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\*Present address: Physikalisches Institut, University of Heidelberg, Heidelberg, Germany.

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<sup>5</sup>This equation corresponds to Eq. (1) in Ref. 3 except that a term in  $A''(\beta_0)$  has been added, and the last term for the interaction energy has been changed from  $D(\beta-\beta_0)$  to  $\frac{1}{2}D(\beta^2-\beta_0^2)$ . Our  $D$  is equivalent to that of Wilets divided by  $\beta_0$ .

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<sup>11</sup>The rotational energies of  $\text{Sm}^{152}$  from a  $\text{Eu}^{152}$  source have been remeasured with a  $\text{Ge}(\text{Li})$  detector as follows:

$I$	$E_I$ (keV)
0	0
2 <sup>+</sup>	121.8 ± 0.1
4 <sup>+</sup>	366.3 ± 0.1
6 <sup>+</sup>	710.6 ± 0.1

Assuming  $E_I = AI(I+1) + B[I(I+1)]^2 + \gamma[I(I+1)]^3$ , we obtain  $A = 21.4$  keV,  $B = -0.20$  keV, and  $\gamma = 0.002$  keV.

<sup>12</sup>M. A. Preston, *Physics of the Nucleus* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1962), Fig. 10-22, p. 280.

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## OBSERVATION OF CENTRIFUGAL STRETCHING IN $\text{Sm}^{152}$

D. Yeboah-Amankwah and L. Grodzins

Laboratory for Nuclear Science\* and Physics Department,  
Massachusetts Institute of Technology, Cambridge, Massachusetts

and

Richard B. Frankel

National Magnet Laboratory,† Massachusetts Institute of Technology, Cambridge, Massachusetts  
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An isomer shift has been observed in the Mössbauer study of the 121.8-keV ( $2^+ \rightarrow 0^+$ ) transition in  $\text{Sm}^{152}$ . A value of  $(\Delta \langle R^2 \rangle / \langle R^2 \rangle)_{2-0} = (10 \pm 3) \times 10^{-4}$  is deduced, based on isotope shift data for  $\text{Sm}^{152}$  and  $\text{Sm}^{154}$ . This value disagrees with present theoretical prediction. In particular, the influence of the  $\beta$  band in  $\text{Sm}^{152}$  can only account for about 20% of the observed stretching.

In a previous Letter<sup>1</sup> we reported the observation that in the deformed nucleus  $\text{W}^{182}$ , the mean square radius of the first  $2^+$  state was greater than that of the  $0^+$  ground state. The observation was based on an unambiguous measurement of the isomer shift (I.S.) in the Mössbauer spectrum between two valence states of W. A meaningful estimate of  $\Delta \langle R^2 \rangle / \langle R^2 \rangle$  for  $\text{W}^{182}$  was, however, impossible due to the large uncertainties in estimating the electronic contribution  $\Delta |\Psi(0)|^2$  to the I.S. Comparison with

nuclear models was further complicated by the sparsity of relevant nuclear information for  $\text{W}^{182}$ .

We report here the observation of the I.S. for the  $2^+ \rightarrow 0^+$  transition in  $\text{Sm}^{152}$ . In this case,  $\Delta \langle R^2 \rangle / \langle R^2 \rangle$  can be reliably estimated from experimental data. Moreover, the rotational and vibrational bands of this pivotal nucleus have been studied in sufficient detail so that a value of  $\Delta \langle R^2 \rangle / \langle R^2 \rangle$  provides a meaningful test of nuclear models. We find that the value of  $\Delta \langle R^2 \rangle /$

$\langle R^2 \rangle$  supports the conclusion that centrifugal stretching is smaller than expected on the basis of the deviations of the energy levels from that of a pure rotor. Only about 20% of the radial change in  $\text{Sm}^{152}$  is accounted for by  $\beta$ -band mixing.

The experiments were carried out at liquid-helium temperatures with a standard electro-mechanical Mössbauer drive system, using a time-mode drive for the multichannel analyzer. Special care was taken to build a highly stable system in which the velocity calibration did not vary for weeks at a time. Most of the various source and absorber matrices that were investigated yielded an impractically small effect. A  $2^+$  valence source of  $\text{Eu}^{152}$  in  $\text{CaF}_2$  and a  $3^+$  valence absorber  $^{152}\text{Sm}_2\text{O}_3$  gave a single line and a relatively large effect. The 5-mC source was prepared by neutron irradiation, in the MIT reactor, of 0.1 mole%  $\text{Eu}^{151}$  in the  $2^+$  valence state in  $\text{CaF}_2$  (as determined by spectroscopic analysis).<sup>2</sup> The Mössbauer spectra obtained with this source and a  $^{152}\text{Sm}_2\text{O}_3$  absorber are shown for two velocity ranges in Figs. 1(a) and 1(b). The line-width is approximately four times broader than

natural. The isomer shift is  $1.65 \pm 0.15$  mm/sec and is so large as to be unambiguous. The results for other source and absorber matrices, as well as for  $\text{Gd}^{154}$  and  $\text{Gd}^{156}$ , will be presented in a more comprehensive paper.<sup>3</sup>

