Foundation.

[1]S. L. Adler, Phys. Rev. Letters 18, 519 (1967).
[2]M. Gell-Mann, Physics 1, 63 (1964).
[3]Y. Nambu, Phys. Rev. Letters 4, 380 (1960); M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960).
[4]D. G. Sutherland, Phys. Letters 23, 384 (1966).
[5]C. Itzykson, M. Jacob, and G. Mahoux, to be published.
[6]S. Barshay, Phys. Letters 17, 78 (1965); J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. 139, B1650 (1965).
[7]H. R. Crouch et al., Phys. Rev. Letters 13, 640 (1964); J. V. Allaby, H. L. Lynch, and D. M. Ritson, Phys. Rev. 142, 887 (1966).
[8]See, for example, H. A. Bethe and F. de Hoffmann, Mesons and Fields (Row, Peterson and Company, Evanston, Illinois, 1955), Vol. II, Sec. 36.
[9]S. D. Drell, Phys. Rev. Letters 5, 278 (1960).

[10]We use particle symbols to denote the corresponding field operators in the interaction densities, with A_μ the electromagnetic vector potential. For theory of double-pion photoproduction see the original work of R. E. Cutkosky and F. Zachariasen, Phys. Rev. 103, 1108 (1956); and also P. Carruthers and H. W. Huang (to be published).

[11]In integrating over the Breit-Wigner shape we make the approximation $\int_{-\delta}^{\delta} dx(\gamma/x^2+\gamma^2)=\pi/2$ (instead of π) for $\delta\approx\gamma$ and therefore $c=\frac{1}{2}$. I thank Professor Adler for pointing out the overestimate given by the zero-width approximation.

[12]This axial-vector current coupled to V_μ is not completely conserved. However, at a total c.m. energy W, the matrix element of its divergence between p and N^* states is proportional to the quantity

$$\frac{W^2+m^{*2}}{4W^2}\frac{\mu^2}{k}+(m^*-m)\left(\frac{m^*+m}{2W}+\frac{q}{m^*}\right)$$

and hence goes to zero when μ^2, the squared pion mass, does so provided one also accepts the limit of the isobar-nucleon mass difference $m^*-m\approx 2\mu$ going to zero [as would be suggested by the static model expression $(m^*-m)^{-1}\cong(4f^2/\mu^2)\int q^3 d\omega_q/\omega_q{}^3$].

[13]These are gauge-invariant at zero pion four-momentum. A strictly conserved current can be represented by

$$\left(\frac{ie}{M}\right)\frac{\partial}{\partial x_\nu}\left[\left(V_\nu{}^+\frac{\partial}{\partial x_\mu}\pi^- - V_\mu{}^+\frac{\partial}{\partial x_\nu}\pi^-\right)\right.$$
$$\left.-\left(V_\mu{}^-\frac{\partial}{\partial x_\mu}\pi^+ - V_\mu{}^-\frac{\partial}{\partial x_\nu}\pi^+\right)\right]=j_\mu.$$

This gives rise to a matrix element $\langle\pi|j_\mu|V\rangle\epsilon_\mu\propto(ie/M)\times[\mu(m^*-m)+\mu^2]\hat{\epsilon}\cdot\hat{\epsilon}_V$ as the pion three-momentum goes to zero, where $\hat{\epsilon}_V$ is the axial-vector meson polarization three-vector and M is an unknown mass parameter, and where V is understood to be virtually emitted with energy $\approx m-m^*$ at the VNN^* vertex. Comparison with the approximate currents suggests $m_{2,3}\approx\mu(m^*-m+\mu)/M$, which means that the effective couplings may be quite small. We emphasize this in this Letter's concluding paragraph.

[14]A similar mechanism with p replacing N^{*+} does not contribute to $\gamma+p\rightarrow\pi^0+p$ at zero pion four-momentum only, because of the vanishing of the lower vertex involving a conserved axial vector current at this point. (See Ref. 1, footnote 27.) Note that m_4 need not be the same as m_3 if K_μ is not an isoscalar.

[15]I am indebted to Professor Weisberger for this remark. All that one can say is that this decay mode has been seen experimentally.

