of strength in the optical transition probability function. A peak in ϵ_2 appears at 13 eV in Robin's data in reasonable agreement with what we would expect from the photoemission data. No reliable data are available in the literature for the energy region from 3 to 4 eV; however, our optical measurements reveal structure in ϵ_2 at about 3.5 eV. These data and their analyses will be presented in a separate publication.

By the analyses presented here we are able to identify all of the structure in the photoemission data of Ag and Pd in terms of energy losses by excitation of the electron plasma and structure in the optical of transition probability function.

We wish to express our appreciation to Dr. R. H. Ritchie of the Oak Ridge National Laboratory for his valuable comments on some theoretical aspects of this problem.

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MICROWAVE ABSORPTION BY MAGNETIC-FIELD-INDUCED SURFACE STATES IN SUPERCONDUCTORS

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It is the prupose of this note to show that the anomalies recently observed in the magnetic field dependence of the microwave absorption of pure Type-I superconductors[1] are consistent with a mechanism involving transitions from thermally excited surface quasiparticle states[2] into the BCS continuum.

In Fig. 1. we show tracings of the microwave absorption derivative in superconducting indium at two frequencies. Characteristic of such data is the pronounced peak followed by a regime of fields where the absorption decreases with increasing field applied parallel to the surface. Studies of the angular dependence of the peaks indicate that they are due to electrons on "cylindrical" pieces of the Fermi surface. We briefly indicate the essential dependence of these peak anomalies on relevant experimental parameters. (1) With increasing frequency the resonant peaks become more pronounced and shift to higher field. Over the range of frequencies studied (10-60 GHz), the

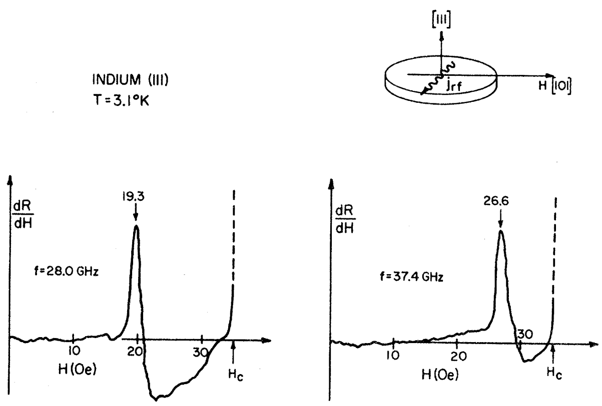

FIG. 1. Experimentally observed variation of the microwave absorption derivative dR/dH, with magnetic field applied parallel to the surface of a disk-shaped specimen of indium. With increasing frequency the peak observed in this spectrum shifts linearly to higher fields. The sharp rise at the end of the trace represents the onset of the transition into the normal state at the critical field H_c.

frequency scaling is linear. (2) As a function of decreasing temperature the amplitude of the signal diminishes rapidly. Peaks shift to increased fields very nearly as $[1-(T/T_c)^4]^{1/3}$. The dependence of the peak signals on rf polarization, field orientation, and surface preparation are discussed at length in Ref. 1.

We would like to interpret this observation in terms of quasiparticle states, bound by the Meissner currents to the surface of the sample. These states, which have energies lying within the zero-field energy gap (Δ_0), are discussed in detail in Ref. 2. Very crudely they arise from the "Doppler"-like shift in the BCS spectrum in the presence of a drift current ($\vec{j}_s = ne\vec{v}_s$, where n is the electron density, $\vec{v}_s$ the superfluid velocity) i.e.,

$$E_k = E_k^{\,0} + \hbar\vec{k}\cdot\vec{v}_s, \tag{1}$$

where $E_k^{\,0}$ is the usual BCS quasiparticle energy. In this case, $|\vec{v}_s|$ falls off as $\exp(-d/\lambda)$, where λ is the penetration depth and d the distance into the sample. This spatial variation of the screening current restricts the states with energy less than Δ_0 to have wave functions localized within a distance $(\epsilon_0/k_F)^{1/2}$ of the surface. Here ξ_0 is the coherence length, k_F the Fermi wave vector. This leads to a quasidiscrete spectrum for the bound states, i.e., if the components of the wave vector (k_x, k_y) lying in the surface are specified, a small discrete number of surface states are determined. As (k_x, k_y) are varied, the binding energies shift giving rise to a quasicontinuum of localized surface modes. The density of bound states is highest for electrons moving parallel to the screening current, because these have maximum binding energy [Eq. (1)]. For cylindrical pieces of Fermi surface, the additional restrictions on the direction of the electronic motion appreciably enhance this peak in the density of states over the case of a spherical Fermi surface. From Ref. 2, the binding energy of states moving parallel to the screening current is given by

$$V_0 \approx \left(\frac{e\hbar}{2mc}\right) k_F \lambda H \approx \left(\frac{3}{8}\right)^{1/2} \left[\frac{\lambda(T)}{\lambda_L(0)}\right] \left[\frac{H}{H_c(0)}\right] \Delta(0), \tag{2}$$

where $\lambda_L(0)$ is the London penetration depth, $H_c(0)$ the critical field, $\Delta(0)$ the energy gap in the absence of a field, all at $T=0$. As the microwave absorption is essentially due to electrons moving parallel to the surface, these states are expected to play an important role. Moreover, the bound states are confined to a depth comparable with the microwave skin layer, further enhancing their contribution to the microwave impedance. In Fig. 2 we show schematically the states with binding energy V_0 together with the BCS continuum.

