# STRUCTURE OF THE $\alpha$ PARTICLE FROM ELASTIC PROTON SCATTERING*

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Elastic electron scattering has provided a wealth of information about the gross structure of nuclei.[1] For light targets first Born approximation leads to a reasonably successful description of the process.

The amplitude for electron scattering from a nucleus $N$ is written

$$A_{eN}(k,q^2)=A_{\rm Mott}(k,q^2)F(q^2), \tag{1}$$

where $A_{\rm Mott}$ is the usual Mott scattering amplitude, $k$ is the momentum, and $q$ is the momentum transfer. The quantity $F(q^2)$ is the charge form factor directly related to the charge density, $\rho(r)$, through

$$F(q^2)=\int\rho(r)e^{i\vec{q}\cdot\vec{r}}d^3r.$$

For $^4$He at large $q^2$ the form factor is found to deviate[2] from the Gaussian found in previous small-momentum-transfer experiments.[3] In fact, a minimum has been found[2] at $q\sim10\ \mathrm{F}^{-2}$.

It might be thought that a similar impulse approximation could be applied to the scattering of very high-energy protons, with $A_{\rm Mott}$ replaced by the appropriate combination of nucleon-nucleon amplitudes. If the matter and charge distributions are similar, the resulting elastic scattering should reflect the form factor as found in electron scattering.

Recently, Palevsky et al.[4] have measured the elastic scattering of 1-GeV protons from $^4$He. The data, illustrated in Fig. 1, show a sharp minimum at $q^2\approx6\ \mathrm{fm}^{-2}$, and possibly a second minimum at $q^2\approx23\ \mathrm{fm}^{-2}$. Since, over the important range of momentum transfer, the nucleon-nucleon cross sections are smoothly varying, these data imply either that the form factor has a structure rather different from that found in electron scattering from $^4$He, or that the simple impulse approximation is not valid here.

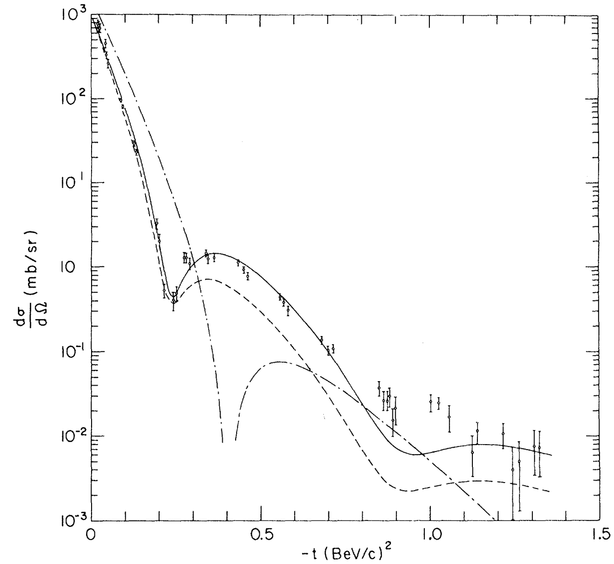


FIG. 1. Differential cross section for elastic $p$-$^4$He scattering. The experimental points are from Ref. 4. The solid curve is the result of the theory with $\alpha^2=0.72$, $\gamma^2=0.2$; the dashed curve has $\alpha^2=0.535$, $\gamma^2=0$; the dot-dashed curve is the impulse approximation with $\alpha^2=0.72$, $\gamma^2=0.2$.

The idea that the matter and charge distributions are so different, i.e., that isospin is not a good quantum number, seems rather drastic for such a light nucleus. On the other hand, we have no reason to believe in Born approximation for strongly interacting particles, that is, multiple scatterings might be important.

A formalism which takes into account these higher order terms has been given by Glauber.[5] The fundamental assumption of this theory is that the phase shifts for a nucleon scattering from a nucleus $N$ are given by the sum of individual nucleon-nucleon phase shifts suitably averaged over the density. For elastic scattering the amplitude is given by

$$A(k,q)=\frac{ik}{2\pi}\int d^3r_1\cdots d^3r_N\rho(r_1\cdots r_N)\delta(r_1+\cdots+r_N)$$
$$\times\int d^2b\,e^{iq\cdot b}\left\{1-\prod_{j=1}^{N}\left[1-\frac{1}{2\pi k}\int d^2q'\exp[-iq'\cdot(b-s_j)]A_j(k,q')\right]\right\}, \tag{2}$$

where $\rho(r_1 \cdots r_N)$ is the many-body density distribution, $b$ the impact parameter, $s_j$ the projection of $r_j$ perpendicular to the beam direction, and $A_j(k,q')$ the nucleon-nucleon amplitude for the $j$th particle.

Away from forward angles, the Glauber theory can be put on a somewhat more rigorous basis by working in the Breit frame. This results in the three-momentum transfer $q^2$ being replaced by the four-momentum transfer $-t$.

The amplitudes are normalized such that

$$\frac{d\sigma}{d\Omega}(k,t)=|A(k,t)|^2. \quad (3)$$

The nucleon-nucleon amplitudes are parametrized as

$$A(k,q)=[k\sigma_T(i+\rho_j)/4\pi]\exp[-\tfrac{1}{2}\beta^2|t|]. \quad (4)$$

For proton-proton scattering we use[4,6] $\sigma_T=48.2$ mb, $\beta^2=5.23$ (BeV/$c$)$^{-2}$, and $\rho_p=-0.325$.

