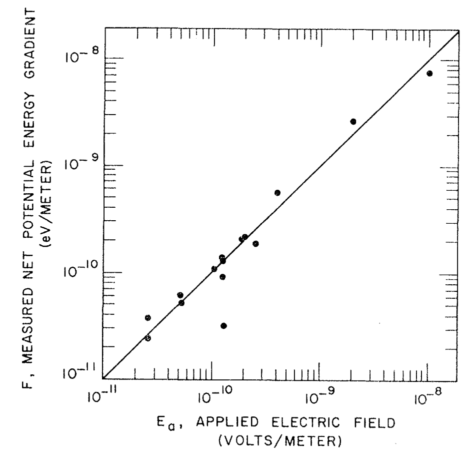

FIG. 2. Experimentally measured potential-energy gradients F versus applied electric fields E_a. The solid line represents $F=eE_a$ for a particle having the inertial mass of the electron.

agrees with the theoretical calculation of Schiff and Barnhill.[12] We conclude that the force of gravity on electrons inside a metal is the same as that on electrons in a vacuum.

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METHOD FOR $\pi\pi$ OR $K\pi$ PHASE-SHIFT ANALYSIS*

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Based on the assumed dominance of one-pion exchange in the reaction

$$\pi^- + p \to \pi^- + \pi^+ + n, \tag{1}$$

several analyses have been reported[1] in which the $T=0$ s-wave $\pi\pi$ elastic-scattering phase shift ($\delta_s{}^0$) was obtained over a range of $\pi\pi$ mass in the ρ region. These analyses generally fall into two classes: (i) those in which the effects of absorption are taken account of incompletely (or not at all), and (ii) those in which the analyses depend on the detailed validity of a theoretical treatment of the absorption. It is the purpose here to point out that the $\pi\pi$ phase shifts may be extracted from the data (at least in the region of the ρ resonance) without complete prior knowledge of the helicity amplitudes in Reaction (1), using therefore only a subset of the assumptions used in the absorption model,[2] all of which have observable consequences and which may be subjected to test. Furthermore, it is shown below that for data in a sufficiently narrow band of total center-of-mass energy E^* and production angle β (or momentum transfer t to the nucleon) in Reaction (1), empirical values of the helicity amplitudes may be extracted from the data. The arguments contained herein should apply equally to the reaction $\pi^+p \to \pi^+\pi^-N^{*++}$ and to the determination of the $K\pi$ phase shifts [at least in the $K^*(890)$ region] if the π-exchange dominated reaction $K^+p \to K^+\pi^-N^{*++}$ is used.

The fundamental assumption common to the one-pion-exchange models with and without absorption is that the amplitude, to reach the final state in (1) with given $\pi\pi$ relative orbital angular momentum l, can be factored into two parts, one of which is the amplitude for l-wave $\pi\pi$ scattering[3] $A_{\pi\pi}{}^{l} \sim \exp(i\delta_l)\sin\delta_l$. Thus we write $A_{\pi\pi}{}^{l} M_{l\mu}{}^{\lambda\lambda'}$ as the amplitude[4] to reach a final state containing a $\pi\pi$ system with internal angular momentum l and helicity μ and a nucleon with helicity λ from an initial state with nucleon helicity λ'. The helicity amplitudes $M_{l\mu}{}^{\lambda\lambda'}$ are in general functions of E^*, t, and the effective mass $m_{\pi\pi}$ of the $\pi\pi$ system. In the case of the one-pion-exchange Born amplitude without absorption, the $M_{l\mu}{}^{\lambda\lambda'}$ are well-defined, relatively real functions of these variables. The absorption model consists essentially of the detailed prescription for modifying the $M_{l\mu}{}^{\lambda\lambda'}$ from these well-known Born amplitudes. In the analysis proposed here, however, we consider the $M_{l\mu}{}^{\lambda\lambda'}$ as unknown parameters to be determined in fitting the data. We make only the tentative assumption (as in the absorption model) that their relative phases are not altered by the absorption; this point is discussed further below. The final-state $\pi\pi$ angular distribution in its rest frame can be written as

$$D(\hat{\pi}_{\text{out}}) = \sum_{\lambda'\lambda} \Big|\sum_{l} A_{\pi\pi}{}^{l} \sum_{\mu} M_{l\mu}{}^{\lambda\lambda'} Y_l{}^{\mu}(\hat{\pi}_{\text{out}})\Big|^2 dt\, dm_{\pi\pi}\, d\Omega_{\hat{\pi}_{\text{out}}} \tag{2}$$

