1(c) for various values of D and $(a_c/a_i)^2$. In all these calculations the structure factor obtained by Henshaw[10] at 84°K was used. The effect of variations in D on the shape of the peak height curve is small and is therefore not shown in Fig. 1.

The full width at half-peak height and the maximum intensity of the observed spectra are shown in Figs. 1(d) and 1(e), respectively, as functions of scattering angle and wave-vector transfer. Q is calculated assuming that both the incident and the scattered neutron wavelength is 4.1 Å. Vertical bars in Fig. 1 indicate errors in the counting statistics and horizontal bars represent the angular resolution of the spectrometer. The solid lines in Figs. 1(d) and 1(e) are the curves obtained from Eq. (3) for the values of the parameters shown.

The value used for the ratio of the scattering lengths is the one reported by Henshaw,[10] and the value used for the diffusion constant should be compared with the values $D=2.43\times10^{-5}$ cm^2/sec and $D=3.5\times10^{-5}$ cm^2/sec, obtained by Naghizadeh and Rice[11] at $T=90$ and $T=100$°K, respectively.

The agreement between the predictions of Eq. (3) and the experimental results must be considered satisfactory. The "theoretical" value for the width depends strongly on the exact shape of $S(Q)$ which was not available at the appropriate temperature. The peaks in $S(Q)$ are expected to broaden when the temperature increases and this will decrease the fluctuations in the calculated width and also in the peak height. Use of a structure factor obtained at 94°K could therefore be expected to improve the agreement.

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PROPAGATION OF SOUND IN MONATOMIC GASES

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Since the days of Kirchhoff, the velocity of propagation and the absorption of sound in gases are usually derived from the Navier-Stokes hydrodynamical equations. For monatomic gases the results are especially simple. One finds for the velocity U and the absorption coefficient in amplitude α, as function of the frequency ω, expansions of the form

$$U_0/U = 1-a_1\xi^2+a_2\xi^4-\cdots,$$

$$\alpha U_0/\omega = b_1\xi-b_2\xi^3+\cdots, \qquad (1)$$

where

$$\xi = \mu\omega/\rho U_0^2, \qquad (2)$$

with ρ and μ the mass density and viscosity coefficient of the gas, respectively, and $U_0 = (5kT/3m)^{1/2}$. The coefficients a_i and b_i are constants if one assumes that the heat conductivity κ is related to μ by

$$\kappa = \frac{15}{4}\frac{k}{m}\mu,$$

which is closely fulfilled for all monatomic gases. One obtains

$$b_1 = 7/6; \quad a_1 = 141/72; \quad b_2 = 1559/432; \qquad (3)$$

etc.

From the point of view of the kinetic theory of gases, the hydrodynamical equations are obtained from the basic Boltzmann equation for the distribution function $f(\vec{r}, \vec{v}, t)$ of the molecules by a successive approximation method, the Chapman-Enskog development, which proceeds in successive powers of a uniformity parameter which is of the order of the relative change of the macroscopic variables (average velocity and/or temperature) over a mean free path λ of the molecules. Now note that the parameter ξ in (1) is of order λ/Λ if Λ is the wavelength of the sound, since $\mu \sim \rho\lambda\langle v\rangle$ with $\langle v\rangle$ a mean thermal velocity, and $\langle v\rangle \sim U_0$. Therefore ξ is just the expansion parameter used in the Chapman-Enskog development; and since from the point of view of this development the Navier-Stokes equations are only valid up to first order in ξ, only the coefficient b_1 can be trusted.

To obtain the correct values for a_1, b_2, etc., one could start from the higher order hydrodynamical equations, of which the second order has been derived in detail by Burnett. This can be and has been done, but it is very involved, and it is much simpler to go back to the Boltzmann equation and to seek solutions for f of the form

$$f = f_0[1 + h(\vec{v})e^{i(\vec{K}\cdot\vec{r} - \omega t)}], \tag{4}$$

where f_0 is the Maxwell distribution and $h \ll 1$. This was pointed out by Wang-Chang and Uhlenbeck,[1] who applied this method to the so-called Maxwell model, which assumes a repulsion between the molecules $\sim 1/r^5$. However, since with (4) the Boltzmann equation becomes

$$[-i\omega + i(\vec{K}\cdot\vec{v})]h = J(h), \tag{5}$$

where $J(h)$ is the linearized collision operator, one can find ω as a function of the wave number K by a direct perturbation calculation in which $K = 1/\Lambda$ is the small parameter. The integral equations occurring in successive order are of the same type as those occurring in the Chapman-Enskog development, and they can be solved for any intermolecular potential by the same successive approximation method.[2] One finds that the dispersion law can again be put into the form (1) and that the coefficients a_i and b_i are almost universal, that is, almost independent of the intermolecular potential and almost independent of the temperature. Their

