RELATIVISTIC MODEL OF A DAUGHTER REGGE TRAJECTORY*

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A model based on ladder diagrams in a $\lambda\varphi^3$ theory is used to study the first daughter trajectory in the scattering of unequal-mass particles. It is found that the daughter pole moves towards the physical region less rapidly than the leading pole, and develops a smaller imaginary part at the two-particle threshold. The motion of the daughter pole is determined primarily by three-particle scattering processes. As a consequence, a model which fails to treat three-particle scattering accurately cannot give a detailed picture of the motion.

Recently Freedman and Wang[1] have shown that for unequal mass kinematics the requirements of analyticity of the scattering amplitude in the energy and momentum-transfer variables s and t coupled with Regge-pole dominance of its asymptotic behavior implies the existence of a whole new set of subsidiary Regge trajectories. For an s-channel Regge pole the positions and residues of the secondary poles, called daughter poles, are completely determined at $s=0$ by the requirement that they exactly cancel the $s=0$ singularities of the leading pole. If $\alpha(0)$ is the position of the leading pole, the nth daughter pole will have $\alpha_n{}^d(0)=\alpha(0)-n$; and its reduced residue will be proportional to s^{-n} and, if n is odd, vanish for coupling to equal-mass particles. Durand[2] had extended the analysis of Freedman and Wang to show that daughter poles are a more general phenomenon which should also occur in the scattering of particles with spin. These discussions of daughter poles provide no information as to their behavior away from $s=0$. The ultimate physical importance of these poles depends on their energy dependence. Preliminary considerations[3] based on the ladder approximation to the Bethe-Salpeter equation indicate that daughters are more approximately parallel to the pole and reach physical values of the angular momentum. If this is really the case, there should be many physical consequences. The work reported in this paper is the first dynamical calculation of the behavior of the daughter poles away from $s=0$, and it shows that these new poles are very different from the leading poles. In particular it shows that arguments previously given on the behavior of daughter poles are invalid because of the neglect of three-particle effects. Several qualitative statements about the motion of the daughter poles can be made; these in turn suggest a possible mechanism that might prevent these new poles from reaching the physical region, and predicting the existence of a whole new set of heretofore unobserved particles.

In this paper we discuss the solution of a high-energy perturbation-theory model which satisfies all the postulates necessary to prove the existence of daughter poles, contains a well-studied leading Regge pole, and leads to analytic expressions for the trajectory and residue of the first daughter pole.[4] Polkinghorne[5] has studied the behavior of an infinite sum of ladder diagrams in a $\lambda\varphi^3$ theory and, by summing all terms of the form $(\ln t)^m/t$, obtained the complete expression for a leading Regge pole which approaches $l=-1$ in the weak-coupling or infinite-s limit. His analysis was carried out for diagrams in which all masses, internal and external, were equal; however, it is trivially extended to ladder diagrams with unequal masses. The external masses are m_1 and m_2, the masses on the sides of the ladder are μ and ν, and the exchanged mass is λ. The ladder diagrams will not satisfy two-particle unitarity unless $m_1=\mu$ and $m_2=\nu$. The existence proof for the daughter poles is independent of unitarity. The diagrams do have the correct analyticity and leading pole. In addition, the daughter pole is presumably a dynamical entity whose coupling to two particles depends on kinematics but whose motion is largely independent of this coupling. We have generalized the approach of Polkinghorne[5] to sum all contributions of the ladder diagrams which are proportional asymptotically to $(\ln t)^m/t^2$ in order to obtain a set of Regge poles near $l=-2$. A lowest order summation (in the coupling constant) has been performed previously in connection with a study of Regge cuts.[6]

