ments.[7,8] (2) $\langle T_{21}\rangle_1\langle T_{21}\rangle_2$ is small compared to $\langle iT_{11}\rangle_1\langle iT_{11}\rangle_2$. (3) $\langle T_{20}\rangle_1\langle T_{20}\rangle_2$ is small compared to one. With these approximations, which are consistent with all available data,[3-8] we can write

$$\langle iT_{11}\rangle_2 = \frac{1}{0.28}\left[\frac{R-L}{R+L+U+D}\right],$$

from which Fig. 2 was prepared. Even when the data are presented in this form, where most of the sensitivity to the first-scattering tensors is removed, agreement between the available predictions and the data is poor.[13]

In conclusion, the present results appear to be in good agreement with earlier measurements (see Fig. 2); however, presently published d-α phase shifts are inconsistent with the present data. It has been demonstrated that a deuteron beam with useful vector polarization can be produced by α-d scattering.

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PHASE SHIFTS FROM THE BETHE-SALPETER DIFFERENTIAL EQUATION*

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A new method of calculating phase shifts for a Bethe-Salpeter equation is presented. The scattering amplitude is calculated below elastic threshold using the differential equation and variational methods, and then continued to the elastic-scattering region to find phase shifts.

Recently Schlessinger and Schwartz presented a method of finding phase shifts in potential theory by solving the Schrödinger differential equation for the scattering amplitude for energies below threshold and continuing it to the scattering region.[1] In this paper we report a variation on their method, involving an on-mass-shell continuation, that has proven successful in solving a Bethe-Salpeter equation.[2] The on-shell amplitude satisfies a simple unitarity relation, and this can be used advantageously in performing the continuation. We calculate below threshold in order to avoid the problems of solving a singular integral equation for the phase shifts.[3]

The differential Bethe-Salpeter equation in the ladder approximation for spinless particles of equal mass m is of the form

$$\mathfrak{D}\psi_k(x) = V(x)\psi_k(x), \tag{1}$$

where $k=(0,\vec{k})$, and $|\vec{k}|^2=\frac{1}{4}E^2-m^2$. We are interested in this equation below elastic threshold ($E^2<4m^2$), where the Wick rotation can be performed.[4] In the four-dimensional Euclidean metric, $\mathfrak{D}$ takes the form[5]

$$\mathfrak{D}=(-\Box-\tfrac{1}{4}E^2+m^2)^2-E^2(\partial/\partial x_4)^2, \tag{2}$$

where

$$\Box=\sum_{\nu=1}^{4}\frac{\partial}{\partial x_\nu}\frac{\partial}{\partial x_\nu}.$$

For mass-μ exchange, the potential is

$$V(x)=(4\mu\lambda/|x|)K_1(\mu|x|). \tag{3}$$

The T matrix is defined as

$$T(k',k)=\int d^4x\, e^{-k'\cdot x}\, V(x)\psi_k(x). \tag{4}$$

Let us define the scattered part of the wave function $\chi_k(x)$ by

$$\psi_k(x)=\varphi_k(x)+\chi_k(x), \tag{5}$$

where $\varphi_k(x)$ is the free wave term $e^{ik\cdot x}$. The differential equation for $\chi_k(x)$ is

$$\mathfrak{D}\chi_k(x)=V(x)\chi_k(x)+V(x)\varphi_k(x). \tag{6}$$

We can write a Kohn-type variational principle[6] for the T matrix based on Eq. (6):

$$T(k',k)=\int d^4x\,\chi_{k'}{}^*(x)\{\mathfrak{D}-V(x)\}\chi_k(x)+\int d^4x\,\chi_{k'}{}^*(x)V(x)\varphi_k(x)$$
$$+\int d^4x\,\varphi_{k'}{}^*(x)V(x)\chi_k(x)+\int d^4x\,\varphi_{k'}{}^*(x)V(x)\varphi_k(x). \tag{7}$$

This variational principle can be applied when the integrals are well defined. For an energy above threshold, the asymptotic behavior of the wave function $\chi_k(x)$ for large x_4 is a growing exponential,[7] and thus the derivative term in Eq. (7) is not well defined. However, below threshold, the wave function is exponentially damped,[7] and there exists an energy region where all the integrals are convergent. In practice, the application of this variational principle is considerably simpler than the Schwinger variational principle based on the integral equation used by Schwartz and Zemach.[5] Our method amounts to solving the bound-state equations using the method of Schwartz[8] but with an inhomogeneous term.

If we do a partial-wave analysis of these equations, we can calculate $T_l(E^2)$ in the region $4(m^2-\mu^2)<E^2<4m^2$, i.e., between threshold and the second Born contribution to the left-hand cut. The integrals diverge below this point, because $V(x)\varphi_k(x)$ grows exponentially there.

