PRECISE MEASUREMENTS OF MUONIC ISOTOPE SHIFTS FOR 29 NUCLIDES, IN PARTICULAR, FOR THE Ca ISOTOPES 40, 42, 44, and 48*

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Precise measurements of the isotope shift for muonic K transitions are reported for $Si^{28,29,30}$, $K^{40,41}$, $Ca^{40,42,44,48}$, $Fe^{54,56,57}$, $Ni^{58,60,61,62}$, $Cu^{63,65}$, $Sr^{86,87,88}$, $Zr^{90,92}$, and $Sn^{116,117,118,119,120,122}$. The shifts Ca^{40}-Ca^{44} and Ca^{40}-Ca^{48} are of special interest, as they confirm the conclusions from electron scattering. The shifts Sr^{86}-Sr^{87} and Sr^{86}-Sr^{88} are found to be negative, indicating that the heavier isotope has a smaller charge radius.

We present here preliminary values of the isotope shifts (I.S.) observed in a recent experiment on the muonic x-ray spectra from a large number of isotopically enriched targets. The apparatus used in this experiment, designed through a collaborative effort between groups at our two institutions, will be described elsewhere[1]; here we list some of its more relevant features: (1) simultaneous detection, in a Ge(Li) crystal, of the photon events from up to four distinct, electronically labeled targets; (2) simultaneous determination, for each photon event, of its pulse height and time of arrival with respect to the muon-stop signal; (3) continuous calibration of the system gain (energy scale), by means of precision pulses and radioactive sources; (4) electronic stabilization of the system gain and on-line computation of the gain-versus-energy relation; (5) use of a large (17-cm^3)[2] Ge(Li) detector surrounded by a cylindrical NaI shield; and (6) possibility of recording events coincident and/or anticoincident with the NaI shield simultaneously.

The advantages of features (1) through (3), in particular, for the measurement of isotope shifts, have already been established by earlier work at the Chicago synchrocyclotron.[3] Feature (4) is primarily useful for precise determinations of transition energies, while (5) and (6) enable one to obtain exceptionally clean data at high event rates. Spectra of transitions of less than ~2500 keV (i.e., from all but the Sn targets) were collected in the anticoincidence mode of (6), the aim being to enhance the full-energy peak to Compton background ratio; more energetic transitions (i.e., K x rays of Sn) were also recorded without the anticoincidence requirement.

Targets to be compared for isotope shifts were always matched in chemical composition and size (generally 5×5 cm), and as far as possible in thickness (generally 2 to 4 g/cm^2). The isotopic enrichment of all but the Sr^{87} (88%) and Sr^{88} (83%) targets exceeded 90%, and for a majority it even exceeded 95%. The four targets to be viewed jointly were juxtaposed in a "massless" vertical frame (placed at 45° to the muon beam), separated from each other by the arms of a cross-shaped scintillator. This cross, integral with a flat anticoincidence counter, prevented cross talk (scattering of muons from one target into another) very effectively. The shift Ca^{40}-Ca^{48} was determined, because of the minute amount of Ca^{48} (3 g of CaO) available, in a special setup that involved two cylindrical samples one above the other, but was functionally similar to the arrangement just described.

The stop rates in the four target positions were carefully equalized by adjusting the beam-transport optics (muon channel and bending magnet). Typical stop rates were ~35 muons/g sec. The Ge(Li) detector was located, at a distance of about 11 cm from the center of the target assembly, on a line perpendicular to the incident beam. Thus, even with equal-mass targets, two targets appeared "weaker" during each run. To eliminate possible systematic shifts due to inequivalent target positions, each comparison of four targets was made by averaging over at least one cycle of four runs. During these, the relative positioning of the targets was kept fixed, but each of the targets was made to assume in turn, through rotation of