The parameters for proton-neutron scattering are unknown. From the proton-deuteron total cross-section measurements,[6] we can infer the total $n$-$p$ cross section, given its relative real part. Accordingly, we treat $\rho_n$ as a free parameter and take $\beta^2$ the same as for proton-proton scattering. We neglect spin dependence but we shall allow for multiple charge exchange.[7]

We choose the four-particle density function as a product of single-particle density functions,

$$\rho(r_1\cdots r_4)=\Pi_i\tilde{\rho}(r_i). \quad (5)$$

As we have stated earlier, the recent electron-scattering results show[2] a deviation from Gaussian at large $t$, hence small $r^2$. Crudely, this can be achieved by choosing the single-particle density of the form

$$\tilde{\rho}(r)=\text{Normalization}\times(e^{-\alpha^2r^2}-Ce^{-\alpha^2r^2/\gamma^2}). \quad (6)$$

$C$ can be interpreted as a measure of the strength of the repulsive part of the shell-model potential and should therefore be small, and $\gamma$ a measure of the range of this core. In general $\gamma \ll 1$.

In the limit $\gamma=0$, the $\alpha$-particle size is given by $\langle R^2\rangle=9/8\alpha^2$. This is the case studied by Cysż and Leśniak.[8] Using this model, and further assuming that the parameters of the neutron-proton amplitude are the same as those for the proton-proton amplitude, these authors were able to reproduce the qualitative features of the data.[4]

For $\gamma$ nonzero, evaluation of Eq. (2) is straightforward though tedious; all integrals can be done analytically. Multiple charge exchange can be allowed for by making the replacement

$$A_jA_i \to A_jA_i-\tfrac{1}{4}(A_j-A_i)^2.$$

The fit shown in Fig. 1 uses the rather unphysical value $C=1$, together with $\sigma_{np}=38.4$ mb, and $\rho_p=\rho_n=-0.325$. The corresponding rms radius is 1.34 fm, which is somewhat smaller than the value found in electron scattering after the proton size is removed.

The description of the data is unexpectedly good, especially in the region of the first minimum and second maximum. Even at large momentum transfer, the qualitative agreement is reasonable.

The impulse approximation, shown as the dot-dashed curve in the figure, predicts a minimum at a position in good agreement with the electron-scattering results.[2]

For contrast, the $\gamma=0$ case is also shown in Fig. 1. Here $\alpha^2$ corresponding to an rms radius of 1.45 fm is used.[8] The theory considerably undershoots the maximum at $|t|=0.35$ (BeV/$c$) ($q^2=9$ fm$^{-2}$). Within this model, no variation of nucleon-nucleon parameters can account for this discrepancy.

The total cross sections corresponding to these three cases are 140, 174, and 142 mb, respectively. These are to be compared with an experimental value of $152\pm8$ mb.

It is difficult to assess the accuracy of the Glauber model. The approximation is explicitly one for small angles.[5] In addition, recoil is not properly allowed for, at least in this form of the model. Our results at large angles may then be fortuitous.

Even better agreement may be obtained at large angles by including spin dependence, by allowing the relative real parts of the nucleon-nucleon amplitudes to vary with angle, and by modifying Eq. (4) to take into account higher orders in $t$. However, the significance of introducing such extra parameters is questionable.

Neglecting the uncertainties in the scattering model, what nuclear information can be deduced? We emphasize that there are no explicit nucleon-nucleon correlations in our density distribution. However, the repulsive part of the "shell-model potential" may be thought of as a consequence of the nucleon-nucleon repulsion.[9] In our model, $C=1$, we over-emphasized this repulsion, as evidenced, say, by the large cross section predicted by the impulse approximation for large momentum transfer, indeed appreciably larger than the electron data. It then seems likely that an equivalent description could be obtained by reducing $C$ and building into the density function the effects of nucleon-nucleon correlations. This possibility is at present being investigated.

The parameters quoted above are not a result of an extensive search. By considering electron scattering and proton scattering together it should be possible to refine the nuclear model. When the final data[4] for scattering from the deuteron, $^{12}C$, and $^{16}O$ become available, we should be able to better determine the parameters of the nucleon-nucleon amplitude.

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## ALGEBRAIC STRUCTURE RESULTING FROM SUPERCONVERGENCE RELATIONS*

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Recently there has been much discussion concerning superconvergent sum rules.[1,2] In the derivation of these rules, one makes an assumption concerning the high-energy behavior of a particular linear combination of helicity amplitudes. The rationale for the assumption usually arises from consideration of Regge exchange in the $t$ channel. The sum rules are then assumed saturated with resonance (or particle) intermediate states and relations between coupling constants are obtained. There are difficulties with the procedure, however. To be consistent one should consider the scattering of particles that are included as intermediate states in the original sum rule. To satisfy the resulting sum rules (without obtaining a null solution) one may have to introduce still more particles in the sum and thus in turn more sum rules. In the case of baryon sum rules the additional particles, in general, have higher spins which necessitate more sum rules including more highly convergent ones.

The apparent requirement of a (infinite?) sequence of resonances for a self-consistent solution of the sum rules suggests that one look for an algebraic structure to classify the particles and their couplings. This we have investigated for the meson-baryon scattering case.

There already exists in the literature a model in which all amplitudes satisfy a superconvergence relation if one restricts the sum to isobar intermediate states. This is the static strong-coupling theory of Goebel.[3] In fact, the "strong-coupling condition" $[g_\alpha, g_\beta]=0$ can be considered a superconvergent sum-rule result. The $g_\alpha$ are the meson currents in meson-baryon scattering. $\alpha$ is an internal symmetry index (and a vector index in the case of $p$-wave mesons). The matrix elements of $g_\alpha$ in isobar space are the coupling constants of the meson. The contribution to the scattering amplitude, for the process meson $\alpha$ plus isobar