The I.S. is related to the change in mean square charge radii by

$$\Delta E = \frac{2\pi}{5} Z e^2 R^2 \frac{[\langle R^2 \rangle_{2^+} - \langle R^2 \rangle_{0^+}]}{\langle R^2 \rangle} \{L_A - L_S\}, \quad (1)$$

where we have used the symbol  $L \equiv \sum |\Psi(0)|^2$ , and S and A stand for source and absorber, respectively.

One can estimate  $L_A - L_S$  within an uncertainty of about 30%. The following two approaches have been used:

(1) The I.S. between the  $0^+$  and  $2^+$  states of  $\text{Sm}^{152}$  can be compared with the isotope shift between the  $0^+$  ground states of  $\text{Sm}^{152}$  and  $\text{Sm}^{154}$ . Striganov, Katulin, and Eliseev<sup>4</sup> have measured the latter shift for a number of atomic transitions. The average of the six largest energy shifts between the  $4f^6 5d 6s^2$  and  $4f^6 6s^2$  configurations was found to be  $41.5 \times 10^{-3} \text{ cm}^{-1}$ . The ratio of the I.S. to the isotope shift is, therefore,

$$\frac{\Delta E(\text{Isomer})}{\Delta E(\text{Isotope})} = 0.13 = \frac{[\langle R^2 \rangle_{2^+} - \langle R^2 \rangle_{0^+}]_{152}}{[\langle R^2 \rangle_{154} - \langle R^2 \rangle_{152}]_{0^+}} \frac{L(4f^5) - L(4f^6)}{L(4f^5 6d 6s^2) - (4f^6 6s^2)}. \quad (2)$$

The radial change deduced from the isotope shift<sup>5</sup> is

$$\frac{[\langle R^2 \rangle_{154} - \langle R^2 \rangle_{152}]}{\langle R^2 \rangle_{152}} = 1.2 \left( \frac{2}{3} \frac{\delta A}{A} \right) = 1.05 \times 10^{-2}.$$

Shielding effects are not very different for similar configurations,<sup>6</sup> so that

$$\begin{aligned} L(4f^5 6d 6s^2) - L(4f^6 6s^2) \\ \simeq L(4f^5 6d) - L(4f^6) \simeq L(4f^5) - L(4f^6). \end{aligned}$$

Thus,

$$(\Delta \langle R^2 \rangle / \langle R^2 \rangle)_{2^+ - 0^+} = (+14 \pm 4) \times 10^{-4}.$$

The uncertainty arises mainly from the assumption about equivalent configurations.

(2) Brix et al.<sup>6</sup> evaluated  $L(4f^6) - L(4f^7)$  for Eu by noting that if one combines isotope shift data in Sm together with hyperfine data on Eu and the assumptions about equivalent orbitals indicated above, then

$$L(4f^6) - L(4f^7) = (1.9 \pm 0.4) \times 10^{26} \text{ cm}^2.$$

This relation should represent,<sup>6</sup> to first approximation, the difference  $\Delta L(4f^5 - 4f^6)$  for the neighboring Sm. Using this value, we obtain

$$(\Delta \langle R^2 \rangle / \langle R^2 \rangle)_{2^+ - 0^+} = (+7.3 \pm 2) \times 10^{-4}.$$

The error is taken from Brix et al. as 30%. The two determinations agree reasonably well and are sufficiently independent to warrant combining them. Therefore, we take as the most reliable estimate for  $\text{Sm}^{152}$

$$(\Delta \langle R^2 \rangle / \langle R^2 \rangle)_{2^+ - 0^+} = (+10 \pm 3) \times 10^{-4}.$$

The stretching may also be deduced from muonic-isomer-shift data<sup>7</sup> in which the energy of the  $2^+ \rightarrow 0^+$  state, excited by the muon, is compared with the transition energy using a radioactive source. If the observed muonic energy shift is ascribed solely to centrifugal stretching, then  $\Delta \langle R^2 \rangle / \langle R^2 \rangle = (5.8 \pm 0.7) \times 10^{-4}$ , in good agreement with the present result.