[16]It should be noted that if L_μ contains a $|\Delta T|=2$ piece, the ratio of the cross section for Reaction (1a) to that for Reaction (1b) can deviate from the value of 3 expected for an isovector L_μ.

UNDERSTANDING "KINEMATIC EFFECTS"*

L. Stodolsky
Brookhaven National Laboratory, Upton, New York
(Received 4 May 1967)

In recent years numerous calculations have been performed for the Drell-Hiida-Deck-type[1] low-mass enhancements for a particle pair in three-particle final states. These mechanisms are interesting on practical grounds for the calculation of certain mass bumps and backgrounds in particle production, and also have some theoretical interest since they should serve as particular models for the more intuitively expressed idea of diffraction dissociation.[2,3] Since production by these mechanisms should go to a constant at high energy (while the conventional one-particle exchanges are decreasing), it seems likely that they play an important role, not only as backgrounds, but also as the production mechanisms for bona fide resonances (as seems to be the case in ρ photoproduction).

In Fig. 1 we establish our notation using the A_1 case for definiteness; at the lower vertex there is a high-energy diffraction scattering and the low-mass bump occurs for the "A_1" pair $q+K'$. The presumed dominance of the exchange of vacuum quantum numbers at the lower vertex ensures the same character for the transition $\pi\rightarrow A_1$. Because of the complicat-

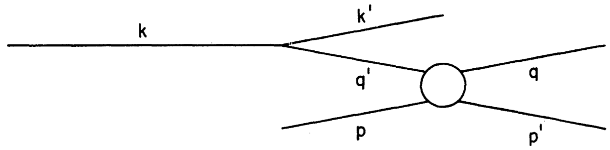

FIG. 1. The dissociation graph. In the case of "A_1" production K is the incident π, K' the ρ, q' the virtual π and q the final π, and P and P' the nucleon. The "A_1" is the combination $K'+q$.

ed kinematics involved it has been difficult to see through this calculation and to extract its general features. Here we would like to note a kinematic simplification which greatly facilitates rough calculations and allows us to examine these general characteristics.

The constant total cross-section diffraction-peak parametrization of the lower vertex results in a matrix element for Fig. 1 whose essential structure is

$$M\sim[|\vec{q}'|/(q'^2-\mu^2)]e^{-\frac{1}{2}a\Delta^2}, \qquad (1)$$

where $|\vec{q}'|$ is the momentum of the virtual particle in the lab, and Δ^2 the square of the invariant four-momentum transfer to the nucleon. If we consider the production of an A_1 system of a definite mass at a definite angle, then Δ^2 is fixed. In particular, let us now specialize to forward production of the A_1 and work in the lab frame so that the nucleon is initially at rest and then recoils purely longitudinally, with a momentum Δ_0. It can be shown that under high-energy conditions (momentum of incident particle and $A_1 \gg$ their masses) the energy transfer to the nucleon, $\Delta_0^2/2M_p$, is negligible and

$$\Delta_0 \cong (M_{A_1}^{\;2}-M_{\rm inc}^{\;2})/2|\vec{K}|; \qquad (2)$$

$M_{\rm inc}$ = mass of incident particle, and $K\cong$ incident momentum $\cong A_1$ momentum to $O(1/K)$, as indicated by Eq. (2). Now (2) holds even for a high-energy scattering by a virtual particle such as at the lower vertex in Fig. 1, except that $M_{\rm inc}^2$ now becomes q'^2. Thus by considering a completely collinear configuration in which the ρ, and therefore the virtual π, also goes straight forward we may compute Δ_0 from the two-body scattering at the lower vertex to get

$$\Delta_0 \simeq (q'^2-\mu^2)/2|\vec{q}|, \qquad (3)$$

where again $|\vec{q}|$ is the momentum of the virtual or final π in the lab. But note that in (3) we have essentially evaluated what is needed in (1), giving the remarkably simple result

$$M\sim(1/\Delta_0)e^{-\frac{1}{2}a\Delta^2}, \qquad (4)$$

which depends only on the mass change in $\pi \to A_1$ and not on any internal variables. A more complicated calculation for the cases in which there is some transverse momentum present shows that (4) is still essentially exact for 0° production (longitudinal proton recoil) and that it is a reasonable approximation for small proton recoil.[4]