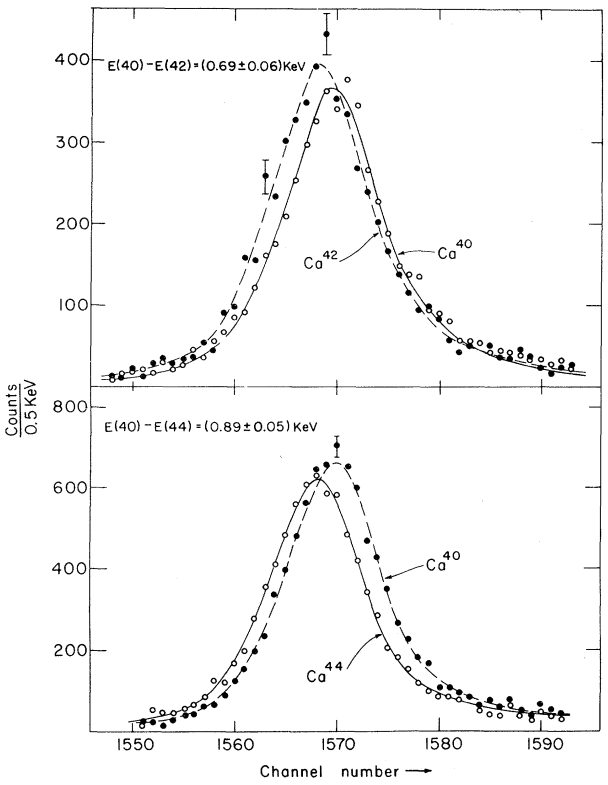

FIG. 1. Muonic $K\alpha$ x-ray spectra from Ca^{40}, Ca^{42}, and Ca^{44}. The curves drawn through the data points are best fits to a numerical line shape derived from an independent set of Ca^{40} data. Note that the fine-structure splitting (1.5 keV) is unresolved.

the frame, all four available positions. A comparison of the spectra from identical targets furthermore showed that position-induced shifts were absent, i.e., below the level of our quoted accuracy.

Typical spectra, corrected for prompt (accidentals) and delayed (capture-gamma) radiations, are shown in Figs. 1-3. Figure 2, obtained with an unusually small sample (Ca^{48}), is presented because of the special interest of the Ca^{40}-Ca^{48} shift. In all cases the accidentals proved to be negligible. The I.S. were extracted from the spectra such as those shown by a novel method (customarily one determines the I.S. by comparing the centroids of the peaks,[3,4] or possibly by means of a multiparameter fit[5] to the observed line shape of each transition). This method consists in (a) using one of the spectra (say Ca^{40}) under comparison, possibly one determined with particularly good statistics, as a "master spectrum"; (b) smoothing this spectrum to obtain a (normalized) "master curve"; (c) fitting this master curve to the

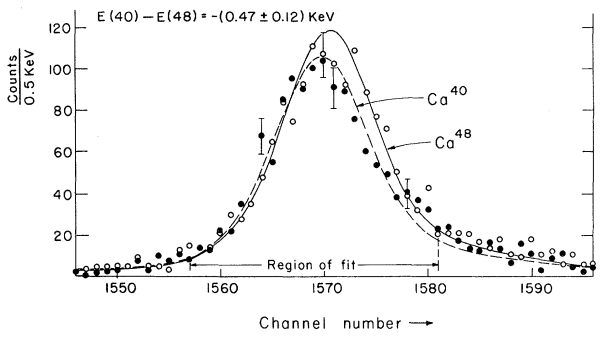

FIG. 2. Muonic $K\alpha$ x-ray spectra from Ca^{40} and Ca^{48}. The curves drawn through the data points are best fits to a numerical line shape derived from an independent set of Ca^{40} data. Note that the fine-structure splitting (1.5 keV) is unresolved.

data points of the comparison spectrum (say Ca^{44}); and (d) determining, if the fit so obtained is statistically acceptable, the relative shift of the "master" and the fitted spectra. Clearly, our method is conceptually equivalent to the fitting by elaborate analytic representations of the presumed line shape; it is, however, far more convenient. The method, valid only for samples of high isotopic purity, was sub-

