

just π exchange alone. This dip arises from the interference between helicity-flip and helicity-nonflip amplitudes.

(ii) Destructive interference between π and ρ exchange tends to decrease the result one would obtain from pure π exchange, in the large-momentum-transfer region.

Several interesting points should be noted about Fig. 2:

(i) The theoretical differential cross section does not shrink with an increase of the antiproton laboratory momentum, in agreement with experiment.

(ii) As the laboratory momentum of the incident antiprotons increases by a factor of 3 the magnitude of the forward differential cross section (theoretical) decreases by a factor of about 10, again in agreement with experiment.

We wish to emphasize that this fit to the data arises from the use of $U(6, 6)$ symmetry and absorptive corrections. Since the coupling strength is taken from π - N scattering, there are no adjustable parameters in the model. The model also gives good agreement with experiment for $p + \bar{p} \rightarrow Y + \bar{Y}$ (to be published shortly).⁷

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COMMENTS ON MEASURING $\text{Re}(A_2/A_0)$ IN THE DECAY $K_S^0 \rightarrow \pi + \pi$

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Two recent experiments^{1,2} have thrown interesting light on the phenomena of K^0 decay. In particular, these experiments yield knowledge about the imaginary part of A_2/A_0 where the amplitudes A_2 and A_0 were defined by Wu and Yang.³

It was pointed out in Ref. 3 that the real part of A_2/A_0 can only be obtained experimentally by measuring accurately the difference

$$\frac{R_S(+ -) - 2R_S(00)}{R_S(+ -) + R_S(00)} = 2\sqrt{2} \text{Re}(A_2/A_0) \cos(\delta_2 - \delta_0). \quad (1)$$

It was also pointed out³ that electromagnetic corrections lead to small changes in the Clebsch-Gordan coefficients and in the phase shifts $\delta_2 - \delta_0$. A partial estimation of the electromagnetic correction was given by Lee and Wu.⁴ We give here an estimate of the electromagnetic correction to Eq. (1) and discuss the feasibility of measuring $\text{Re}(A_2/A_0)$.

We found the electromagnetic correction to Eq. (1) to be

$$\frac{R_S(+-, \omega)}{R_S(00)} - 2 = 6\sqrt{2} \operatorname{Re}(A_2/A_0) \cos(\delta_2 - \delta_0) + 2 \left[\frac{3\alpha}{\pi} \ln \frac{\Lambda}{m_\pi} + a \ln \frac{\omega}{m_\pi} - 0.015 + C\alpha \right], \quad (2)$$

where $R_S(+-, \omega)$ is the rate of $K_S \rightarrow (\pi^+ + \pi^- + \pi^+ + \pi^- + \gamma)$ for γ energy $\leq \omega$ in the K rest system,

$$\alpha = e^2/\hbar c = 1/137,$$

$$a = \frac{2\alpha}{\pi} \left[-1 + \frac{1+v^2}{2v} \ln \frac{1+v}{1-v} \right],$$

and v is the velocity of π^+ in the K rest system. In Eq. (2) the four terms in the square bracket arise, respectively, from the following:

(a) The electromagnetic renormalization of the $K\pi\pi$ vertex and the charged pion wave functions. This term is logarithmically divergent and Λ is the ultraviolet cutoff momentum.

(b) The soft-photon emission⁵ accompanying $K \rightarrow \pi^+ + \pi^-$.

(c) The correction⁴ due to the phase-space-volume difference resulting from the mass difference of the charged and neutral pions.

(d) Finite effects plus the strong-interaction correction to the electromagnetic correction.

In evaluating the renormalization of the $K\pi\pi$ vertex one considers five Feynman diagrams. In the gauge in which the photon propagator is $(k^2 - i\epsilon)^{-1}(\delta_{\mu\nu} - k^{-2}k_\mu k_\nu)$, only the "wave-function renormalization" part of these diagrams contributes to the ultraviolet divergence. If one uses instead $(k^2 - i\epsilon)^{-2}\delta_{\mu\nu}$ for the photon propagator, then the "wave-function renormalization" part and the "vertex" part both contribute to the ultraviolet divergence, in the ratio of 2 to 1, yielding the same total result. Some finite parts of the contributions (a) and (b) have been lumped into the term $C\alpha$.