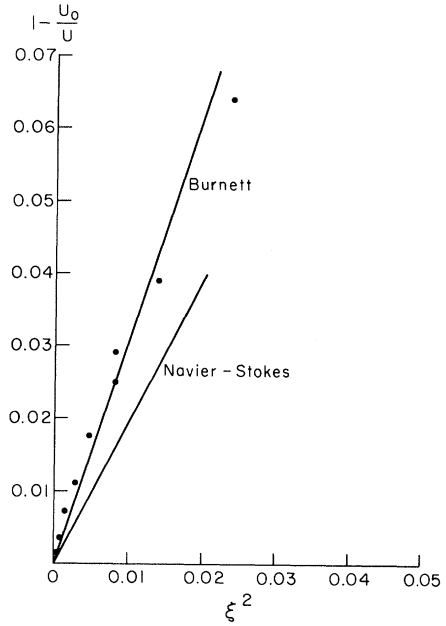

FIG. 1. Comparison between theory and experiment for the initial frequency dependence of the phase velocity U in neon.

approximate values are

$$b_1 = 7/6; \quad a_1 = 215/72; \quad b_2 = 5155/432; \tag{6}$$

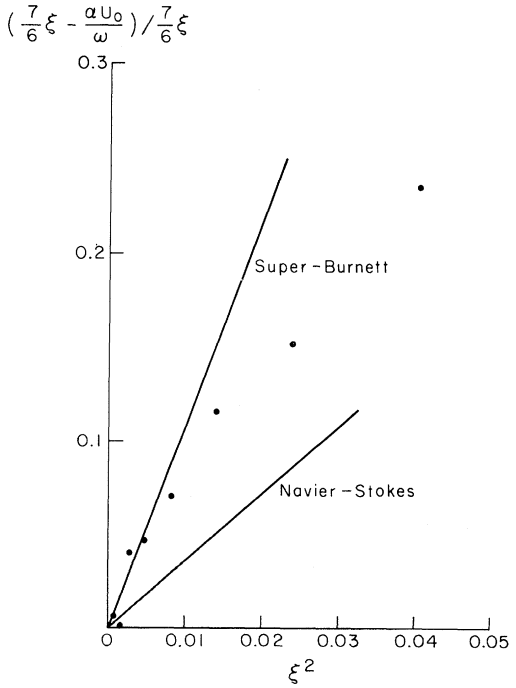

FIG. 2. Comparison between theory and experiment for the initial non-Kirchhoffian frequency dependence of the absorption coefficient α in neon.

etc. These are in fact the values for the Maxwell model and they agree with the results of Wang-Chang and Uhlenbeck. For a $1/r^s$ repulsive potential, the coefficients are temperature independent, and they are slowly increasing functions of the force index s, differing from the values (6) by less than 10% for $s = \infty$ (hard-sphere model). For a Lennard-Jones (12-6) potential, the coefficients are weakly temperature dependent but the deviations from (6) are never more than 7%. This insensitivity to the intermolecular potential makes a significant comparison with experiment possible, especially since the correct values of a_1 and b_2 are considerably different from the Navier-Stokes values (3).

In Figs. 1 and 2, the experimental values[3] for the initial frequency dependence of U and for the deviation from the first (Kirchhoff) value of the absorption coefficient in neon are plotted against ξ^2. The lines labeled Burnett and Super-Burnett refer to the values (6); they clearly represent the initial slope of the experimental curves much better than the Navier-Stokes values (3).

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OBSERVATION OF ANOMALOUS ELECTRON HEATING IN PLASMA SHOCK WAVES*

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Direct observations are reported of the electron velocity distribution throughout the region traversed by a magnetic shock wave in a plasma. In one case, the electron heating rate is greater than 10 times that predicted from normal and compressional effects.

In ordinary magnetohydrodynamic shock waves, known resistive (electron-ion collisions) and viscous (ion-ion collisions) processes account for the dissipation of the initial shock-frame streaming energy. In low-density plasmas such binary collision processes may be unable to provide the dissipation, and a form of anomalous dissipation due to collective plasma interactions takes over. We report here measurements that show the occurence of such anomalous dissipation.

A technique for obtaining electron velocity distributions, and hence heating rates, is the analysis of light scattered from the plasma when it has been illuminated by an intense source. The source used is a 200-MW Q-spoiled ruby laser, focused into the plasma to probe a small ($\sim$1 mm^3) volume with 20-nsec time resolution. Spectral analysis of the scattered light yields the one-dimensional electron velocity distribution in a direction defined by the difference between the wave vectors of the incident and scattered light. The electron density profile through the shock follows from the area under the scattered light profiles.

Scattering measurements were made on the nonlinear compressional waves produced in an electrodeless discharge tube. Results of magnetic-probe and emission-spectroscopic investigations of waves in this device have been reported.[1] A large-amplitude, radially converging compressional wave is induced in a previously formed plasma by a rapidly rising current flowing in a wide, single-turn strap wrapped around a 10-cm-diam quartz tube. The laser beam is parallel to, and 1.8 cm off, the axis. Scattered light collected at 90° to the incident beam direction is analyzed by tun-