Equation (4) has several interesting features:

(A) The "ρ-π" low-mass enhancements (when spin and form factors at the dissociation vertex are ignored) are simply given by $(1/\Delta_0)^2e^{-a\Delta_0^2}$ $\times$(phase space). In Fig. 2 we compare this with the numerical calculations of Maor and O'Halloran[1] (who use experimental cross sections for the lower vertex) and of Allard et al.[5] for A_1 production at low and high energy, respectively. In the low-mass peak region where the exponential is not important, simply $(1/\Delta_0^2)$ $\times$(phase space) works quite well. This kind of distribution should also hold for the density of points on the Dalitz plot as well as for the integrated mass distributions, except for the sector where the lower vertex energy is below the diffraction region. At lower energies, as in the Maor-O'Halloran case, this region, where our approximation does not apply, is not entirely negligible compared to the total phase space, but with increasing energy it becomes less important.

(B) Equation (4) contains no reference to the

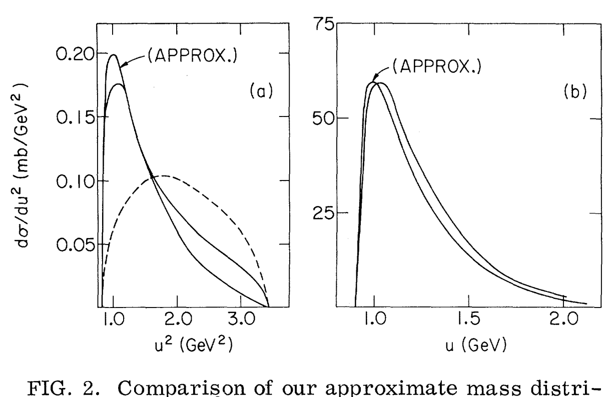

FIG. 2. Comparison of our approximate mass distribution $(1/\Delta_0)^2e^{-a\Delta_0^2}\times$(phase space) with (a) Maor and O'Halloran for 3.65 BeV and (b) Allard et al. for 16 BeV for "A_1" production. The quantity denoted by u in this figure is called M_{A_1} elsewhere in the paper.

mass of the exchanged particle. The low-mass enhancement results not from the closeness of the propagator pole to the physical region, but on the closeness of the final "A_1" mass to the initial mass. Thus we find, contrary to what seems to have been the general assumption, that the scattering of the heavier virtual constituent is just as important as that of the lighter. If now we incorporate our approximation into the general formula for the cross section we obtain, in fact,

$$d\sigma \cong \left(\frac{1}{2\pi}\right)^4 \frac{1}{32}\left(\frac{1}{M_{A_1}{}^2 - M_{\text{inc}}{}^2}\right)^2 |(f_{\rho\pi\pi} 2K_\mu \epsilon_\mu)[\sigma(\pi N)e^{-[\frac{1}{2}a(\pi N)]\Delta^2} + \sigma(\rho N)e^{-[\frac{1}{2}a(\rho N)]\Delta^2}]|^2 \times \frac{\kappa}{M_{A_1}} d\Omega_\kappa d\Delta^2 dM_{A_1}{}^2. \quad (5)$$

M_{A_1} is the mass of the $\rho\pi$ combination and κ the momentum of one of the particles in the A_1 rest frame.

(C) Since (4) essentially only depends on the mass change in going from M_{inc} to M_{A_1} and not on any internal angular variables, no angular momentum (or parity) is contributed by the process of scattering the virtual particle onto the mass shell and all angular information referring to the "decay" of the A_1 comes from the dissociation vertex. This explains the angular independence in the "A_1" rest frame noted by workers[6] on the A_1 and $N^*(1400)$ problems and shows that it is, in fact, a general feature of the dissociation process.