In the above evaluation we assumed that the unrenormalized (by electromagnetic effects) $K\pi^+\pi^-$ and $K\pi^0\pi^0$ vertices are the quantities on which one makes the isospin separation. It seems to us that this is the only reasonable definition.

Putting in numerical values we obtain

$$\begin{aligned} & \left[\frac{3}{\pi} \alpha \ln \frac{\Lambda}{m_\pi} + a \ln \frac{\omega}{m_\pi} - 0.015 + C\alpha \right] \\ & = \frac{0.70}{100} \ln \frac{\Lambda}{m_\pi} + \frac{0.65}{100} \ln \frac{\omega}{m_\pi} - \frac{1.5}{100} + \frac{C}{137}. \quad (3) \end{aligned}$$

As an estimation of the orders of magnitude

we take $\Lambda = 2$ BeV, $\omega = 10$ MeV. The total of the first three terms of (3) is then about -0.015 . The finite term αC can be evaluated exactly if we "switch off" the strong interactions. In this case C is given by

$$\begin{aligned} C = \frac{1}{\pi} & \left\{ \left(\frac{1+v^2}{2v} \right) \left[\ln \frac{1+v}{1-v} + 2 \ln \frac{1+v}{1-v} \ln \frac{1-v^2}{v^2} \right. \right. \\ & + f\left(\frac{2}{1-v}\right) - f\left(\frac{2}{1+v}\right) + 2f\left(\frac{1}{1+v}\right) - 2f\left(\frac{1}{1-v}\right) \\ & \left. \left. + f\left(\frac{1-v}{1+v}\right) - f\left(\frac{1+v}{1-v}\right) \right] - \frac{1}{4} - 2 \ln 2 \right\}, \quad (4) \end{aligned}$$

where $f(x)$ is the Spence function as defined and tabulated by, for example, Mitchell.⁶ The numerical value of αC , with C given by Eq. (4), is -0.01 . We guess that (3) is $\sim (-2 \pm 2) \times 10^{-2}$. Thus it is useful⁷ to measure the left-hand side of Eq. (2) up to an accuracy of $\sim \pm 0.04$ [greater accuracy would lead to a better knowledge of $\operatorname{Re}(A_2/A_0)$ only when the cutoff Λ and the strong interaction effect on C are better understood]. Such a measurement would yield a value of $\operatorname{Re}(A_2/A_0)$ with an absolute accuracy of $\sim \pm 0.01$ [where we have taken $|\sqrt{2} \cos(\delta_2 - \delta_0)| \sim 1$ from Refs. 1 and 2].

From Refs. 1 and 2 one obtains⁸

$$|\operatorname{Im}(A_2/A_0)| < 3 \times 10^{-3}. \quad (5)$$

Knowledge of $\operatorname{Re}A_2/A_0$ would⁹ contribute toward separating the $K \rightarrow \pi + \pi$ ($I=2$) amplitude into the $|\Delta I| = \frac{3}{2}$ and $|\Delta I| = \frac{5}{2}$ components $\alpha_{3/2}$ and $\alpha_{5/2}$:

$$\begin{aligned} A_2^+ & = \left(\frac{3}{4}\right)^{1/2} \alpha_{3/2} - \left(\frac{1}{3}\right)^{1/2} \alpha_{5/2}, \\ A_2 & = \left(\frac{1}{2}\right)^{1/2} \alpha_{3/2} + \left(\frac{1}{2}\right)^{1/2} \alpha_{5/2}, \quad (6) \end{aligned}$$

where

$$|A_2^+|/A_0 = 0.055 \quad (7)$$

from the rate of $K^+ \rightarrow \pi^+ + \pi^0$. A comparison of the magnitudes of Eqs. (7) and (5) indicates⁹ that probably A_2 is mostly real. If that is true then all evidence is consistent with the assumption that (the strong, electromagnetic and $|\Delta I| < \frac{3}{2}$ weak interactions are CP conserving^{3,10} and that) the $|\Delta I| \geq \frac{3}{2}$ weak interactions manifest

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