in which we assume production from an unpolarized target and sum over the final-state nucleon variable, and where the argument of the spherical harmonic $Y_l{}^{\mu}(\hat{\pi}_{\text{out}})$ is the outgoing π^- unit vector expressed in the $\pi\pi$ rest frame. Selecting a coordinate system in the $\pi\pi$ rest frame such that the $\hat{y}$ axis is along the normal to the production plane $\hat{n} \sim \hat{\pi}_{\text{in}} \times \vec{e}_z$, where $\vec{e}_z$ is a unit vector along the z axis which is taken to be the direction of motion of the $\pi\pi$ system, the consequence of parity conservation on the helicity amplitudes is the relation $M_{l,-\mu}{}^{-\lambda,-\lambda'} = (-1)^{\mu+\lambda+\lambda'} M_{l\mu}{}^{\lambda\lambda'}$, valid for any l.[5] This reduces the number of independent helicity amplitudes for each l from $4(2l+1)$ to $2(2l+1)$.

The $l=0$ and 1 expansion of Eq. (2), appropriate to the experimental data[1] in the ρ region, can be simply expressed by considering the forms of the measurable $Y_L{}^M$ moments of $D(\hat{\pi}_{\text{out}})$. In writing these out, we consider the nucleon helicity-nonflip and -flip amplitudes for a given $\pi\pi$ helicity as the x and y components of a vector. Thus: $(M_{11}{}^{++}, M_{11}{}^{-+}) \equiv \vec{p}_1$, $(M_{1-1}{}^{++}, M_{1-1}{}^{-+}) \equiv \vec{p}_{-1}$, $(M_{10}{}^{++}, M_{10}{}^{-+}) \equiv \vec{p}_0$, and $(M_{00}{}^{++}, M_{00}{}^{-+}) \equiv \vec{s}$. With this notation the measured moments for data in the jth $\pi\pi$ mass bin have the forms

$$N = |A_{\pi\pi}{}^{P}|^2\{|\vec{p}_1|^2 + |\vec{p}_0|^2 + |\vec{p}_{-1}|^2\} + |A_{\pi\pi}{}^{S}|^2\{|\vec{s}|^2\}, \tag{3a}$$

$$N\langle Y_1{}^0\rangle = 2(4\pi)^{-\frac{1}{2}}\,\mathrm{Re}(A_{\pi\pi}{}^{S} A_{\pi\pi}{}^{P*})\{\vec{p}_0\cdot\vec{s}\}, \tag{3b}$$

$$N\langle \mathrm{Re}\,Y_1{}^1\rangle = (4\pi)^{-\frac{1}{2}}\,\mathrm{Re}(A_{\pi\pi}{}^{S} A_{\pi\pi}{}^{P*})\{\vec{s}\cdot(\vec{p}_1 - \vec{p}_{-1})\}, \tag{3c}$$

$$N\langle Y_2{}^0\rangle = 2(20\pi)^{-\frac{1}{2}}|A_{\pi\pi}{}^{P}|^2\{|\vec{p}_0|^2 - \tfrac{1}{2}(|\vec{p}_1|^2 + |\vec{p}_{-1}|^2)\}, \tag{3d}$$

$$N\langle \mathrm{Re}\,Y_2{}^1\rangle = (3/20\pi)^{\frac{1}{2}}|A_{\pi\pi}{}^{P}|^2\{\vec{p}_0\cdot(\vec{p}_1 - \vec{p}_{-1})\}, \tag{3e}$$

$$N\langle \mathrm{Re}\,Y_2{}^2\rangle = (6/20\pi)^{\frac{1}{2}}|A_{\pi\pi}{}^{P}|^2\{-\vec{p}_1\cdot\vec{p}_{-1}\}, \tag{3f}$$

where N is the intensity ($\pi\pi$ mass spectrum), and where we have suppressed the E^* and t dependence of the $\vec{p}_i$ and $\vec{s}$ vectors. Equations (3) are generalizations of the quantities[6] A_l used in conventional two-body scattering experiments and can be obtained experimentally for a discrete sample of events by evaluating[7]