The analytic continuation is performed using the K matrix defined by

$$K_l(E^2)=\frac{T_l(E^2)}{2+2i\rho T_l(E^2)}, \tag{8}$$

where

$$T_l(E^2)=[\exp(i\delta_l)\sin\delta_l]/\rho,$$
$$\rho=(\tfrac{1}{4}E^2-m^2)^{1/2}/8\pi E. \tag{9}$$

The analytically continued unitarity equation implies that $K_l(E^2)$ is analytic in E^2 at threshold, and thus by employing the K matrix, we have removed the threshold branch point.[9] Figure 1 shows the cut structure of $K_l(E^2)$ in the E^2 plane in the region of interest.

Before doing the continuation to the scattering region, we first remove the cut contribution $K_{\rm cut}(E^2)$ between $4m^2-4\mu^2$ and $4m^2-\mu^2$, thus enlarging the region of analyticity. The continuation is done using a Padé form as in Ref. 1,

$$K_l(E^2)-K_{\rm cut}(E^2)$$
$$=\sum_{i=0}^{n}a_i(E^2)^i\left[1+\sum_{i=1}^{n}b_i(E^2)^i\right]^{-1}. \tag{10}$$

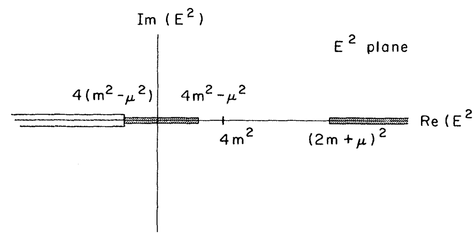

FIG. 1. Cut structure of $K_l(E^2)$ showing the first inelastic threshold and the first two contributions to the left-hand cut.

Table I. A sample of the convergence of the extrapolation for two attractive potentials upon increasing the order of fitting. The S.Z. values were taken from the Schwartz and Zemach calculation described in Ref. 5 (private communication). Cancellations in the fitting generally limit the meaningful size of fitting functions to $n=5$ or 6 for the accuracy of our input numbers for the extrapolation.

n	$(\delta_0/\pi)-1$ [a]	δ_0/π [b]
2	−0.2420	0.2703
3	−0.3147	0.2674
4	−0.3277	0.2672
5	−0.3048	0.2666
6	−0.3083	0.2670
7	−0.3093	0.2663
8	−0.3119	0.2731
S.Z. values	−0.3097	0.2684

[a] $E^2=5.6$, $\lambda=3$, $\mu=m=1$.
[b] $E^2=5.2$, $\lambda=0.7$, $\mu=m=1$.

We extrapolate with these functions and then add $K_{\text{cut}}(E^2)$ back in.

For a strong attractive potential with a deeply bound state, i.e., $\lambda=3$, $\mu=m$, the S-wave phase shift was obtained to at least 2 % in the entire elastic-scattering region. Close to elastic threshold and for weaker potentials, the accuracy was considerably better. The input numbers for the extrapolation were good to four or five places. Table I gives a sample of the stability of the extrapolation.

In conclusion, we find that because of the high accuracy of the variational method below threshold, it is possible to get phase shifts in the elastic region by extrapolation using the simpler differential-equation methods and modest computer time. Roughly two significant figures are lost in the extrapolation.

I wish to thank Professor Charles Schwartz for suggesting this problem and for many enlightening discussions. Also I would like to thank Leonard Schlessinger for helpful discussions on the numerical continuation.

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TEST OF CURRENT ALGEBRA AND ELECTROMAGNETIC C NONINVARIANCE IN THE REACTIONS $\gamma+p\rightarrow\pi+N^*(1238)$ NEAR THRESHOLD

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In a recent Letter Adler[1] has remarked that, assuming the validity of the hypotheses of (a) current algebra[2] and (b) partially conserved axial-vector current,[3] the experimental similarity of the K_2^0 and η decays into three pions within their Dalitz plots "must be regarded as an accident" in the usual picture of η decay. This is because whereas in the linear-matrix-element approximation the above hypotheses give the correct prediction of the $K_2^0\rightarrow\pi^++\pi^-+\pi^0$ matrix element (under the assumption that the observed linear matrix element inside the Dalitz plot can be extrapolated to the points where the pion four-momenta vanish), the same approximation and hypotheses imply that $\eta\rightarrow3\pi$ with total isospin $T=1$ (or 3) is forbidden.[4,5] This forbiddenness exists not only in second order in the usual CP-invariant electromagnetic current, but also in fourth order.[1] The current-algebra success in correlating $K_2^0\rightarrow\pi^+ +\pi^-+\pi^0$ and the experimental similarity of the three-pion decays of K_2^0 and η led Adler[1] to the possibility of circumventing this "accident" by investigating the consequences of an addition to the usual electromagnetic current, J_μ, of a piece K_μ which violates C_{strong} (and T_{strong}) invariance.[6] With suitably postulated equal-time commutation relations for K_μ with the