ent analysis shows no such enhancement in the $I=1$ state at that energy. If a plausible background is subtracted, a resonance could exist only at a mass of ~1860 MeV/c^2 (~0.575 GeV/c, center-of-mass momentum), and would be less than 10% elastic.

The author wishes to acknowledge helpful discussions with Dr. D. V. Bugg and Dr. K. F. Riley, and to thank the Rutherford Laboratory for computing facilities.

[1]R. L. Cool, G. Giacomelli, T. F. Kycia, B. A. Leontic, K. K. Li, A. Lundby, and J. Teiger, Phys. Rev. Letters 17, 102 (1966).

[2]Birmingham University-Cambridge University-Rutherford Laboratory Collaboration, "K^+-nucleon total cross sections in the range 0.5 to 2.5 GeV/c," to be published.

[3]M. Lusignoli, M. Restignoli, G. A. Snow, and G. Violini, Phys. Letters 21, 229 (1966); Nuovo Cimento 45A, 792 (1966).

[4]Using the notation of Ref. 3, we obtain $18.93-Z_p(Y_1^*) = 2.12g_{p\Lambda K}^2+1.83g_{p\Sigma^0K}^2$ and $12.98-Z_n(Y_1^*)=1.83g_{n\Sigma^-K}^2$, in units of 10^{-7} MeV^{-2}; which gives $g_{p\Lambda K}^2=5.87\pm\sim1.0$. We assume a ratio for $g_{p\Sigma^0K}^2/g_{p\Lambda K}^2=1/27$ as given by an f/d ratio of $\frac{2}{3}$ (see Ref. 3, Phys. Letters).

[5]R. W. Bland, M. G. Bowler, J. L. Brown, G. Goldhaber, S. Goldhaber, J. A. Kadyk, V. H. Seeger, and G. H. Trilling, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (to be published); R. W. Bland, M. G. Bowler, J. L. Brown, G. Goldhaber, S. Goldhaber, A. A. Hirata, B. C. Shen, V. H. Seeger, and G. H. Trilling, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (to be published).

[6]A. T. Lea, B. R. Martin, and G. C. Oades, Phys. Letters 23, 380 (1966).

NEW SUM RULES FOR SUPERCONVERGENCE

R. Gatto
Istituto di Fisica dell'Università, Firenze, Italy, and Sezione di Firenze dell'Istituto Nazionale di Fisica Nucleare, Firenze, Italy
(Received 20 February 1967)

A new sum rule for the pion-scattering amplitude $B^+(\nu,t)$ is derived from a superconvergence assumption and its saturation is discussed. A similar sum rule for the amplitude $F^-(\nu,t)$ is also obtained and its consequences are examined.

In this note we write two new sum rules for pion-nucleon scattering and examine their saturation. The sum rules are derived from a superconvergence assumption.[1] The first sum rule, for the amplitude $B^+(\nu,t)$, when saturated with N and $N^*(1236)$, leads to a surprisingly close value for the width of N^*.

Let us consider the pion-nucleon scattering amplitude $-A(\nu,t)+\frac{1}{2}i\gamma_\mu(q_\mu+q_\mu')B(\nu,t)$, where q_μ and q_μ' are the momenta of the incoming and outgoing pion, $t=-(q_\mu-q_\mu')^2$, and $4M\nu=-(p_\mu+p_\mu')\cdot(q_\mu+q_\mu')$, p_μ and p_μ' being the momenta of the initial and final nucleon. We first consider the amplitude $B^+(\nu,t)=\frac{1}{3}B^{(1/2)}(\nu,t)+\frac{2}{3}B^{(3/2)}(\nu,t)$. For its asymptotic form at large ν we assume the Regge form

$$B_R^{+}(\nu,t)=\gamma_P(t)\frac{P_{\alpha_P}'(-Z)-P_{\alpha_P}'(Z)}{\sin[\pi\alpha_P(t)]}+\gamma_{P'}(t)\frac{P_{\alpha_{P'}}'(-Z)-P_{\alpha_{P'}}'(Z)}{\sin[\pi\alpha_{P'}(t)]},\tag{1}$$

where z is the cosine of the scattering angle in the crossed annihilation channel, $z=\nu/\nu_0(t)$, and $M\nu_0(t)=p_tq_t$, p_t and q_t denoting the c.m. momenta in the crossed channel. In (1) we have included the P and P' contributions. We assume that the difference $B^+(\nu,t)-B_R^+(\nu,t)$ superconverges. The amplitude $B_R^+(\nu,t)$ satisfies a formal dispersion relation directly obtainable for the integral representation

$$P_\alpha'(-Z)=\frac{\sin\pi\alpha}{\pi}\left(\frac{1}{1-Z}+\int_1^\infty P_\alpha'(Z')\frac{d_{Z'}}{Z'-Z}\right),\tag{2}$$

possibly continued analitically to values Re$\alpha\geq1$. From the unsubtracted dispersion relation[2] for