$$N_j\langle Y_L^M\rangle_j = \sum_{i=1}^{N_j} Y_L^M(\theta_i, \varphi_i),$$

$$\delta(N_j\langle Y_L^M\rangle_j) \simeq N_j\delta\langle Y_L^M\rangle_j$$

$$= [\sum (Y_L^M)^2 - N_j\langle Y_L^M\rangle_j^2]^{\frac{1}{2}}, \qquad (4)$$

for the N_j events in the jth $m_{\pi\pi}$ bin. If we were performing a real-particle $\pi\pi$ scattering experiment describable by a plane-wave initial state e^{ikz}, the bracketed $\{\,\}$ coefficients of $|A_{\pi\pi}^S|^2$ and $|A_{\pi\pi}^P|^2$ in Eq. (3a) and of $\mathrm{Re}(A_{\pi\pi}^S A_{\pi\pi}^{P*})$ and $|A_{\pi\pi}^P|^2$ in Eqs. (3b) and (3d) occur in the ratios of $1{:}3{:}\sqrt{3}{:}3$, respectively. In addition, $\langle Y_L^M\rangle = 0$ for $M \neq 0$. Here, however, all these coefficients are treated as unknown parameters.

Owing to the fact that the helicity amplitudes occur quadratically in the moment expressions for the quantities $N\langle Y_L^M\rangle$, averaging over E^* or t in any analysis destroys the relationships between the moments implied in the equations. Let us first consider an analysis in which such averages are made. It is expected that the helicity amplitudes depend very weakly on $m_{\pi\pi}$ compared, say, with the dependence on $m_{\pi\pi}$ of the resonant p-wave amplitude $A_{\pi\pi}^P$. Hence in the ρ region, or in general for a sufficiently small range of $m_{\pi\pi}$, helicity amplitudes may be assumed constant. (We consider below, however, how a moderate energy dependence may be allowed for and included in an analysis.) With this assumption, the entire $m_{\pi\pi}$ dependences of the $N\langle Y_L^M\rangle$ reside in the $\pi\pi$ scattering amplitudes. In this case, the following conditions result on the measurable $N\langle Y_L^M\rangle$ quantities of Eqs. (3): (a) $N\langle Y_2^0\rangle$, $N\langle\mathrm{Re}(Y_2^1)\rangle$, and $N\langle\mathrm{Re}(Y_2^2)\rangle$ should have the same dependence on $m_{\pi\pi}$. (b) $N\langle Y_1^0\rangle$ and $N\langle\mathrm{Re}(Y_1^1)\rangle$ should have the same dependence on $m_{\pi\pi}$. Note that (a) must be true even if the helicity amplitudes are not relatively real[8] and therefore tests the basic factorization assumption directly. However, (b) requires, in addition to the factorization assumption, that $\vec{p}_0\cdot\vec{s}$ and $\vec{s}\cdot(\vec{p}_1-\vec{p}_{-1})$ be real. Thus, the success of (b) may be thought of as simultaneously testing both the factorization and the reality assumptions.

Agreement of experimental data with requirements (a) and (b) implies knowledge of the $m_{\pi\pi}$ dependences of $K_1|A_{\pi\pi}^P|^2$, $K_2\,\mathrm{Re}(A_{\pi\pi}^S A_{\pi\pi}^{P*})$, and $K_3|A_{\pi\pi}^P|^2+K_4|A_{\pi\pi}^S|^2$, where K_1, K_2, K_3, and K_4 are unknown constants. Selection of a trial value for K_1 determines $A_{\pi\pi}^P$ as a function of $m_{\pi\pi}$.[9] (Note that in the ρ region $K_1|A_{\pi\pi}^P|^2$ should have the dependence of a Breit-Wigner distribution without background and should provide a more reliable determination of the ρ parameters than a fit to the mass spectrum.) Subsequent selection of a trial value for K_2 then gives $A_{\pi\pi}^S$ as a function of $m_{\pi\pi}$.[10] K_1 and K_2 are determined by the requirement that some linear combination of these $|A_{\pi\pi}^P|^2$ and $|A_{\pi\pi}^S|^2$ fit the mass spectrum. In other words, for data at Q different $\pi\pi$ mass values, the determination of δ_S and δ_p at these Q energies is a $(Q-4)$-constraint problem. The resultant χ^2 probability is a test of the validity of the assumptions used. In the ρ region, an alternative fitting procedure could involve assuming a Breit-Wigner form for the ρ with E_ρ and Γ_ρ as free parameters. This would be a $(2Q-6)$-constraint fit. With the large number of constraints possible with even moderate statistics, it would also be possible to assume a second- (or even third-) order dependence of the multiplicative functions $\{\vec{p}_i, \vec{s}\}$ on $m_{\pi\pi}$. Thus, the fit could be extended over a larger range of $m_{\pi\pi}$.