We work with the Mellin transform of the scattering amplitude $L(\alpha,s)$, where

$$L(\alpha,s)=\int_0^\infty d\tau\,\tau^{-\alpha-1}f(s,-\tau). \tag{1}$$

The summation near $\alpha=-2$ produces a recurrence of the leading pole due to the use of Mellin rather than the Legendre transforms,[6] a daughter pole, and two new Regge poles. The procedure for carrying out the summation is straightforward, though considerably more complicated than that near $\alpha=-1$ because of the lack of a simple factorization property for the amplitudes. Since the method of solution is so involved algebraically, the details will be presented elsewhere and we will only discuss the solution here. The poles of the amplitude appear as roots of a 4×4 determinant. There are no simplifications in the full solution unless the masses on the sides of the ladder are taken to be equal ($\mu=\nu$). In that case, the Mellin-transformed amplitude to lowest order in the coupling constant is given by

$$L(\alpha,s)=G^2\left\{\frac{-2p^2}{\alpha+2-g^2K(s)}+\frac{(m_1^2-m_2^2)^2/2s}{\alpha+2-g^2/\mu^2}-\frac{N(\alpha,s)}{D(\alpha,s)}\right\}, \qquad (2)$$

where

$$g^2=G^2/16\pi^2,\ 4p^2=s-2(m_1^2+m_2^2)+(m_1^2-m_2^2)^2/s,\ 4q^2=s-4\mu^2,$$

and $K(s)$ is the trajectory function for the leading Regge pole [$K(0)=1/\mu^2$].[5,6] The first term in $L(\alpha,s)$ is just the recurrence of the leading Regge pole. The second term has the correct residue and position at $s=0$ to be the daughter pole.[1] The third term, $N(\alpha,s)/D(\alpha,s)$, contains two new poles near $\alpha=-2$; they are present in the equal-mass limit. The significance of these new poles is uncertain; they will not be considered hereafter. To lowest order in g^2 the daughter pole is a fixed pole. This is the first indication that the daughter pole behaves very differently from the leading pole.

If $\mu=\nu$, the pole, which in lowest order was identified as the daughter pole, factors out very neatly from the 4×4 determinant and both its trajectory function and its residue can be obtained to all orders in the coupling constant. The complete expression for the daughter trajectory function is given by the solution of an equation of the form $\alpha+2=F(\alpha,s)$. The function $F(\alpha,s)$ is given by an infinite series in powers of the coupling constant with coefficients given by integrals over the Feynman parameters associated with ladder diagrams having both ends contracted. A term by term analysis shows that at $s=0$, $\alpha_d=\alpha_l(0)-1$ where $\alpha_l(0)$ is the position of the leading pole. The position of the daughter pole first moves with s in order g^4. The slope of the leading pole at $s=0$ is $+g^2/6\mu^4$ and that of the daughter pole is $+0.09g^4(\mu^6)$. More important, however, is the fact that as functions of s, the integrals in $F(\alpha,s)$ have no two-particle cuts. The singularity structure of the trajectory function is identical to that arising from the diagram sum of Fig. 1(a). The corresponding sum for the leading trajectory is given in Fig. 1(b). Because of the minus signs in Fig. 1(a), all two-particle cuts cancel out term by term from the daughter trajectory. We show below that if $\mu\neq\nu$ the two-particle cuts enter with coefficients proportional to $(\mu-\nu)^2$. The residue of the daughter pole, at least through g^4, has the correct $s=0$ value relative to the leading pole and vanishes to all orders for all s if either the incoming or outgoing pair of masses are equal. The general arguments of Freedman and Wang[1] only show that the singular portion of the first daughter residue vanishes for equal masses. In addition the residue is a product of two form factors each of which has its first singularity at the three-particle threshold due to a cancellation similar to that involved in the trajectory function. Since the three-particle contributions of ladder diagrams are not those which would be expected to dominate a three-particle scattering amplitude, we conclude that our model is useless for determining the actual motion of the daughter pole. Such a determination would require an accurate treatment of three-particle scattering. This same objection applies

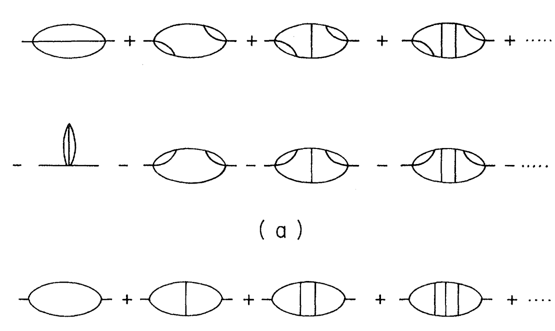

FIG. 1. (a) The diagram sum which describes graphically the analytic structure of the daughter trajectory function. (b) The corresponding diagram sum for the leading trajectory.

to the Bethe-Salpeter calculations of the daughter-pole motion using the ladder approximation.