$B^+(\nu,t)$ and from the above superconvergence assumption one obtains the sum rule

$$\frac{G^2}{2M}+\frac{1}{\pi}\int_{\mu+t/4M}^{\nu_0(t)} d\nu' \,\mathrm{Im}B^+(\nu',t)$$

$$+\frac{1}{\pi}\int_{\nu_0(t)}^{\infty} d\nu'\left\{\mathrm{Im}B^+(\nu',t)-\gamma_P(t)P_{\alpha_P}'\left[\frac{\nu'}{\nu_0(t)}\right]-\gamma_{P'}(t)P_{\alpha_{P'}}'\left[\frac{\nu'}{\nu_0(t)}\right]\right\}=(1/\pi)\nu_0(t)[\gamma_P(t)+\gamma_{P'}(t)], \quad (3)$$

where M is the nucleon mass and $G^2/4\pi=14.5$.

For an approximate evaluation of Eq. (3) we assume that $\mathrm{Im}B^+(\nu,t)$ can be approximated by a sum $\mathrm{Im}B_R^+(\nu,t)+\mathrm{Im}B_{\mathrm{res}}^+(\nu,t)$, where the latter term consists of resonant contributions in the direct channel. Such a decomposition is often made in work on Regge poles[3] and appears to be valid down to low energies (e.g., laboratory momenta ~700 MeV/c for charge-exchange scattering).[4] The extension down to the region of the first resonance is expected to be an acceptable first approximation. The value of $\mathrm{Im}B_R^+(\nu,0)$, calculated from the Phillips-Rarita parameters,[5] is about 8% of the value ue of $\mathrm{Im}B^+(\nu,0)$ computed by Hamilton and Woolcock.[6] One can also make the following remark. The superconvergence assumption does not only apply to the difference $B^+-B_R^+$, but rather to any expression $B^+-\bar{B}^+$, where $\bar{B}^+$ tends asymptotically to B_R^+ and is analytic except for a cut on the real axis. The discontinuity of $\bar{B}^+$ can thus be suitably constructed within the general limitations such as to make the difference $\mathrm{Im}(B^+-\bar{B}^+)$ approximable by the resonant contribution. Under such assumptions the sum rule (3) becomes simply a sum rule on $\mathrm{Im}B_{\mathrm{res}}^+(\nu,t)$, reminiscent in content of the condition of vanishing sum of residues for the Pauli magnetic form factor.[7] In the narrow-width approximation one obtains

$$\frac{G^2}{4\pi}=\frac{1}{3}\sum_R(I_R+\tfrac{1}{2})\eta_R\frac{MM_R\Gamma_R}{q_R^{\,3}}\left\{2\left(\tfrac{1}{2}\cos^2\theta+\frac{E_R}{M}\tfrac{1}{2}\sin^2\theta\right)(-1)^{J_R-l_R-\frac{1}{2}}P_{l_R}'(\cos\theta)-\left(\frac{E_R}{M}-1\right)(J_R+\tfrac{1}{2})P_{l_R}(\cos\theta)\right\}, \quad (4)$$

where the sum is extended over all N-π resonances R, of mass M_R, width Γ_R, elasticity η_R, total angular momentum J_R, and decay orbital angular momentum l_R, and where E_R and q_R are the c.m. energy and momentum of the nucleon emitted in $R\to N+\pi$. If we keep only the contribution from the 3-3 resonance $\Delta(1236)$, in Eq. (4), at $\theta=0$, we obtain the comforting result

$$\Gamma_{3-3}=\frac{G^2}{4\pi}\frac{3}{4}\frac{q_R^{\,3}}{MM_R}\left[1-\frac{(M_R-M)^2}{2MM_R}+\frac{\mu^2}{2MM_R}\right]^{-1}$$

$$\cong\frac{G^2}{4\pi}\frac{3}{4}\frac{q_R^{\,3}}{MM_R}=115\ \mathrm{MeV}. \quad (5)$$

A more complete calculation can be made, inserting the N and Δ resonances included in the recent tabulation by Rosenfeld et al.,[8] with the values of the parameters as given in the tables. The contribution from each resonance to the right-hand side of Eq. (4), at $\theta=0$, is shown in Table I. One must note that, in spite of the peculiar closeness of the 3-3 contribution to the actual value of $G^2/4\pi$, the convergence of the series may in fact be rather slow.