Let us now consider that the experimental moments (3) have been obtained with sufficiently narrow E^* and t selections so that use may be made of the explicit functional dependence of the bracketed quantities in Eq. (3) and the helicity amplitude quantities $\vec{p}_i$ and $\vec{s}$ extracted from the data. Consider for purposes of illustration that δ_p and δ_S have been previously obtained from an analysis using E^*- and t-averaged data; in practice, however, they may also be variables in the fit along with the quantities $\vec{p}_i$ and $\vec{s}$ and any possible t dependence of δ_p and δ_S could be studied. Note that the functional form of the brackets in Eq. (3) is such that only dot products of the vectors occur. Thus the equations are invariant under rotations and reflections in the plane of the vectors and no generality is lost if we assume $\vec{s} = (|\vec{s}|, 0)$. In addition, the equations are invariant under the transformation $-\vec{p}_1 \leftrightarrow \vec{p}_{-1}$. These invariances are due to the fact that no polarizations are measured in the type of experiment discussed here. The following seven quantities may be obtained from the experimental $N\langle Y_L^M\rangle$ moments: $|\vec{s}|$, $|\vec{p}_0|$, $\vec{p}_0\cdot\vec{s}$, $|\vec{p}_1-\vec{p}_{-1}|$, $(\vec{p}_1-\vec{p}_{-1})\cdot\vec{s}$, $|(\vec{p}_1-\vec{p}_{-1})-\vec{p}_0|$, and $(|\vec{p}_1|^2+|\vec{p}_{-1}|^2)$. There are seen to be five independent variables[11] which determine the first six experimental quantities and thus one constraint exists between the experimental quantities. The quantity $|\vec{p}_1|^2+|\vec{p}_{-1}|^2$ simply requires the vectors $\vec{p}_1$ and $\vec{p}_{-1}$ to originate at an arbitrary point on a circle whose center is at the midpoint of the vector $\vec{p}_1-\vec{p}_{-1}$. Subject to the ambiguity in $\vec{p}_1$ and $\vec{p}_{-1}$ and the stated rotation and reflection invariances, the helicity amplitudes can be obtained as functions of E^* and t, thereby allowing comparison with various theoretical models.

An important test of the correctness of the method of δ_s and δ_p determinations suggested here is whether or not independent analyses at different E^* regions and in different reactions will yield the same results. In particular, the reaction $\pi^+p\to\pi^+\pi^-N^{*++}$ will be of great interest in this respect, as it is known that this reaction is dominated by ρ^0N^{*++} production via π exchange. This reaction is also of interest in connection with the possibility of utilizing the correlations[12] between the scattering angular distributions at the $\pi\pi$ vertex and the $p\pi^+$ vertex. It is straightforward to write down the joint distribution function starting from a form analogous to Eq. (2). Parity conservation allows only correlation moments of the form $N\langle \mathrm{Re}(Y_L^M)\mathrm{Re}(\mathcal{Y}_{\mathcal{L}}^{\mathfrak{M}})\rangle$ or $N\langle \mathrm{Im}(Y_L^M)\times\mathrm{Im}(\mathcal{Y}_{\mathcal{L}}^{\mathfrak{M}})\rangle$ to occur in this joint distribution function. There are 16 such independent moments all of which must have the $m_{\pi\pi}$ dependence of $|A_{\pi\pi}^P|^2$ and ten independent moments which must have the $m_{\pi\pi}$ dependence of $\mathrm{Re}(A_{\pi\pi}^S\times A_{\pi\pi}^{P*})$. These arguments apply equally to the use of the production reaction $K^+p\to K^*N^{*++}$ and the study of $K\pi$ scattering.

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[3]We consider δ_l here as an effective phase shift to be empirically determined as a function of momentum transfer t. The observed (Ref. 1) Breit-Wigner shape of the ρ peak in the $\pi\pi$ mass spectrum of Reaction (1) and its weak dependence on t (if any) argue that the use of $\sin\delta$ for the magnitude of $A_{\pi\pi}^l$ is a good approximation. See also Ref. 4. The near absence of events with 4π mass less than 1 GeV in the final state $n\pi^+\pi^+\pi^-\pi^-$ [S. U. Chung et al., Lawrence Radiation Laboratory Report No. UCRL-16881 (revised), 1967 (unpublished)] supports the assumption of no inelasticity in the ρ region.

