average energy of the electrons, calculated from the Doppler broadening of the $I_2 > I_1$ peak (4.35 keV FWHM) compared to the 569.7-keV peak from Bi^{207} (353 keV FWHM), is 6.3 eV. That of the $I_1 > I_2$ peak (3.98 keV FWHM) is 3.3 eV.

If the lifetime spectra are resolved into three components, we obtain the following results:

(1) The long-lifetime component ($\tau_L = 2.01 \pm 0.04$ nsec, ~20 %) has a broad angular distribution.

(2) The intermediate-lifetime component ($\tau_I = 0.64 \pm 0.10$ nsec, ~13 %) has an angular distribution narrower than that of the long-lifetime component. It is believed that this component is due to free-positron annihilations with outer atomic electrons.[6] The extra momentum contribution in the long-lifetime component may be due to the orbital momentum of the positron bound in positronium.

(3) The short-lifetime component ($\tau_S = 0.33 \pm 0.02$ nsec, ~67 %) has a complex origin; besides a narrow component due to the annihilation of singlet positronium, there is some other decay mechanism with large momentum but short lifetime.

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DISPERSION IN SECOND SOUND AND ANOMALOUS HEAT CONDUCTION AT THE LAMBDA POINT OF LIQUID HELIUM*

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The absence of a characteristic length at the lambda temperature T_λ of liquid helium is used to determine the wave-number dependence of the phase fluctuations and the second-sound dispersion relation $\omega = ak^{3/2}$ where ω and $\vec{k}$ are the frequency and wave number and $a \approx 0.1$ cm$^{3/2}$ sec^{-1}. Further predictions are $|T-T_\lambda|^{-1/3}$ singular temperature variation for second-sound damping ($T < T_\lambda$) and the thermal conductivity ($T > T_\lambda$).

The purpose of this note is to point out some dynamical consequences of the absence of any characteristic length beyond the atomic dimensions in an extended homogeneous system at its phase transition. This similarity property holds neither below nor above the transition temperature, where a temperature-dependent correlation length, which becomes much greater than atomic dimensions, is manifested. It is precisely at the transition temperature that this length becomes infinite and is no longer relevant as a characteristic unit. The similarity property then provides a useful means of connecting the critical behavior of the system above the transition with that below. In this way we predict an anomalous dispersion of second sound at the liquid-helium lambda temperature, T_λ, of the form $\omega \propto k^{3/2}$ (where ω and k are the frequency and wave number, respectively), and a relation between second-sound damping and the heat conductivity, for temperatures T below and above T_λ, respectively. Both of these quantities are predicted to vary as $|T-T_\lambda|^{-1/3}$, and recent experimental data are consistent with this singular behavior.[1,2]

A complete description of the hydrodynamics of the superfluid helium at long wavelengths entails the kinetically conjugate variables of quantum mechanical phase and mass density for the superfluid, and the corresponding variables of normal-fluid velocity and entropy density for the normal fluid. But because of the finite compressibility of the liquid, the quantum mechanical phase fluctuations dominate the behavior of the Bose field at long wavelengths and consequently determine the single-particle Green's function at separations much larger than atomic dimensions according to the following manner:

$$G(r) \propto \langle e^{-i\varphi(\vec{x}')} e^{i\varphi(\vec{x})} \rangle \propto \exp[F(r)-F(0)]. \quad (1)$$

$\varphi(\vec{x})$ is the phase of the superfluid condensate at point $\vec{x}$. Mass-density fluctuations affect $G(r)$ at small values of r but do not contribute to its large-separation dependence. Because the compressibility remains finite as the lambda point is approached, the long-wavelength density fluctuations are limited and remain uncritical. Consequently, the phase and entropy flucutations are effectively the only variables free for contributing to critical fluctuations. The correlation between two points in the liquid at $\vec{x}'$ and $\vec{x}$, separated by the distance $\vec{r} = \vec{x}' - \vec{x}$, is weakened by the differing phases at these two points.[3,4] Assuming that the higher order correlations can be neglected,[5] we can identify the function F with the correlation function for the phase field itself and can write it in terms of the mean-square fluctuations of the Fourier components,

$$F(r) = \langle \varphi(\vec{x}')\varphi(\vec{x}) \rangle = \int f(\vec{k}) e^{i\vec{k}\cdot\vec{r}} d\vec{k}, \quad (2)$$

