d-α DOUBLE SCATTERING*

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Azimuthal asymmetries produced in d-α double scattering have been measured at ten second-scattering angles. Both scatterings take place at the same center-of-mass energy. Published sets of phase shifts are inconsistent with the present measurements. It has been demonstrated that α-d scatterings can be used to produce a deuteron beam with useful vector polarization at energies near 10 MeV.

d-α elastic scattering remains of great interest because of its potential usefulness as a source of polarized deuterons and as a deuteron-polarization analyzer. A large amount of work has been done on d-α scattering in the past two or three years.[1-8] However, the number of phase-shift parameters is large, and it appears impossible to derive even approximately correct phase shifts without using a large amount of polarization data.

Prior to the present work, all studies of d-α polarization have made use of (1) the s-wave reaction ^{3}He$(d,p)^4$He as an analyzer or (2) scattering of polarized deuterons obtained from a polarized ion source whose calibration depended, to some extent, on the mirror reaction T$(d,n)^4$He.

Unfortunately the ^{3}He$(d,p)^4$He or T$(d,n)^4$He s-wave angular distributions are sensitive only to the second-rank tensor components of the deuteron polarization and not the vector polarization. Also, at present there is some uncertainty in the analyzing power of this reaction.

Deuteron double-scattering measurements, although experimentally quite difficult, offer an independent method of obtaining polarization information. In view of the rapid development of polarized-ion source techniques, we view our measurements as primarily of value in (1) helping to provide a standard for deuteron-polarization calibration for ion sources, and (2) determination of the d-α analyzing power for possible use in "triple-scattering" type experiments.

The differential cross section for the second scattering in a double-scattering experiment is given by

$$\sigma(\theta_2, \varphi_2) = \sigma_0(\theta_2)[a(\theta_2) + b(\theta_2)\cos\varphi_2 + c(\theta_2)\cos 2\varphi_2],$$

where

$$a(\theta_2) = 1 + \langle T_{20}\rangle_1\langle T_{20}\rangle_2,$$
$$b(\theta_2) = 2(\langle iT_{11}\rangle_1\langle iT_{11}\rangle_2 + \langle T_{21}\rangle_1\langle T_{21}\rangle_2),$$
$$c(\theta_2) = 2\langle T_{22}\rangle_1\langle T_{22}\rangle_2.$$

The unpolarized cross section is $\sigma_0(\theta_2)$ and θ_2 is the c.m. angle. The parameters T_{qk} are those defined by Lakin[9] and further elucidated by Satchler.[10] The tensors with subscript 1 characterize the once-scattered beam and are referred to the incident laboratory direction for the second scattering (as z axis). The tensors with subscript 2 are the polarization values which would have been produced by an unpolarized beam and are referred to the outgoing center-of-mass direction (as z axis). The y axis for both sets is in the direction $\vec{k}_{in} \times \vec{k}_{out}$.

In our experiment, we bombarded a liquid-nitrogen—cooled deuterium gaseous target with 18-MeV α particles from the Los Alamos variable-energy cyclotron. The ~12-MeV deuterons recoiling at 29° (center-of-mass angle $\theta_1 = 122°$) were then passed through a quadrupole focusing lens,[11] slowed with foils to 9 MeV (so that the first and second scatterings would occur at the same c.m. energy) and allowed to enter a ^{4}He gas-filled chamber. Scattered deuterons were detected with four pairs of E-ΔE semiconductor detectors set at a given θ_2 and at $\varphi_2 = 0$, 90, 180, and 270°. (The total counts recorded in each of these φ directions will be referred to as L, U, R, and D, respectively.) An angular resolution of ±4° (c.m.) was used for both first and second scatterings. Coincidence events were recorded in three-parameter form (counter number, E, ΔE) event by event on magnetic tape. The second scattering chamber was designed to be rotated about the beam axis. Thus, a measurement at a single angle θ_2 consisted of four runs with the chamber rotated through 90° after each run. Counter efficiencies, small differences in θ_2, and differences in total integrated current cancel out if the following ratios are defined:

$$\frac{R}{L}=\frac{(a+c)-b}{(a+c)+b}=\frac{(R_1R_2R_3R_4)^{1/4}}{(L_1L_2L_3L_4)^{1/4}}$$

and

$$\frac{U+D}{R+L}=\frac{a-c}{a+c}=\frac{(U_1U_2U_3U_4)^{1/4}+(D_1D_2D_3D_4)^{1/4}}{(R_1R_2R_3R_4)^{1/4}+(L_1L_2L_3L_4)^{1/4}},$$

where, for example, R_1 is the number of counts recorded in telescope 1 when it is at $\varphi_2=180°$. The angles $\varphi_2=90°$ and 270° (U and D) are in principle equivalent; however, the ratio $(U+D)/(R+L)$ is essentially independent of beam position and direction and therefore has a very small systematic error. As in the spin-$\frac{1}{2}$ case, R/L is sensitive to the alignment and position of the beam in the horizontal plane. Accordingly, the chamber was aligned to within 0.15° by a combination of (1) angle determination by Rutherford scattering from gold and (2) position determination by means of nuclear emulsions. The machine instability limited the alignment quality to the value quoted. This uncertainty could result in an error in R/L as large as 3 % for the three smallest angles but no more than 1 % for the remaining angles.

In Fig. 1 the results obtained for the two ratios are shown together with the predictions from the phase shifts obtained by McIntyre and Haeberli.[5] Although geometrical corrections have not been made, runs with better angular resolution gave consistent results.[12] The agreement between the predicted and measured asymmetries is poor. It should be pointed out that the asymmetries are essentially proportional to the tensors which characterize the first scattering. Thus, small changes in the phase shifts can produce a large change in scale factor without appreciably changing the shape of the curves. For example, although the predicted values of $(U+D)/(R+L)$ are all within 1 % of unity, the shape of the curve is similar to the observed curve.