The analysis presented here for pion-nucleon scattering can be extended quite straightforwardly to the $\pi\Lambda$, $\pi\Sigma$, and $\pi\Xi$ scattering amplitudes. It yields, for the $(\Sigma\Sigma\pi)$, $(\Lambda\Sigma\pi)$, and $(\Xi\Xi\pi)$ coupling constants, results quite consistent with the SU(3) predictions. Also, application to photoproduction shows a quite accurate saturation of the relation for the crossing-odd (nonsuperconvergent) amplitude A_4^- (in the usual notation).

Table I. Saturation of Eq. (4) with the known resonances. Equation (4) is written, at $\theta=0$, as $G^2/4\pi=\sum_R C_R$, where C_R is the contribution from the resonance R.

R	C_R
$\Delta(1236)$	15.3
$N'(1400)$	−2.5
$N(1525)$	−1.9
$N(1570)$	−0.02
$\Delta(1670)$	+0.07
$N(1670)$	+0.95
$N(1688)$	−2.3
$N'(1700)$	−0.11
$\Delta(1920)$	+3.9
$N(2190)$	−1.1

Let us next consider the amplitude

$$F^-(\nu,t)=A^-(\nu,t)+\frac{4M^2\nu}{4M^2-t}B^-(\nu,t). \tag{6}$$

From its asymptotic form

$$F_R^-(\nu,t)=\gamma(t)\frac{P_{\alpha_\rho}(-Z)-P_{\alpha_\rho}(Z)}{\sin[\pi\alpha_\rho(t)]}, \tag{7}$$

using the representation[9]

$$P_\alpha(-Z)=-\frac{\sin\pi\alpha}{\pi}\int_1^\infty P_\alpha(Z')\frac{dZ'}{Z'-Z}$$

analytically continued for $\mathrm{Re}\alpha\geq 0$, together with the unsubtracted dispersion relations for $A^-(\nu,t)$ and $B^-(\nu,t)$, one easily finds, from similar reasoning as before, the superconvergence relation

$$\frac{2M\nu_B}{4M^2-t}G^2+\frac{1}{\pi}\int_{\mu+t/4\pi}^{\nu_0(t)}d\nu'\,\mathrm{Im}F^-(\nu',t)+\frac{1}{\pi}\int_{\nu_0(t)}^{\infty}d\nu'\left\{\mathrm{Im}F^-(\nu',t)+\gamma(t)P_{\alpha_\rho}\left[\frac{\nu'}{\nu_0(t)}\right]\right\}=0. \tag{8}$$

Here $\nu_B=(-\mu^2+\frac{1}{2}t)/2M$ and $\alpha_\rho(t)$ denotes the ρ trajectory. Saturation of this sum rule by the assumption $F^-(\nu,t)=F_R^-(\nu,t)+F_{\mathrm{res}}^-(\nu,t)$, as was done before the $B^-(\nu,t)$, leads in the narrow-width approximation to the equation

$$\frac{-\mu^2-\frac{1}{2}t}{4M^2}\frac{G^2}{4\pi}+\frac{1}{6}\sum_R(-1)^{I_R-\frac{1}{2}}\frac{\eta_R\Gamma_R}{q_R}\left(\frac{M_R}{M}\right)^2\left\{(J_R+\tfrac{1}{2})\left[1+\frac{(M_R-M)^2-\mu^2}{2MM_R}-\tfrac{1}{2}\sin^2\theta\right]\right.$$
$$\left.\times P_{l_R}(\cos\theta)+\tfrac{1}{2}(-1)^{J_R-l_R-\frac{1}{2}}\sin^2\theta\frac{(M_R-M)^2-\mu^2}{2MM_R}P_{l_R}'(\cos\theta)\right\}=0. \tag{9}$$

We have calculated the sum in Eq. (9) including, as before, the resonant states of Rosenfeld tables, but found no general sign of convergence except in the (hoped fast) decrease of the elasticity parameters η_R. In particular, saturation with only the pole term and the 3-3 contribution is impossible, as both add with the same sign in the vicinity of $t=0$.

In the forward direction $\mathrm{Im}F^-(\nu,0)=\mathrm{Im}F^-(\omega,0)=q_L\Delta(\omega)$, where

$$\Delta(\omega)=\tfrac{1}{2}[\sigma_{\mathrm{tot}}^-(\omega)-\sigma_{\mathrm{tot}}^+(\omega)] \tag{10}$$

and ω and q_L are the pion energy and momentum in the laboratory. From the validity of Eq. (8) one easily derives, for sufficiently large ω, the equation

$$\frac{\alpha_\rho+1}{\omega^2}\left[-\mu^2\frac{G^2}{4\pi}+\int_\mu^\omega d\omega'(\omega'^2-\mu^2)^{1/2}\Delta(\omega')\right]=\Delta(\omega). \tag{11}$$

The Regge behavior is implicit in Eq. (11), as can be seen directly by differentiating the equation. Numerical evaluation of Eq. (11) for $\omega\geq 6$ BeV, using the current data[10] on $\Delta(\omega)$ and for $\alpha_\rho=0.57$[11] shows that the relation is well verified. Unfortunately, because of the large experimental errors, such an analysis is only an alternative verification of the Regge behavior.