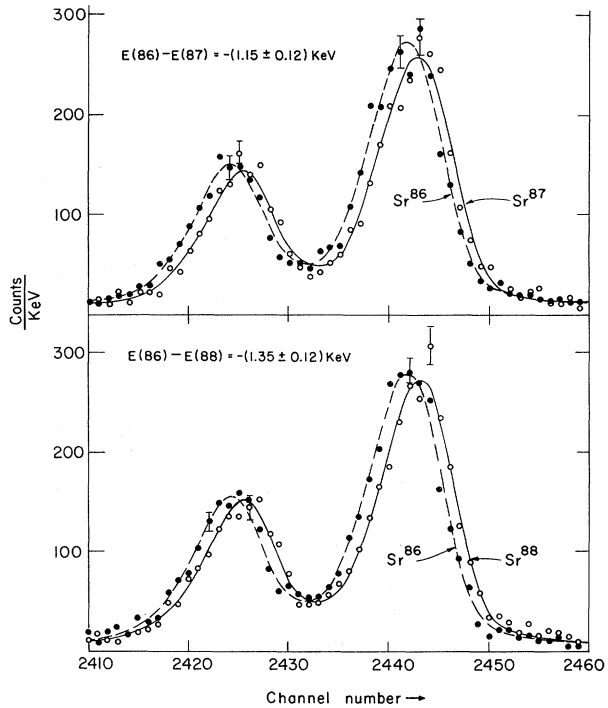

FIG. 3. Muonic $K\alpha$ x-ray spectra from Sr^{86}, Sr^{87}, and Sr^{88}. The curves drawn through the data points are best fits to a numerical line shape derived from Sr^{86} data. The fine-structure splitting of ~17.3 keV is clearly resolved.

Table I. Isotope shifts in Ca muonic K x rays (keV).

Isotope pair	Observed shift[a]	Reduced mass shift	Volume shift Observed	Volume shift e^- scattering prediction	$A^{1/3}$ predictions[b] $c=c_0A^{1/3}$	$A^{1/3}$ predictions[b] $R=R_0A^{1/3}$
40-42 ($K\alpha$)	$+0.69\pm0.06$	-0.11	$+0.80\pm0.06$		+1.20	+1.61
40-44 ($K\alpha$)	$+0.89\pm0.05$	-0.20	$+1.09\pm0.05$	$+0.90\pm0.20$[c]	+2.40	+3.22
($K\beta$)	$+1.04\pm0.20$	-0.24	$+1.28\pm0.20$			
40-48 ($K\alpha$)	-0.47 ± 0.12	-0.37	-0.10 ± 0.12	-0.15 ± 0.02[d]	+4.80	+6.44

[a]Corrected for isotopic impurities.
[b]$c_0=1.08$ F, $R_0=1.20$ F.
[c]Ref. 5.
[d]Ref. 6.

jected to many checks: interchange of the roles of master and fitted spectra, variation of the range of fitting, etc., but in all cases the derived shifts were within the range of expected statistical fluctuations.

Table I summarizes our results for the Ca isotopes. We deal with these separately, because detailed studies of the differences in the charge distributions of these particular isotopes are available from the work of Hofstadter and collaborators.[6,7] The shifts of the K lines depend, of course, essentially only on the isotopic differences in $\langle r^2\rangle$; our results are however useful to corroborate the rather astonishing conclusions of Ref. 7, viz. that Ca^{48} has a smaller $\langle r^2\rangle$ than Ca^{40}. For both Ca^{40}-Ca^{48} and Ca^{40}-Ca^{44}, our volume shifts are seen to be in good agreement with the electron scattering predictions; as regards the 40-44 volume I.S., our observed value confirms an earlier result obtained (with NaI) at Chicago [(0.9 ± 0.3) keV][8] rather than a similar Columbia measurement [(0.5 ± 0.3) keV].[4] It also seems to differ substantially from a very recent preliminary Columbia result [(1.21 ± 0.08) keV].[9]

Table II lists our results for elements other than Ca. Next to the "observed" I.S., we have indicated the predictions from two simple models: (a) The rms charge radius increases as $A^{1/3}$, (b) the nuclear charge distribution is of the Fermi type, with a half-density radius c varying as $A^{1/3}$, but with a constant skin thickness t (= 2.4 F). The second model is in somewhat better agreement with the data.