The mechanism which we believe is responsible for the observed peaks in the microwave impedance is the following: At finite temperature the quasiparticle surface states are occupied and can be excited into the continuum by the absorption of a microwave photon. With increasing magnetic field we expect initially a slowly increasing absorption, reflecting the increasing density of final states into which the surface state can be scattered. We should observe a peak in the absorption at the field where the bound state energy V_0 equals the microwave energy. The theoretical expressions for this absorption are given in Eqs. (3.27) and (3.31) of Ref. 2. In Fig. 2 we show the variation of absorption derivative with field calculated on the basis of this model.

With reference to Fig. 1 we see that the observed variation of the absorption derivative is qualitatively similar to the theoretical curve.

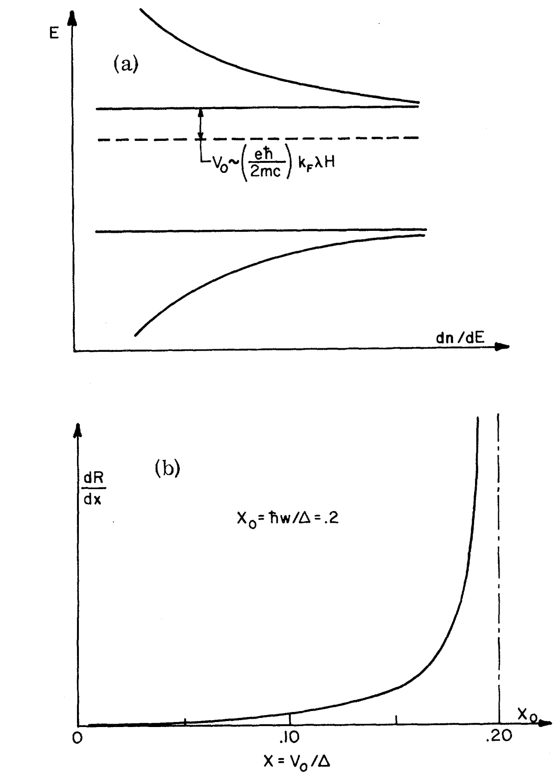

FIG. 2. (a) Schematic of the BCS density of states together with the magnetic-field-induced surface state. (b) A calculation [based on Eq. (3.31) of Ref. 2] of the microwave absorption derivative dR/dH, as a function of magnetic field. The variation of the absorption is due to excitation of the surface states into the BCS continuum. While the derivative curve has a square-root singularity at $\hbar\omega - V_0$ (reflecting the corresponding singularity in the BCS density of states), the total absorption remains finite.

The decrease of surface resistance at fields above the peak we attribute to the fact that the surface-state binding energy now exceeds $\hbar\omega$ and that electrons in these states can no longer contribute to the absorption. In addition to the resonant excitation from states due to well-defined cylindrical Fermi surface sections, we expect that there should also be a broad background contribution due to other electrons.

In terms of our model and with reference to Eq. (2), using $k_F \approx 10^8$ cm^{-1} and $\lambda \approx 10^{-5}$ cm, we expect to observe the peak in Fig. 1 at $H \approx 30$ Oe in good agreement with the experiment. The linear dependence of the peak on the microwave frequency is as expected. The decrease in amplitude with temperature we attribute to a decrease in the quasiparticle population, roughly as $\exp(-T_c/T)$. The shift to higher fields with decreasing temperatures is due to the variation of λ with temperature. In the nonlocal limit

$$\lambda(T) \approx \lambda_L^{2/3}(T)\xi_0^{1/3} \approx \lambda_L^{2/3}(0)\xi_0^{1/3}[1-(T/T_c)^4]^{1/3}, \qquad (3)$$

in good agreement with the experimentally observed dependence of the peak field on temperature. We conclude that the systematics of the experimental behavior of the peaks in the microwave absorption appear to be consistent with our surface state picture.

We would like to suggest that at frequencies where $\hbar\omega > \Delta(T)$, another type of absorption maximum may occur arising from the excitation of a ground-state pair into a surface state and a continuum state at the gap edge. The peak would be expected to occur at a field such that $\hbar\omega = 2\Delta(T) - V_0$. However, preliminary estimates indicate that for indium the intensity of this absorption would be only 10^{-3} of the high-temperature mechanism.

Finally, we would like to note that in the normal state of metals qualitatively similar surface quantum states exist.[3] These account for the microwave absorption oscillations observed in weak magnetic fields.[4]

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