In Fig. 2 we show the values of $\langle iT_{11}\rangle_2$ extracted from the data of Fig. 1. Several assumptions are made in order to do this: (1) The beam polarization $\langle iT_{11}\rangle_1$ was determined to be −0.28 by normalization to the Wisconsin measure-

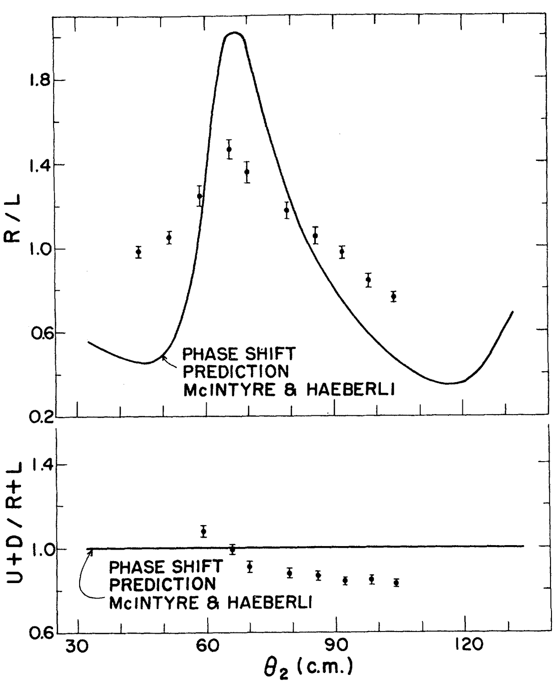

FIG. 1. The azimuthal asymmetries R/L and $(U+D)/(R+L)$ produced in $^4He(d,d)^4He$ double scattering. The energy for both scatterings corresponds to $E_d=9.0$ MeV. The first scattering angle is 122° (c.m.). The predicted asymmetries from the phase shifts of McIntyre and Haeberli are shown.

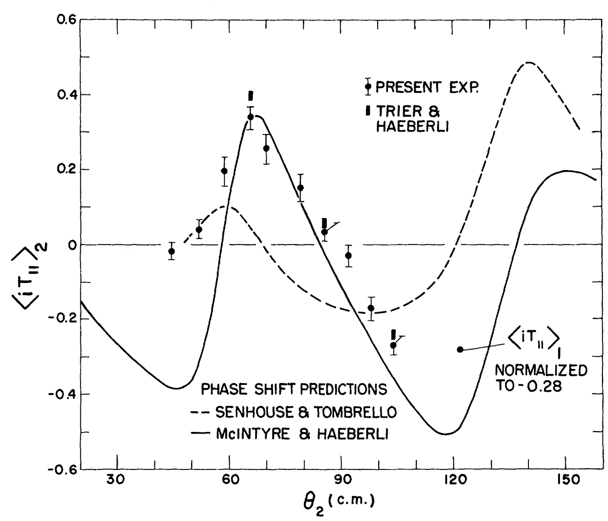

FIG. 2. Deuteron vector polarization $\langle iT_{11}\rangle_2$ extracted from the asymmetry data shown in Fig. 1. The predictions from the phase shifts of McIntyre and Haeberli and of Senhouse and Tombrello are shown. The value (−0.28) of $\langle iT_{11}\rangle_1$ was obtained by normalization to the measurements of Trier and Haeberli (solid rectangles).

ments.[7,8] (2) $\langle T_{21}\rangle_1\langle T_{21}\rangle_2$ is small compared to $\langle iT_{11}\rangle_1\langle iT_{11}\rangle_2$. (3) $\langle T_{20}\rangle_1\langle T_{20}\rangle_2$ is small compared to one. With these approximations, which are consistent with all available data,[3-8] we can write

$$\langle iT_{11}\rangle_2 = \frac{1}{0.28}\left[\frac{R-L}{R+L+U+D}\right],$$

from which Fig. 2 was prepared. Even when the data are presented in this form, where most of the sensitivity to the first-scattering tensors is removed, agreement between the available predictions and the data is poor.[13]

In conclusion, the present results appear to be in good agreement with earlier measurements (see Fig. 2); however, presently published d-α phase shifts are inconsistent with the present data. It has been demonstrated that a deuteron beam with useful vector polarization can be produced by α-d scattering.

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[13]It has already been shown in Refs. 5 and 7 that the phase shifts of Ref. 2 and those of L. S. Senhouse, Jr. [thesis, California Institute of Technology, 1964 (unpublished)], are incorrect.

PHASE SHIFTS FROM THE BETHE-SALPETER DIFFERENTIAL EQUATION*

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A new method of calculating phase shifts for a Bethe-Salpeter equation is presented. The scattering amplitude is calculated below elastic threshold using the differential equation and variational methods, and then continued to the elastic-scattering region to find phase shifts.

Recently Schlessinger and Schwartz presented a method of finding phase shifts in potential theory by solving the Schrödinger differential equation for the scattering amplitude for energies below threshold and continuing it to the scattering region.[1] In this paper we report a variation on their method, involving an on-mass-shell continuation, that has proven successful in solving a Bethe-Salpeter equation.[2] The on-shell amplitude satisfies a simple unitarity relation, and this can be used advantageously in performing the continuation. We calculate below threshold in order to avoid the problems of solving a singular integral equation for the phase shifts.[3]

The differential Bethe-Salpeter equation in the ladder approximation for spinless particles of equal mass m is of the form

$$\mathfrak{D}\psi_k(x) = V(x)\psi_k(x), \tag{1}$$