where

$$f(k) = \langle |\varphi_{\vec{k}}|^2 \rangle. \quad (3)$$

Because of the isotropy of the liquid, both $F(r)$ and $G(r)$ are functions only of the magnitude of the separation r. The similarity property is expressed by

$$G(r) \propto r^{-1-\eta}, \quad (4)$$

where the exponent $1+\eta$ is some constant.[6] Substituting into Eq. (1) we find that the phase fluctuations have to be such that

$$F(0) - F(r) = (1+\eta) \ln r + \text{const}. \quad (5)$$

It is easily verified that the following wave-number dependence for the phase correlation function satisfies Eqs. (2) and (5):

$$f(k) = [2\pi^2(1+\eta)]/k^3. \quad (6)$$

At temperatures slightly below the lambda point the above analysis breaks down at very long wavelengths and the phase fluctuations at these long wavelengths have to be computed differently. Here, the similarity principle does not apply and we turn instead to standard linearized two-fluid hydrodynamics.[7,8] The two-fluid model requires a kinetic energy associated with superfluid flow of the form

$$E_K = \tfrac{1}{2}\rho_s v_s^2 V = \tfrac{1}{2}\sum_{\vec{k}} (\rho_s k^2/m^2) |\varphi_{\vec{k}}|^2. \quad (7)$$

Here ρ_S is the superfluid mass density, and V is the volume; v_S has the usual gradient relationship with the phase, and we are employing units in which Boltzmann's constant is unity and Planck's constant equals 2π. The classical equipartition theorem requires the following strength of phase fluctuation:

$$f_T(k) = m^2 T/\rho_s k^2 = 4\pi\xi/k^2, \quad (8)$$

where

$$\xi(T) = m^2 T/4\pi\rho_s = (45\ \text{Å})(T_\lambda - T)^{-2/3}, \quad (9)$$

and the temperatures are in millidegrees. For convenience we have introduced the correlation length ξ of Josephson,[9] Tyson and Douglass,[10] and Kane and Kadanoff,[11] representing the reciprocal of the wave number for which the short-wavelength and long-wavelength expressions, Eqs. (6) and (8), become approximately equal.

We can characterize the dynamics of the problem without entering into all of the details of the equations of motion by introducing a field $\pi(\vec{x})$ canonically conjugate to $\varphi(\vec{x})$, whose fluctuations define a correlation function

$$g(k) = \langle |\pi_{\vec{k}}|^2 \rangle. \quad (10)$$

In the long-wavelength region where two-fluid hydrodynamics applies, we see from the equation for the time rate of change of phase and from the continuity equation for entropy that $\pi(\vec{x})$ is to be identified with the fluctuations in entropy density measured in units of S/n, the average entropy per particle. The correlation functions correspond to static susceptibilities for the two fields individually, and their product determines a characteristic frequency $\omega_{\vec{k}}$ by

$$\omega_{\vec{k}} = T[f(\vec{k})]^{-1/2}. \quad (11)$$

Substitution of the entropy fluctuation-dissipation theorem for the specific heat $c_p(T)$,

$$g_T(0) = n^2 c_p(T)/S^2, \quad (12)$$

into Eq. (11) gives

$$c_2^{\ 2} = \frac{\omega_k^{\ 2}}{k^2} = \frac{TS^2}{\rho c_p}\frac{\rho_s}{\rho} \quad (13)$$

for the velocity of second sound. Instead of ρ_S/ρ, the standard expression[12] contains ρ_S/ρ_n as a factor. The difference is proportional to ρ_S/ρ_n, which measures the strength of the noncritical density fluctuations relative to the critical fluctuations of the second-sound modes and becomes negligibly small at T_λ.