Since the motion of the daughter poles should be independent of external masses, the fact that our model violates two-particle unitarity in the external particles should not affect the above conclusions. However, unitarity does tell us that the trajectory function for the daughter pole cannot remain real above the two-particle threshold. We have considered the effect of keeping $\mu \neq \nu$. In this case the daughter pole mixes in a very intimate way with the two other dynamical poles near $\alpha = -2$. In the lowest order it is one of three roots of a cubic equation which cannot conveniently be solved analytically; it can be solved in various limits: (1) If the exchanged mass λ vanishes, the equation can be solved and the daughter trajectory is parallel to the leading pole; in this limit the three-particle threshold is degenerate with the two particle threshold. (2) For $\Delta = (\mu - \nu)^2$ small and s near 0 and well below the two-particle threshold, we have for the daughter pole

$$(\alpha+2)_d = \frac{g^2}{\mu^2}\left[1 + \frac{1}{12}\frac{\Delta^2}{\mu^2} + \frac{\Delta^2}{80\mu^4} + \frac{s\Delta}{\mu^4}\left(\frac{1}{45} - \frac{\mu^2}{18\lambda^2}\right)\right], \quad (3)$$

while the leading pole is given by

$$(\alpha+1)_l = \frac{g^2}{\mu^2}\left[1 + \frac{1}{\mu^2}\left(\frac{s}{6} + \frac{\Delta}{12}\right) + \frac{1}{\mu^4}\left(\frac{s^2}{30} + \frac{s\Delta}{60} + \frac{\Delta^2}{80}\right)\right]. \quad (4)$$

Note the singular dependence on λ^2 in Eq. (3). For s approaching $\pm\infty$ we find

$$(\alpha+2)_d = \frac{g^2}{\mu\nu}\left\{1 + \frac{\lambda^2\Delta}{2s\mu\nu}\left[1 + \frac{\Delta}{s} + \frac{\lambda^2(\mu+\nu)^2}{4s\mu\nu} + \frac{\Delta^2}{s^2} + \frac{\lambda^2(\mu+\nu)^2}{2s^2\mu\nu}(\mu^2+\nu^2) - \frac{\lambda^4(\mu+\nu)^2}{4s^2\mu\nu}\right] + \frac{\lambda^4\Delta}{s^3}\left[\ln\frac{s^2}{\mu^2\nu^2} + 2\pi i\theta(s)\right]\right\}. \quad (5)$$

The corresponding expression for the leading pole is

$$(\alpha+1)_l = \frac{g^2}{s}\left[\ln\frac{s^2}{\mu^2\nu^2} + 2\pi i\theta(s)\right]. \quad (6)$$

Both of these limits show that although the daughter pole moves with s in lowest order, it behaves very differently from the leading pole. The daughter pole develops an imaginary part at the two-particle threshold but at least asymptotically it is smaller than that of the leading pole by a factor s^{-2}. Thus, even in a unitary model the daughter pole does move parallel to the leading pole. Three-particle scattering should still dominate its motion. Qualitatively we can say that the daughter pole approaches the two-particle threshold less rapidly than the leading pole and develops a smaller imaginary part there.

