). This fact, our large value (~60°) of δ_S^0 at 600 MeV, the value of $\delta_S^0 - \delta_S^2 = \pm(66 \pm 13)°$ at the K-meson mass deduced from $K \to 2\pi$ decay,[15,16] plus the absence of structure observed in $\pi\pi$ mass spectra between 400 and 600 MeV together suggest that δ_S^0 is large (~60°–90° or so) throughout this region, perhaps being due to the simultaneous existence of the $\sigma(\sim400)$ meson.

The measured helicity amplitude quantities are shown in Table II for the preferred "Up-Up" solution, normalized to unit $\cos\theta_{\text{c.m.}}$ interval, such that

$$d^2\sigma/dm_{\pi\pi}d\cos\theta_{\text{c.m.}} = |A_{\pi\pi}^S|^2\{|\vec{s}|^2\} + A_{\pi\pi}^P|^2\{\textstyle\sum_i|\vec{p}_i|^2\}\ \mu b/\text{MeV}. \quad (3)$$

In evaluating these numbers we use an approximate track length for the event sample of 5.3 events/μb (±10% possible systematic uncertainty). Thus the $|\vec{s}|^2$ values of Table II can be used to predict $d^2\sigma/dm_{\pi\pi}d\cos\theta_{\text{c.m.}} = (|\vec{s}|^2/18) \times |\sin\delta_S^0\exp(i\delta_S^2) - \sin\delta_S^2\exp(i\delta_S^2)|^2$ μb/MeV for the reaction $\pi^-p \to \pi^0\pi^0 n$ near ~2.7 GeV/c, the average π^- beam momentum for our sample.

As discussed in I, the quantities in columns 4-7 of Table II may be compared with the ratios 1:3:$\sqrt{3}$:3 expected for a real particle plane-wave initial state scattering experiment. The entries in columns 4-6 are seen to agree fairly well with these ratios, which is remarkable in view of the possible distortions due to absorption and the virtual nature of the exchanged pion. The similarity between the δ_S^0 solutions presented here and those of earlier analyses[5] which ignored the effects of absorption must be related to this agreement. $|\vec{p}_0| - \frac{1}{2}(|\vec{p}_1|^2 + |\vec{p}_{-1}|^2)$ is most strongly affected by the absorption; the last column shows how the intensity ratio of ρ helicity states $(m = \pm1)/(m = 0)$ varies with momentum transfer. A $\pi\pi$ phase-shift analysis[17] using the reaction $\pi^+p \to \pi^+\pi^-N^{*++}$ yields the same three δ_S^0 solutions presented in this paper and therefore lends great support to the assumption that we are actually measuring properties of the $\pi\pi$ system rather than properties of the reaction as a whole.

We wish to express our appreciation to the groups who permitted us to use their data in this analysis and particularly to L. Jacobs, J. Kirz, and D. Miller (Berkeley); W. Selove (Pennsylvania-Saclay); F. Loeffler, D. Miller, and G. Tautfest (Purdue); and W. A. Cooper (Argonne-Toronto-Wisconsin) for their help.

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[1] Peter E. Schlein, preceding Letter [Phys. Rev. Letters 19, 1052 (1967)].

[2] The following laboratories and collaborations have generously contributed their data to this analysis: Argonne-Toronto-Wisconsin [D. R. Clear et al., Nuovo Cimento 49A, 399 (1967)], Pennsylvania-Saclay [V. Hagopian et al., Phys. Rev. 145, 1128 (1966); V. Hagopian and Y. Pan, Phys. Rev. 152, 1183 (1966)], Purdue [D. H. Miller et al., Phys. Rev. 153, 1423 (1967)], and Lawrence Radiation Laboratory [L. Jacobs, University of California Radiation Laboratory Report No. UCRL-16877, 1966 (unpublished)]. The average beam momentum for the entire sample is 2.69 GeV/c.

[3] The y axis is taken, as usual, to lie along the normal to the production plane $\hat{n} \sim \hat{\pi}_{\text{in}}^- \times \vec{e}_z$.

[4] The z-direction cosine of $\hat{\pi}_{\text{out}}^-$ in the coordinate system used is identical to the "longitudinal decay co-

Table II. Helicity quantities for the "Up-Up" solution.

$\cos\theta_{\text{c.m.}}$ interval	Approximate t interval (GeV^2)	No. of events 600-1000 MeV	(600-900 MeV)	Coefficients in $d^4\sigma/dm_{\pi\pi}d\cos\theta_{cm}d\Omega_{\hat{\pi}^-_{out}}$ (μb/MeV): $\lvert\vec{s}\rvert^2$	$\Sigma\lvert\vec{p}_i\rvert^2$	$\vec{p}_o\cdot\vec{s}$	$\lvert\vec{p}_o\rvert^2 - \frac{1}{2}(\lvert\vec{p}_1\rvert^2+\lvert\vec{p}_{-1}\rvert^2)$	$\dfrac{\lvert\vec{p}_1\rvert^2+\lvert\vec{p}_{-1}\rvert^2}{\lvert\vec{p}_o\rvert^2}$
0.900-0.957	0.090-0.175	2225	(1894)	38± 5	132± 4	54± 4	51± 4	0.69±0.05
0.957-0.983	0.045-0.090	2224	(1940)	101±11	287±11	146± 9	174±10	0.35±0.03
0.983-1.000	0.010-0.045	2291	(2003)	166±21	448±18	246±16	324±16	0.23±0.03
0.900-1.000	0.010-0.175	6740	(5837)	76± 5	226± 5	110± 4	129± 4	0.40±0.03

sine" which scales linearly along a line of constant $\pi\pi$ mass on a Dalitz plot. Background due to crossing N^* bands will appear as localized distortions of the distribution due to $\pi\pi$ scattering alone. This situation becomes aggravated when several discrete beam momenta are used and can lead to the appearance of higher order moments.