[4]Note here that any multiplicative off-mass-shell effects on $A_{\pi\pi}^l$ as, for example, the $(q_{\mathrm{off}}/q_{\mathrm{on}})^l$ factor suggested by F. Selleri [Phys. Letters 3, 76 (1962)] would be included in our definition of the $M_{l\mu}^{\lambda\lambda'}$.

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[6]The A_l's are defined by the expansion $d\sigma/d\Omega=\lambda^2\sum_l A_lP_l$.

[7]φ is the conventional Treiman-Yang angle and θ is the $\pi\pi$ rest frame scattering angle using the helicity direction and not the direction as the z axis. For discussions of the validity of these moment expressions, see P. E. Schlein et al., Phys. Rev. Letters 11, 167 (1963), and P. M. Dauber, thesis, University of California, Los Angeles, 1966 (unpublished).

[8]If the $\vec{p}_i$ and $\vec{s}$ are not real, then the moments $N\langle Y_2^M\rangle$ and $N\langle Y_1^M\rangle$ in Eq. (3) have the forms $\sim|A_{\pi\pi}^P|^2|\{\}|^2$ and $\sim\mathrm{Re}(A_{\pi\pi}^SA_{\pi\pi}^{P*})\,\mathrm{Re}\{\}+\mathrm{Im}(A_{\pi\pi}^SA_{\pi\pi}^{P*})\,\mathrm{Im}\{\}$, respectively. In terms of the analysis described below, this means that if averages are not made over E^* and t, the moments $N\langle Y_1^M\rangle$ can be expressed as $\sim\sin\delta_p\sin\delta_s\times|\{\,\}|\cos[\delta_p-(\delta_s+K)]$ and δ_s can still be determined to within the unknown additive constant K, which is the phase of $\{\,\}$.

[9]If the p wave is resonant, as it is in both the $\pi\pi$ and $K\pi$ interactions, the association of the peak in $N\langle Y_2^M\rangle$ with $\delta_p=90°$ determines K_1, and the Wigner condition of counter-clockwise rotation with increasing energy serves to uniquely determine δ_p at each $m_{\pi\pi}$ value.

[10]Actually, knowledge of $\mathrm{Re}(A_{\pi\pi}^SA_{\pi\pi}^{P*})$ and $A_{\pi\pi}^P$ yields, in general, two such solutions for $A_{\pi\pi}^S$. This twofold ambiguity, as well as the determination of K_2, should be resolved in the fit to the mass spectrum.

[11]These may be taken as $|\vec{s}|$, $|\vec{p}_0|$, $\theta_{s,\,p_0}$ (the angle between $\vec{s}$ and $\vec{p}_0$), $|\vec{p}_1-\vec{p}_{-1}|$, and $\theta_{p_1-p_{-1},\,s}$ (the angle between $\vec{s}$ and $\vec{p}_1-\vec{p}_{-1}$).

[12]See H. Pilkuhn and B. E. Y. Svensson, Nuovo Cimento 38, 518 (1965), who include references to earlier work. The effects of absorption on correlation moments are discussed by B. E. Y. Svensson, Nuovo Cimento 39, 667 (1965); J. T. Donohue, thesis, University of Illinois, 1967 (unpublished); and J. D. Jackson et al., Phys. Rev. 139, B428 (1965).

$\pi\pi$ PHASE-SHIFT ANALYSIS FROM 600 TO 1000 MeV*

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A method[1] has been proposed (referred to hereafter as I) for extracting the $\pi\pi$ elastic scattering phase shifts from data on $\pi N \to \pi\pi N$. It is shown in I that complete prior knowledge of the helicity amplitudes is not necessary in the analysis. Although some of these become additional free parameters in fitting the data, a large number of constraints remain which test the validity of the model. We present here an analysis of this type for $\pi\pi$ effective mass $0.6 < m_{\pi\pi} < 1.0$ GeV and $\cos\theta_{\rm c.m.} > 0.9$ (nucleon momentum transfer $t \lesssim 0.175$ GeV^2), using a sample of data with beam momenta 2.1-3.2 GeV/c compiled from several laboratories[2]:

$$\pi^- + p \to \pi^- + \pi^+ + n \text{ (6740 events)}, \tag{1}$$

$$\pi^- + p \to \pi^- + \pi^0 + p \text{ (3656 events)}, \tag{2}$$

where the numbers of events are those remaining after the $m_{\pi\pi}$ and $\theta_{\rm c.m.}$ selection. The detailed analysis, described below, is concerned mainly with Reaction (1), with Reaction (2) used to obtain independent information on the $T=2$ s-wave interaction. Aside from demonstrating that the data satisfy well the tests suggested in I, the $T=0$ s-wave phase shift (δ_s^0) is shown to increase from ~60° to ~90° in the range $600 < m_{\pi\pi} \lesssim 730$. For 730 MeV $< m_{\pi\pi}$, δ_s^0 most likely continues to increase, implying the existence of a $T=0$ scalar meson $\sigma(730)$.

We show in Fig. 1 the spherical harmonic moments $\langle Y_l^0\rangle$ of the $\pi_{\rm out}^-$ angular distribution in the $\pi\pi$ rest frame of Reaction (1) for $l \le 10$. As explained in I, the coordinate system used has its z axis along the direction of motion of the $\pi\pi$ system for reasons of simplifying the extraction of the helicity amplitudes in the subsequent analysis.[3] For both Reactions (1) and (2) (similar to Fig. 1, but not shown), small but significant (negative) moments exist for

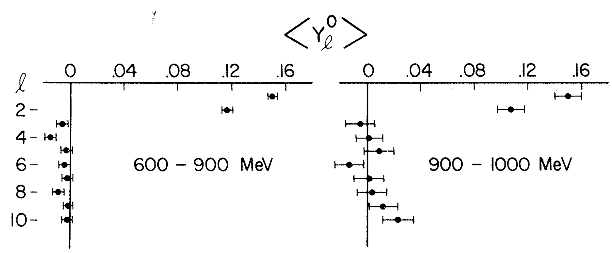

FIG. 1. Moments $\langle Y_l^0\rangle$ of the outgoing π^- in the $\pi\pi$ rest frame of $\pi^- p \to \pi^-\pi^+ n$ with $\cos\theta_{\rm c.m.} > 0.9$. The polar axis is the helicity axis of the $\pi\pi$ system. The moments are separately given for $0.6 < m_{\pi\pi} < 0.9$ and $0.9 < m_{\pi\pi} < 1.0$ GeV.

l as high as 8. We take these to be due to πN^* background[4] but henceforth ignore their presence compared with the large $l = 1, 2$ moments. As in earlier analyses,[5] we assume that only s- and p-wave scattering need be considered for the $\pi\pi$ interaction in this region.

The moments $N\langle Y_1^m\rangle$ and $N\langle Y_2^m\rangle$ (N is the $\pi\pi$ effective-mass spectrum) evaluated[6] every 20 MeV for $600 < m_{\pi\pi} < 1000$ MeV are given in Fig. 2 for Reactions (1) and (2). As shown in Eqs. (3a)-(3f) of I, these quantities have a dependence on the effective $\pi\pi$-scattering amplitude functions which is determined only by l. Thus, $N\langle Y_1^m\rangle \sim \{\ \}\,{\rm Re}(A_{\pi\pi}^S A_{\pi\pi}^{P*})$ and $N\langle Y_2^m\rangle \sim \{\ \}\,|A_{\pi\pi}^P|^2$, where the brackets $\{\ \}$ denote functions of the helicity-amplitude vectors (defined in I) $\vec{p}_1$, $\vec{p}_0$, $\vec{p}_{-1}$, and $\vec{s}$. To the extent that these bracket quantities can be considered independent of $m_{\pi\pi}$, the data in Fig. 2 directly display the $m_{\pi\pi}$ dependence of the scattering-amplitude functions shown; the more rapidly varying the phase shifts, the better this approximation. Dirict tests of the fundamental factorization and reality assumptions of the formalism in I are that $N\langle Y_1^0\rangle$ and $N\langle {\rm Re}Y_1^1\rangle$ have the same $m_{\pi\pi}$ dependence and that $N\langle Y_2^0\rangle \sim N\langle {\rm Re}Y_2^1\rangle \sim N\langle {\rm Re}Y_2^2\rangle \sim$ (p-wave Breit-Wigner). Applying