Conclusion (A) is very gratifying since it shows how a particular model realizes exactly the same $1/\Delta_0{}^2$ behavior derived previously on other grounds for the dissociation process.[3] This lends weight to the explanation in Ref. 3 that this factor causes the ρ mass skewing in photoproduction[7] and the suggestion that the ρ mass will return to normal in electroproduction with highly virtual photons.[3] Although the treatment of resonance final-state interactions (for the "A_1" system) with a nonlocal production process as in Fig. 1 is somewhat subtle, it also indicates that we should expect downward mass skewing for broad resonances produced predominantly by dissociation.

We see, then, that Fig. 1 does indeed correspond to the physical picture for the dissociation process. Masses for the "A_1" system closer to that of the beam particle are reached more easily (as follows from wave-function overlap arguments), and [conclusion (B)] the probability for scattering a component of the beam particle depends on the probability of its being present and not on its mass.[8] Furthermore, (5) shows that production into a given mass interval is energy independent, as expected for a diffraction process.

Conclusion (C) leads to a number of results for dissociation with spin changes. First of all, for incident spin-0 particles (but not for spin$\neq 0$) we get the "natural parity change" $\pi, K \to 0^-, 1^+, 2^-, \cdots$ but $\pi, K \not\to 0^+, 1^-, 2^+, \cdots$. Secondly, since Eq. (4) takes care of the propagation and scattering of the virtual particle, the spin couplings and form factor at the upper vertex may be conveniently evaluated by going into the A_1 rest frame. (We assume, as usual, that at the lower vertex, scattering is completely spin independent.) In this frame, choosing the incident-particle direction as the quantization axis, we see that the only helicity is brought in by the beam particle. Thus the precise form of the qualitative statement that the "helicity is conserved" in dissociation is that the "A_1" has the same helicity as the "π," in the A_1 rest frame. This also means that there is a Treiman-Yang–type isotropy test for rotations around the beam direction in the A_1 frame.[8]

Despite conclusion (C), spin states different from those of the beam particle are generated due to the Lorentz transform in going from the "π" to "A_1" rest frames, if the "π" has a finite size and some internal spin structure. For instance, if we consider spin-0 dissociating into spin-0 particles, the form factor $F((K-K')^2)$, when evaluated in the A_1 rest frame, becomes dependent on the angle between K and K' and thus $F((K-K')^2) \sim \sum a_l P_l(\cos\theta)$, generating the entire natural-parity series. Thus the finite size of the system (form factor) is correlated with the transfer of angular momentum into its center of mass, as we would expect. Similarly, for dissociation into particles with spin, the spin couplings, which in the "π" rest frame correspond to a state with the same spin as the "π," are turned into other states by the Lorentz transformation. (It should be realized that these spin couplings, which are usually neglected, introduce powers of momentum which will tend to broaden the "A_1" peak.) These features are suggestive of a purely kinematical picture (of forward high-energy diffraction reactions with spin change) in which the incident particle has some overlap with a final state of a different spin and mass simply due to the transformation from one frame to the other. Finally, it should perhaps be re-emphasized that processes of type Fig. 1 via dissociations like $\pi \to (\bar{\Lambda}, \Sigma)$, $(\bar{\Xi}, \Xi)$, or even $K^- \to (\bar{\Xi}^0, \Omega^-)$ [or maybe $\to (\bar{q}q)$?], offer a mechanism for the production of high strangeness free of strangeness exchange or statistical limitations, once the energy is sufficiently high.

I would like to thank M. Ross, R. Huson, V. Barnes, D. Miller, and R. Panvini for many helpful discussions and assistance on this matter.

*Work performed under the auspices of U. S. Atomic Energy Commission.

[1]S. D. Drell and K. Hiida, Phys. Rev. Letters 7, 199 (1961); R. T. Deck, Phys. Rev. Letters 13, 169 (1964); U. Maor and T. A. O'Halloran, Jr., Phys. Letters 15, 281 (1965); L. Resnick, Phys. Rev. 150, 1292 (1966); U. Maor, Ann. Phys. (N.Y.) 41, 456 (1967). See also D. R. O. Morrison, Phys. Letters 22, 226 (1966).

[2]M. L. Good and W. D. Walker, Phys. Rev. 120, 1857 (1960). These authors emphasized the importance of the parameter Δ_0 introduced below.

[3]M. Ross and L. Stodolsky, Phys. Rev. 149, 1172 (1966), give a more recent discussion and a detailed application to photoproduction of vector mesons.