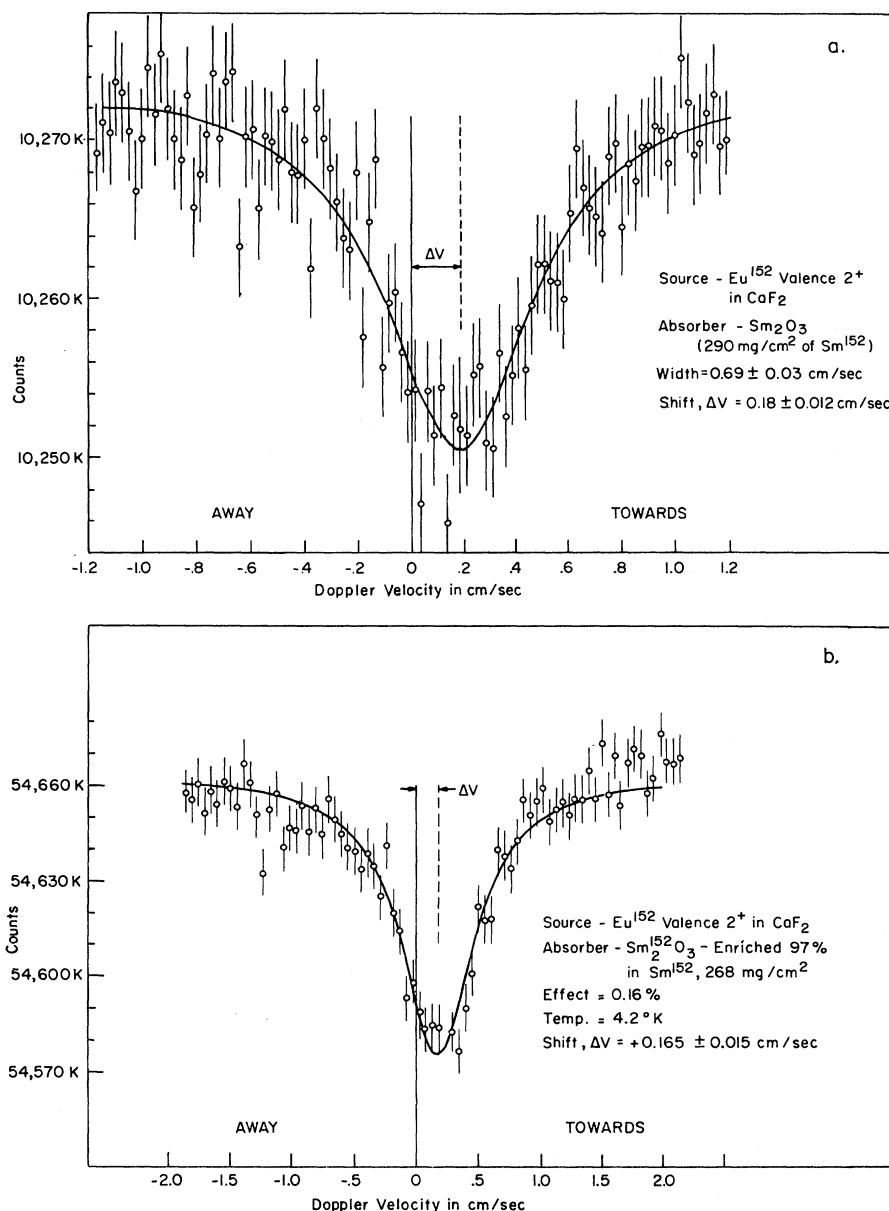


FIG. 1. (a) Mössbauer absorption resonance in the 122 keV,  $2^+ \rightarrow 0^+$  transition in  $\text{Sm}^{152}$ , for a  $^{152}\text{Eu}^{2+}$  source in  $\text{CaF}_2$  and a  $^{152}\text{Sm}_2\text{O}_3$  absorber. (b) Same as (a), except covering a wider velocity range.

The radius change can be related to the change in deformation by assuming that the nucleus is axially symmetric and incompressible. In that case,  $\Delta\langle R^2 \rangle / \langle R^2 \rangle = 5/(2\pi)\beta\Delta\beta$ . For  $\text{Sm}^{152}$ ,  $\beta = 0.304$ ,<sup>8</sup> so that  $\Delta\beta_{2-0} = 4.2 \times 10^{-3}$  and  $(\Delta\beta/\beta)_{2-0} = 1.4 \times 10^{-2}$ .