The results for the Sr isotopes deserve special attention. Both observed I.S. are negative, in contrast with the general trend. There exists little previous information concerning the

Table II. Summary of isotope-shift results in muonic K x rays (keV).

Isotope pair	Isotope shift (observed)[a]	Reduced mass effect	Volume effect Observed	Volume effect $A^{1/3}$ predictions[b] $c=c_0A^{1/3}$	Volume effect $A^{1/3}$ predictions[b] $R=R_0A^{1/3}$
Si^{28}-Si^{29}	-0.08 ± 0.04	-0.05	-0.03 ± 0.04	+0.25	+0.35
Si^{28}-Si^{30}	$+0.00\pm0.04$	-0.10	$+0.10\pm0.04$	+0.50	+0.70
K^{39}-K^{41}	$+0.35\pm0.06$	-0.10	$+0.45\pm0.06$	+1.19	+1.60
Fe^{54}-Fe^{56}	$+2.97\pm0.12$	-0.09	$+2.76\pm0.10$	+2.10	+2.75
Fe^{56}-Fe^{57}	$+1.25\pm0.20$	-0.04	$+1.14\pm0.17$	+1.05	+1.40
Ni^{58}-Ni^{60}	$+3.14\pm0.14$	-0.09	$+3.23\pm0.14$	+2.80	+3.65
Ni^{60}-Ni^{61}	$+0.97\pm0.18$	-0.04	$+1.01\pm0.18$	+1.40	+1.85
Ni^{60}-Ni^{62}	$+2.00\pm0.17$	-0.09	$+2.09\pm0.17$	+2.80	+3.65
Cu^{63}-Cu^{65}	$+2.31\pm0.16$	-0.09	$+2.40\pm0.16$	+2.85	+3.70
Sr^{86}-Sr^{87}	-1.15 ± 0.10	-0.04	-1.11 ± 0.10	+2.40	+3.00
Sr^{86}-Sr^{88}	-1.35 ± 0.12	-0.08	-1.27 ± 0.12	+4.80	+6.05
Zr^{90}-Zr^{92}	$+8.2\pm0.4$	-0.07	$+8.3\pm0.14$	+5.30	+6.65
Sn^{116}-Sn^{120}	$+11.81\pm0.20$	-0.12	$+11.93\pm0.20$	+14.4	+17.6
Sn^{117}-Sn^{120}	$+9.28\pm0.15$	-0.09	$+9.37\pm0.15$	+10.8	+13.2
Sn^{118}-Sn^{120}	$+5.17\pm0.10$	-0.06	$+5.23\pm0.10$	+7.2	+8.8
Sn^{119}-Sn^{120}	$+3.42\pm0.17$	-0.03	$+3.45\pm0.17$	+3.6	+4.4
Sn^{120}-Sn^{122}	$+4.95\pm0.15$	-0.06	$+5.06\pm0.15$	+7.2	+8.8

[a]Corrected for isotopic impurities.
[b]$c_0=1.08$ F, $R_0=1.20$ F.

I.S. in Table II, except for the case of Sn. For this element, optical[10] and muonic[11] I.S. data are available. The optical I.S. for Sn, while based on accurate measurements, are affected by dubious corrections[12] invented to allow for the specific mass shift, of essentially unknown magnitude. While our results agree qualitatively with Stacey's,[10] it appears preferable to accept our results, and to use them to compute the much-needed specific mass shifts from Stacey's data. We shall discuss this point elsewhere.

Our experiments became possible only through the cooperative attitude of Professor H. L. Anderson, who arranged for the sharing of the equipment, and the untiring help of Dr. G. Rogosa (U. S. Atomic Energy Commission), who coordinated the procurement of isotopically enriched targets. We are also indebted to Professor E. Shrader and Professor H. Newson, who generously lent us isotopes which they needed themselves. We are grateful to Mr. D. White (Argonne National Laboratory) and Dr. K. Haefner for much assistance in preparing usable targets. We gratefully acknowledge the valuable help of Mr. Frank Castorf and Mrs. Serena Torres with programming. Finally, we wish to thank those members of the National Research Council-Chicago collaboration who did not directly participate in this work for sharing their apparatus and programs with us.