[1]V. De Alfaro, S. Fubini, G. Rossetti, and G. Furlan, Phys. Letters 21, 576 (1966).
[2]G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. 106, 1337 (1957).
[3]See, for instance, R. K. Logan and L. Sertorio, Phys. Rev. Letters 17, 834 (1966).
[4]V. Barger and M. Olsson, Phys. Rev. 151, 1123 (1966).
[5]R. J. N. Phillips and W. Rarita, Phys. Rev. 139B, 1336 (1965).
[6]J. Hamilton and W. S. Woolcock, Rev. Mod. Phys.

35, 737 (1963).
[7]See D. Amati, R. Jengo, and E. Remiddi, Phys. Letters 22, 674 (1966).
[8]A. H. Rosenfeld et al., Rev. Mod. Phys. 39, 1 (1967).
[9]G. F. Chew, S. C. Frautschi, and S. Mandelstam, Phys. Rev. 126, 1202 (1962).
[10]For a compilation see N. N. Focacci and G. Giacomelli, to be published.
[11]See, for instance, G. Höhler, J. Baacke, and G. Eisenbeiss, Phys. Letters 22, 203 (1966).

TEST OF TIME-REVERSAL INVARIANCE IN $K_{\mu 3}^0$ DECAY*

Kenneth K. Young and Michael J. Longo
The University of Michigan, Ann Arbor, Michigan

and

Jerome A. Helland
University of California, Los Angeles, California
(Received 27 February 1967)

We report here the results of an experiment to test time-reversal invariance in the decay $K_L^0 \to \mu^+ + \pi^- + \nu$ ($K_{\mu 3}^0$ decay) by searching for a component of the muon polarization that is transverse to the decay plane. The observation of such a component would indicate a nonzero expectation value of the operator $\vec{\sigma}_\mu \cdot (\vec{p}_\pi \times \vec{p}_\mu)$ which is odd under T, and would constitute a direct proof of a violation of T invariance.[1]

All experimental evidence to date is consistent with CPT invariance, which is generally believed to be an exact symmetry principle. The observation of the CP-nonconserving decay[2] $K_L^0 \to 2\pi$ therefore constitutes indirect evidence for a violation of T invariance. However, no direct evidence for the violation of T invariance has ever been found. This makes a sensitive test of T invariance particularly interesting at this time. Some models for CP nonconservation predict a large transverse polarization of the muon in $K_{\mu 3}^0$ decay.[3,4] For example, in Sachs' model, maximal interference between the $\Delta S = \Delta Q$ and the $\Delta S = -\Delta Q$ amplitudes would give an average transverse polarization of approximately 20%.[3] Our experiment could detect a polarization 15 times smaller than this.

In the usual $V-A$ theory for $K_{\mu 3}$ decay, all observables except absolute rates can be expressed in terms of the ratio $\xi(q^2)$ of the two form factors $f^-(q^2)$ and $f^+(q^2)$. If T invariance holds, ξ is a real number. The relation between Imξ and the transverse polarization has been given by several authors.[5]

This experiment employed conventional scintillation-counter techniques. Neutral K mesons were allowed to decay in flight, and the polarization of the decay muons was measured by bringing the muons to rest in graphite and observing the asymmetry of the positrons from the μ^+ decay. A plan view of the experimental arrangement is shown in Fig. 1, and an isometric view showing the arrangement of the counters in more detail appears in Fig. 2. A neutral beam was taken off at an angle of 4° from a copper target in the external proton beam of the Bevatron. The neutral beam was carefully designed so that once beyond the defining aperture it was well clear of any material that might produce background problems. After the defining aperture the beam traveled in a helium bag. The flux of K_L^0 just downstream of the defining aperture was about one-half million per pulse of 5×10^{11} protons in the external beam, for K's in the momentum range 1 to 4 GeV/c.

Positive muons from K_L^0 decays occurring in the decay region were bent out of the neutral beam by the analyzing magnet, slowed down in a copper degrader, then stopped in the graphite stack. The apparatus was designed to accept decays with muons going generally forward in the K_L^0 rest frame and with the pions going off at roughly 90°. The decay plane of the K_L^0 was constrained on the average to be a horizontal plane by requiring that the pion from the decay be detected in either of the counter pairs L_1L_2 or R_1R_2. The vertical component of the muon polarization was then determined by measuring the vertical asymmetry of the positrons from the muon decay by means of counters above and below the graphite (counters T_1T_2 and B_1B_2 in the insert of Fig. 1). The requirement for an "event" was