The measured value is not in accord with theoretical predictions. Thus we have the following:

(1) Udagawa and Sheline<sup>9</sup> have calculated  $\Delta\beta_{2-0}$  for  $\text{Sm}^{152}$  on the basis of a microscopic model involving Coriolis antipairing forces. They predict  $\Delta\beta_{2-0} = 1.3 \times 10^{-2}$ , approximately three times the observed value.

(2) One can estimate  $\Delta\beta/\beta$  from the deviations of the ground-state rotational band from that of a pure rotor. The derivation of the relationship between  $E$  and  $\Delta\beta/\beta$  assumes that the mo-

ment of inertia  $\mathcal{I}$  is proportional to  $\beta^2$ . From Ref. 1,

$$E_I = \frac{\hbar^2 I(I+1)}{2\mathcal{I}} \left[ 1 - \frac{\Delta\beta}{\beta} \right]. \quad (3)$$

Using the accurately measured values<sup>10</sup> of the  $2^+$  and  $4^+$  excited states of  $\text{Sm}^{152}$ , 121.78 and 366.44 keV, respectively,  $(\Delta\beta/\beta)_{2-0} \approx 4.2 \times 10^{-2}$ , again considerably larger than the observed value.

(3) The rotational-vibrational interaction will cause centrifugal stretching. The largest contribution might be expected to come from the  $\beta$ -band ( $K=0$ ) mixing, which has been well studied in  $\text{Sm}^{152}$ . One can estimate the contribution of the  $\beta$ -band mixing to the change in mean radius in a model-independent way in terms of the mixing parameter<sup>11</sup>  $\epsilon$ , obtaining

$$\Delta\langle R^2 \rangle / \langle R^2 \rangle = 2\epsilon I(I+1)\rho/Z, \quad (4)$$

where  $\rho = \langle I_{\text{gr}} | (\sum p) (r_p^2/R^2) | I_{\beta} \rangle$ , defined by Church and Weneser.<sup>12</sup> From the measurements of the  $(0_{\beta}^+ \rightarrow 0_{\text{gr}}^+)$  transition<sup>13</sup> and the  $B(E2)$  values,<sup>11,14</sup> one obtains  $\rho \approx 0.16$  and  $\epsilon = (6 \pm 1) \times 10^{-3}$ . Thus,  $\Delta\langle R^2 \rangle / \langle R^2 \rangle \approx 2 \times 10^{-4}$ , only one fifth of the measured value.<sup>15</sup>

We conclude that present theoretical predictions do not explain the observed change in nuclear radius.

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<sup>11</sup>Y. Yoshizawa, B. Elbek, B. Herskind, and M. C. Olesen, *Nucl. Phys.* **73**, 273 (1965); G. G. Seaman, J. G. Greenberg, D. A. Bromley, and F. K. McGowan, *Phys. Rev.* **149**, 925 (1966).

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<sup>13</sup>The ratio of  $K$  electrons depopulating the  $0_{\beta}^+$  state has been taken from I. Marlund, O. Nathan, and O. B. Nielsen, *Nucl. Phys.* **15**, 199 (1960), who report  $N_K(0_{\beta}^+ \rightarrow 0_{\text{gr}}^+) / N_K(0_{\beta}^+ \rightarrow 2_{\text{gr}}^+) = 1.4 \pm 0.15$ . G. Ewan and coworkers have recently remeasured this ratio and report (private communication) a preliminary value of 1.8.

<sup>14</sup>According to G. D. Symons and A. C. Douglas, *Phys. Letters* **24B**, 11 (1967), who quote private communications, the measured value of  $B(E2)(0_{\text{gr}}^+ \rightarrow 2_{\beta}^+)$  from Yoshizawa, Elbek, Herskind, and Olesen (Ref. 11), should be reduced by nearly a factor of 3, which would reduce the contribution of the  $\beta$ -band-mixing value to  $\Delta\langle R^2 \rangle / \langle R^2 \rangle$  by about a factor of 3.

<sup>15</sup>One can, of course, invoke the weak coupling model to eliminate  $\rho/Z$  and obtain

$$\frac{\Delta\langle R^2 \rangle}{\langle R^2 \rangle} = \frac{3}{\pi} \epsilon I(I+1) \beta_0^2 \times \left( \frac{B(E2, 0_{\text{gr}}^+ \rightarrow 2_{\beta}^+)}{B(E2, 0_{\text{gr}}^+ \rightarrow 2_{\text{gr}}^+)} \right)^{1/2} \text{Intrinsic} = 5.9 \times 10^{-4},$$

where we have again used the data of Ref. 11. The near agreement of this value with experiment is probably not too meaningful, but rather points up the fact that the weak coupling model does not yet account for the experimental  $\rho$  value in  $\text{Sm}^{152}$ .