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[1]H. L. Anderson, R. J. McKee, R. Barton, F. Castorf, C. K. Hargrove, E. P. Hincks, and J. McAndrew, to be published.

[2]Kindly loaned to us by Atomic Energy of Canada, Ltd., Chalk River Nuclear Laboratories.

[3]V. L. Telegdi, in Proceedings of the Williamsburg Conference on Intermediate Energy Physics, February, 1966 (unpublished), p. 77; R. D. Ehrlich, D. Fryberger, D. A. Jensen, C. Nissim-Sabat, R. J. Powers, B. A. Sherwood, and V. L. Telegdi, Phys. Letters 23, 468 (1966). The idea of viewing several targets simultaneously with a single detector is due to R. C. Cohen, S. Devons, A. D. Kanaris, and C. Nissim-Sabat, Phys. Rev. 141, 48 (1966).

[4]Cohen et al., Ref. 3; C. Nissim-Sabat, thesis, Columbia University Nevis Report No. 129, 1965 (unpublished).

[5]H. L. Anderson, R. J. McKee, C. K. Hargrove, and E. P. Hincks, Phys. Rev. Letters 16, 434 (1966).

[6]R. Hofstadter, G. K. Nöldeke, K. J. van Oostrum, L. Suelzle, M. R. Yearian, B. C. Clark, R. Herman, and D. G. Ravenhall, Phys. Rev. Letters 15, 758 (1965).

[7]K. J. van Oostrum, R. Hofstadter, G. K. Nöldeke, M. R. Yearian, B. C. Clark, R. Herman, and D. G. Ravenhall, Phys. Rev. Letters 16, 528 (1966).

[8]J. A. Bjorkland, S. Raboy, C. C. Trail, R. D. Ehrlich, and R. J. Powers, Phys. Rev. 136, B341 (1964).

[9]E. Macagno, R. Barrett, S. Bernow, S. Devons, I. Duerdoth, D. Hitlin, J. Kast, J. Rainwater, K. Runge, and C. S. Wu, Bull. Am. Phys. Soc. 12, 75 (1967).

[10]D. N. Stacey, Proc. Roy. Soc. (London) A280, 439 (1964). Note that the reduced-mass correction is done with the wrong sign.

[11]T. T. Bardin, R. C. Barrett, R. C. Cohen, S. Devons, D. Hitlin, E. Macagno, C. Nissim-Sabat, J. Rainwater, K. Runge, and C. S. Wu, Phys. Rev. Letters 16, 429 (1966).

[12]W. H. King, J. Opt. Soc. Am. 53, 638 (1963).

DETECTION OF THE SINGLET DEUTERON ($\hat{d}$) AND THE REACTION $Be^9(p,\hat{d})Be^8$†

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When a singlet deuteron—a neutron and proton in the $S=0$, $T=1$ state, which we designate $\hat{d}$—is emitted in a nuclear reaction, it breaks up almost instantaneously, but not until it is outside the range of interaction with the residual nucleus. Thus, the momentum of the n-p system is conserved, and the momenta of the neutron and proton are the vector sum of the momentum of the original $\hat{d}$ plus the momentum of the breakup which must be equal and opposite for the neutron and proton. The total energy available to the system may be divided between the energy of the $\hat{d}$ ($E_{\hat{d}}$) and the energy available in the break-up (E_{BU}) in a variety of ways; this is governed by a density-of states function which has been calculated by Simpson,[1] using a theory due to Phillips, Griffy, and Biedenharn.[2]

Attempts to detect singlet deuterons by observing only the proton or only the neutron have been reported by Temmer.[3] In the experiments reported here, the reaction $Be^9(p,pn)Be^8$ is