[4]By calculating the quantity s for both the reaction as a whole and the lower vertex, both before and after the scattering, and comparing, we get

$$\frac{M_{A_1}^{\;2}-M_{\mathrm{inc}}^{\;2}-\vec{P}'^2}{K_0}-\frac{\mu^2-q'^2-\vec{P}'^2}{q_0'}=2\vec{P}'\cdot(\hat{Q}-\hat{q}).$$

The quantity on the right, where $Q=q+K'$ is the momentum of the "A_1," is essentially the scalar product of the "decay" angle of the A_1 and the recoil proton momentum $\vec{P}'$, and thus is zero for forward production. Similarly, P' is very small for forward production and so our approximation, viz. $(M_{A_1}^{\;2}-M_{\mathrm{inc}}^{\;2})/K_0=(\mu^2-q'^2)/q_0'$, is excellent (except for the small strip on the Dalitz plot where the lower vertex energy is very small —we always neglect this region). For nonforward production the equation may be examined in detail, but the general condition is roughly that the recoil momentum $P'^2 \ll M_{A_1}^{\;2}-M_{\mathrm{inc}}^{\;2}$.

[5]Orsay-Milan-Saclay-Berkeley Collaboration, Nuovo Cimento 46A, 737 (1966). I would like to thank R. Huson and V. Barnes for their help in making this comparison.

[6]R. Huson, private communication; see also the comments of Resnick (Ref. 1, Sec. 4).

[7]The ρ mass spectrum recently observed in photoproduction experiments at Deutches Elektronen-Synchrotron shows good agreement with $(1/\Delta_0^2)\times$(Breit-Wigner), using normal values for the ρ mass and width (S. Ting, private communication).

[8]That these features ought to be present in diffraction production has been noted by Marc Ross (unpublished).

SEARCH FOR $\eta \to \pi^+ + \pi^- + \pi^0 + \gamma$†

Stanley M. Flatté
Lawrence Radiation Laboratory, University of California, Berkeley, California

(Received 1 May 1967)

Singer has made a detailed theoretical analysis[1] of the decay mode $\eta \to \pi^+ + \pi^- + \pi^0 + \gamma$, and predicts $\Gamma(\eta \to \pi^+ + \pi^- + \pi^0 + \gamma)/\Gamma(\eta \to \pi^0 + \gamma + \gamma) \approx 0.23\,\%$. Since recent results indicate[2] $\Gamma(\eta \to \pi^0 + \gamma + \gamma) \approx \Gamma(\eta \to \pi^+ + \pi^- + \pi^0)$, we shall take his prediction as $\Gamma(\eta \to \pi^+ + \pi^- + \pi^0 + \gamma)/\Gamma(\eta \to \pi^+ + \pi^- + \pi^0) < 1\,\%$ for the purposes of this paper. On the other hand, Singer shows[1] that on the basis of order-of-magnitude arguments on powers of α, as well as the A-quantum-number arguments of Bronzan and Low,[3] one would expect $\Gamma(\eta \to \pi^+ + \pi^- + \pi^0 + \gamma)/\Gamma(\eta \to \pi^+ + \pi^- + \pi^0) \approx 1$. And aside from this, simple models fail to account for the branching ratios of the η by factors like 10^3, so that a priori we cannot assume that $\eta \to \pi^+ + \pi^- + \pi^0 + \gamma$ is small. We therefore have a clear-cut experimental question: Is the mode $\eta \to \pi^+ + \pi^- + \pi^0 + \gamma$ comparable in magnitude to the mode $\eta \to \pi^+ + \pi^- + \pi^0$, or is it very much smaller?

Our experimental result is $\Gamma(\eta \to \pi^+ + \pi^- + \pi^0 + \gamma)/\Gamma(\eta \to \pi^+ + \pi^- + \pi^0) < 0.07$. Although this result appears to be in mild disagreement with the A-quantum-number calculations, one should remember that their prediction is only order-of magnitude. However, our result serves to reassure physicists that no large $\eta \to \pi^+ + \pi^- + \pi^0 + \gamma$ decay mode is lurking in the background.