[5]See Ref. 1 of the preceding Letter. L. Jacobs has a more complete summary of work on this subject. See also A. Rosenfeld et al., Rev. Mod. Phys. 39, 1 (1967).

[6]In the evaluation of these moments, we use the formula

$$N\langle Y_l^m\rangle = \sum_{i=1}^{N} Y_l^m(\hat{\pi}_{\text{out}}^-)_i$$

and $\delta[N\langle Y_l^m\rangle] = \{[\sum (Y_l^m)^2 - N\langle Y_l^m\rangle^2][1+(2/\pi N)^{1/2}]\}^{1/2}$. We thank Derek Hudson for valuable discussions concerning this error expression. In evaluating the moments we find no evidence for the presence of any illegal moments $\langle \mathrm{Im}\, Y_l^m\rangle$.

[7]The decision to use $N\langle Y_1^0\rangle$ and not $N\langle \mathrm{Re}Y_1^1\rangle$ for this purpose was based on the assumption that ω exchange would contribute less to $m=0$ helicity states of the ρ than to the $m=\pm 1$ states. The observed good compatibility between the moments $\langle Y_1^0\rangle$ and $\langle \mathrm{Re}Y_1^1\rangle$ for $\pi^-\pi^0 p$ somewhat obscures the wisdom of this choice, however.

[8]See Refs. 4 and 5 of the preceding Letter.

[9]J. D. Jackson, Nuovo Cimento 34, 1644 (1964).

[10]Note that in constructing the χ^2, the strongly correlated quantities $N\langle Y_l^m\rangle$ are not used, but rather the quantities N and $\langle Y_l^m\rangle$. The searches were performed using the Berkeley program MINFUN; see W. E. Humphrey, Alverez Group Programmers Note No. P-6, 1962 (unpublished).

[11]We thank E. Abers and M. Parkinson for helpful discussions concerning this point.

[12]I. F. Corbett, C. J. S. Damerell, N. Middlemas, D. Newton, A. B. Clegg, W. S. C. Williams, and A. S. Carroll, Phys. Rev. 156, 1451 (1967).

[13]Z. S. Strugalski, I. V. Chuvilo, I. A. Ivanovska, L. S. Okhrimenko, B. Niczyporuk, I. Kanarek, B. Stowinski, and Z. Jabłonski; quoted by G. Goldhaber in Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966 (University of California Press, Berkeley, California, 1967), p. 108.

[14]M. Wahlig, E. Shibata, D. Gordon, D. Frisch, and I. Mannelli, Phys. Rev. 147, 941 (1966).

[15]T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. 16, 530 (1966).

[16]$\delta_S{}^2$ is determined to be small throughout the region considered in our analysis because of the characteristic interference with the ρ shown in Fig. 2(b) for $\pi^-\pi^0 p$, although it should be commented that the fits are rather insensitive to its inclusion; the absence of any explicit constraint on $\{\vec{s}\cdot\vec{p}_0\}_{\pi^-\pi^0 p}$ in the fits implies that the quoted errors on $\delta_S{}^2$ in Fig. 3 may be somewhat underestimated.

[17]E. Malamud, P. E. Schlein, T. G. Trippe, D. Brown, and G. Gidal, in Proceedings of the International Conference on Elementary Particles, Heidelberg, Germany, 1967 (to be published).

IS THE POMERANCHON A FIXED POLE?

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It is shown that the Mandelstam cut mechanism, which allows the existence of fixed poles at negative values of angular momentum, is not sufficient to allow the Pomeranchon to have zero slope. It is suggested that this fact makes it unlikely that the Pomeranchon is a fixed pole.

The Pomeranchuk trajectory has had a rather interesting history. Some time ago it was thought to be a trajectory much like any other, giving rise to the f^0 when it went through spin 2, and having a slope similar to the other trajectories.[1] More recently, the observed nonshrinkage of diffraction peaks has indicated that the Pomeranchon has an anomalously small slope, so that at present it is the only trajectory generally accepted by Regge phenomenologists which has no particles assigned to it. It is understood that, in the absence of any cuts in the angular-momentum plane, no trajectory can be flat (i.e., a fixed pole); however, the realization that cuts can and probably do allow flat trajectories at negative values of l has led to speculation that the Pomeranchon is also flat. This possibility has been suggested in a recent paper by Oehme,[2] who pointed out that it would provide a simple way to construct a model having both nonshrinking diffraction peaks and asymptotically constant cross sections. It would also eliminate the unpleasant feature, present if the Pomeranchon is not flat, of the amplitude having an infinite number of branch points, corresponding to the exchange of all numbers of Pomeranchuk poles, converging at $J=1$ in the forward direc-