The empirical logarithmic singularity in the specific heat corresponds at T_λ to the homogeneous function[13]

$$g(k) = -A\ln(k/k_c), \quad (14)$$

where $A = 0.95n^3/S^2 = 1.48n$ and $k_c \approx 20$ Å^{-1}. Substituting Eqs. (14) and (6) into Eq. (11) now determines the critical dispersion for second sound as

$$\omega_k = ak^{3/2}, \quad (15)$$

where $a = T_\lambda[3\pi^2 A\ln(k_c/k)]^{-1/2} \approx 0.1$ cm$^{3/2}$ sec^{-1}. Thus, although the velocity of second sound vanishes as the lambda temperature is approached, this is true only in the limit of very long wavelengths. Equation (15) means that at any finite wavelength the corresponding frequency of second sound should be expected to interrupt its decrease as the lambda point is approached. It is important to note that this expected dispersion is of an anomalous type,[14] i.e., the velocity increases with increasing frequency or wave number, rather than decreasing as is common in the familiar problems of lattice dynamics, optics of transparent media, etc. It is also essential to recognize that the second-sound modes must become critically damped at T_λ in order for the frequency spectrum of the π fluctuations to pass continuously into a nonresonant diffusion spectrum representing ordinary heat conduction[15] for $T > T_\lambda$. The breadth of this spectrum as well as the width of the second-second resonance for $T < T_\lambda$ are given by a diffusion term Dk^2. Critical damping at T_λ requires from Eqs. (15) and (9)

$$Dk^2 \approx 2\omega_k \approx 2ak^{3/2}, \quad (16)$$

$$D \approx 2ak^{-1/2} \approx 2a\xi^{1/2} \approx 1.2\times10^{-4}|T-T_\lambda|^{-1/3}\ \mathrm{cm^2\ sec^{-1}\ mdeg^{1/3}}, \quad (17)$$

where, as before, we have taken the joining of the different curves to occur at a wave number approximately equal to the reciprocal of the correlation length. For a given wave number $\vec{k}$, critical behavior occurs only within a temperature width[16] of $\Delta T \approx 3\times10^{-10}k^{3/2}$ cm$^{3/2}$ mdeg.

Recent measurements of second-sound attenuation[2] and of thermal conductivity[1] are consistent both in relative temperature dependence and in absolute magnitude with the singular behavior predicted by Eq. (17). Direct experimental verifications, by cold-neutron scattering or Brillouin scattering, of the basic dispersion relation Eq. (15) would be extremely valuable. The shape of the frequency spectrum of the entropy density fluctuations at T_λ is expected to be that of a critically damped oscillator and has to be the same at all wave numbers k (simply expanding or contracting as $k^{3/2}$). Otherwise an absolute length could be determined from the spectra, which would be inconsistent with the similarity property.

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NUCLEAR COOLING APPLIED TO MEASUREMENTS IN He^3†

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An apparatus for the use of nuclear adiabatic demagnetization has now produced and maintained submillidegree temperatures for as long as seven hours. It has cooled 0.45 cm^3 of He^3 at various pressures to 4 mdeg K in the liquid state and to approximately 7 mdeg K in the solid state. Measurements of the nuclear-spin susceptibility, specific heat, and thermal-boundary impedance have been made to these temperatures in the liquid and some preliminary observations made in the solid.

The first measurements[1] of the properties of He^3 below 8 mdeg K indicated a specific-heat anomaly at 5.5 mdeg K which was interpreted as evidence of a predicted superfluid transition[2] in liquid He^3. A large number of measurements made since that time by the Illinois group[3] have exhibited no anomalies and no radical departure from the predictions of the Landau theory of Fermi fluids. The measurements reported here use a different cooling method, different thermometers, and different measuring techniques from those of the other experiments. The results indicate that the spin susceptibility is independent of temperature to within ±5% between 4 and 30 mdeg K. Thus, there is no transition of the type predicted in which the spins align antiparallel and the susceptibility decreases. The specific-heat measurements are less accurate but if there is an anomaly it must be much smaller than that reported by Peshkov.

The apparatus, similar to one reported earlier,[4] involves three cooling stages below 1°K: a He^3 refrigerator at 0.35°K, a cerium magnesium nitrate salt at 0.013°K, and the nuclear cooling stage. Figure 1 is a schematic diagram of the nuclear stage. There are two important thermal barriers between the cold nuclei in the high-field magnet at the bottom of the sample and the nuclei of the same copper wires in the Helmholtz pair where the temperature is measured. The first is the spin-relaxation process by which energy can be transferred from the conduction electrons to the cold nuclei. The second is the electronic conduction along the copper wires. The relaxation is determined by the Korringa relation which says that the relaxation time, T_1, times the temper-