Finally the slow motion of the daughter pole and its small imaginary part above the two-particle threshold suggest a mechanism which might prevent the daughter from becoming physical. Just as each leading Regge pole apparently generates a set of daughter poles, heuristic arguments indicate that it will also generate a series of Regge cuts.[7] The leading cuts arising from three-particle unitarity have the feature that the physical-sheet branch point at zero energy is at the same position as the daughter pole and it remains fixed for $s < \mu^2$. For $s > \mu^2$, the leading branch point moves according to the trajectory function of the leading Regge pole, displaced by one unit to the left in the angular momentum plane, and with an energy $s_1 = (s^{1/2} - \mu)^2$. Hence as s increases from 0, the daughter pole will initially move to the right of the branch point. At the two-particle threshold, the daughter pole becomes complex and lies above the leading cut, whose branch point remains real for $s_1 < 4\mu^2$. Because of the slow motion of the daughter pole with increasing s compared to the trajectory function of the leading pole, there is the possibility that, as s approaches the three-particle threshold (or s_1 the two-particle threshold), the branch point can lie to the right of the daughter pole. Then, if the branch point moves upward in the complex plane faster than the daughter pole, the daughter pole will disappear from the physical sheet of the scattering amplitude.[8] Verification of this conjectured mechanism for the elimination of daughter poles from the physical scattering amplitude would require an accurate estimate of the position of the daughter pole near the three-particle threshold.

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[3]D. Freedman and J. Wright, private communication.

[4]For a complete discussion of the techniques and jus-

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[8]This possibility was first suggested by Professor L. F. Cook.

PROOF THAT THE NEAR-FORWARD MINIMUM AND SECONDARY PEAK IN $\pi^- p$ ELASTIC SCATTERING ARE RESONANCE EFFECTS*

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It is deduced using a very general and simple approach that the differential cross-section minimum near the forward direction and secondary diffraction peak in $\pi^- p$ elastic scattering in the region from 1.7 to 2.5 BeV/c are resonance phenomena. Experiments of simple interpretation are proposed to determine if this is the general nature of the dip-secondary-peak sequence observed in various reactions.

In a recent Letter, Frautschi[1] suggested that the minima of the differential cross section in the reactions $\pi^\pm + p \to \pi^\pm + p$ near $t = -0.6$ (BeV)^2 are due to the passage of the P', T_8, and ρ trajectories through a zero near this value of squared momentum transfer in conjunction with the existence of Chew's "ghost-killing" mechanism[2] for the 2^+ nonet. Under these conditions the helicity-flip amplitude (in the t channel) is expected to vanish in this t region and thus give rise to a minimum in the differential cross section in agreement with experiment. Frautschi considered the fact that the polarization changes sign in that vicinity at 2.1 BeV/c to be a confirmation of his ideas.[3]

In the present note we wish to present the results of an analysis of the $\pi^- p$ elastic differential cross section and polarization in the region from 1.7 to 2.5 BeV/c in the spirit of a very general method we recently proposed.[4,5] According to our results the near-forward minimum of the cross section, the related change of sign of the polarization, and the secondary maximum in this reaction are due to the presence of a resonant amplitude.[6]

Our method is based on the following two considerations:

(1) In the energy region of a resonance, any set of amplitudes that determine a given process may be written in all generality as the sum of two terms: the resonant term plus "the rest," which from now on we will simply call "background." This decomposition has the advantage over the classical partial-wave decomposition that the resonant and background contributions behave quite differently as a function of energy in the region in consideration. Similarly, if there is more than one resonant amplitude contributing to a given enhancement, we may separate them from "the rest."

(2) The behavior of the phase and magnitude of the contribution from a resonant eigenstate to a partial-wave amplitude as functions of the energy is expected to be satisfactorily described by a Breit-Wigner form.[7] Therefore, in pseudoscalar meson-spin-$\frac{1}{2}$ baryon elastic scattering, we may write

$$\tan\delta_l^J = \Gamma_l^J / 2[(W_R)_l^J - W], \quad (1)$$

$$|f_l^J| = x_l^J \sin\delta_l^J / k, \quad (2)$$

where f_l^J is the resonant eigenstate contribution to the partial-wave amplitude of orbital (total) angular momentum l (J), δ_l^J its phase or the eigenphase, Γ_l^J the total width, x_l^J the elasticity, $(W_R)_l^J$ the resonant energy, and k the c.m. momentum.

Although only the existence of one resonance [$N^*(2190)$ of spin-parity $\frac{7}{2}^-$] in the vicinity of 2.07 BeV/c has been established,[5] as Yokosawa et al. have reported that there might be at least one other resonating partial wave near this energy we prefer not to ignore a priori the possibility of several resonant partial waves in our analysis. We assume, however, that if there is more than one partial wave contrib-