volve not only the velocity but also the numbers of dislocations generated at the indentation.

We mentioned earlier that the respective authors in Ref. 6 do not agree on the location of the dislocation acceptor level. Our analysis and results would imply that the level lies above the center of the energy gap as proposed by Logan, Pearson, and Kleinman.[6] If, as Broudy suggests, the dislocation acceptor level lies near the valence band, the level would certainly be almost completely filled in intrinsic germanium and we should observe negligible effects with n doping.

The mass action theory which led to (5) resembles the classical work on the heterogeneous solubility in semiconductors by Reiss, Fuller, and Morin.[7] Indeed, many predictions of the latter theory can be easily transcribed to apply to our problem. First, counterdoping should essentially eliminate any effects due to charged impurities. Secondly, impurities of valencies larger than one behave in an obviously different way. Thirdly, as the temperature is increased $v_D \to v_D^0$ since $m \to K_i^{-1/2}$. This appears to be the case at about 725°C, as can be seen from an extrapolation of the data in Fig. 6 of Ref. 1. Finally, one would expect to observe a dislocation velocity of about 10^{-4} cm/sec (at 500°C in Ge) under sufficiently large electric fields $E \sim \tau b/q\theta\rho = 9\times10^6$ V/cm, where τ is the shear stress.

We are indebted to C. Herring for many valuable discussions in connection with this work and to A. R. Hutson for comments.

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MEASUREMENT OF THE NUCLEAR GAMMA RAY IN MUONIC Sm^{152} †

S. Bernow, S. Devons, I. Duerdoth, D. Hitlin, J. W. Kast,
E. R. Macagno, J. Rainwater, K. Runge,* and C. S. Wu
Columbia University, New York, New York
(Received 27 March 1967)

The energy of the first rotational nuclear gamma ray of Sm^{152} in the presence of a muon in a 1s state has been found to be 1.03 ± 0.15 keV greater than the normal transition energy. This type of measurement provides a method of determining the change in mean square radius between ground and excited states (isomer shift) in deformed nuclei.

It has been shown that in strongly deformed nuclei, the first few rotational levels can be excited by the dynamic $E2$ interaction between a muon and the nucleus. For example, in a muonic atom of Sm^{152}, the probability that the nucleus is left in the state $I = 2^+$ is 0.30, and in the muonic atom of U^{238}, it is 0.56. The lifetime of the 2^+ state is much shorter than the lifetime of the muon for capture or decay. Therefore, the $2^+ \to 0^+$ rotational transition takes place in the presence of a muon in the 1s state. In this Letter, we wish to report the observation of a shift in the energy of the nuclear gamma ray from the first rotational to ground state in the muonic atom Sm^{152} when a muon is in a 1s state compared with the same transition energy without the muon. It should be pointed out that the measured effect—a difference between the energy of the transition with and without the muon present—can, in the case of Sm^{152}, be attributed almost entirely to the difference in the mean square radius of the nucleus in the ground (0^+) and first excited (2^+) states in the absence of the muon. The effect is analogous to the so-called "isomer shift" (change in rms charge radius with excitation) that has been measured by Mössbauer techniques.[1,2] Theoretical investigations of the effect of the muonic atom on nuclear levels were carried out by Chinn and Wilets[3] and Hüfner,[4] who attributed the shift to a change of the nuclear energy due to the change of deformation of the nucleus by the muon. By proper ordering and examination of the terms in the Hamiltonian it can be shown that the muon-produced shifts in the energies of the nuclear states are zero to first order in the muon-produced nuclear distortion.

The exact Hamiltonian of the nucleus-muon system has three parts:

$$H = H_N + H_\mu + H_{\text{int}},$$

where H_μ is for the muon kinetic energy, H_{int} is for the muon-nucleus electromagnetic interaction (potential energy), and H_N is the (exact) purely nuclear Hamiltonian which may be approximated using particular nuclear models.

Consider any eigenstate ψ_j of energy E_j when the muon is not present, and where $\langle\psi_j|H_N|\psi_j\rangle = E_j$. Suppose that this state has a small modification in the presence of the muon so the nuclear wave function becomes

$$\varphi_j = \{1-\sum|a_k|^2\}^{1/2}\psi_j + \sum a_k\psi_k,$$

where $k \neq j$ in the sum and the a_k are taken as small. The magnitude of the change $(\varphi_j - \psi_j)$ is porportional to the a_k. For φ_j we have

$$\langle\varphi_j|H_N|\varphi_j\rangle = E_j + \sum|a_k|^2(E_k - E_j),$$

so the change in $\langle H_N\rangle$ is zero to first order in the muon-produced nuclear distortion, but may not be negligible for a large enough distortion when quadratic effects are important. The application of this basic theorem to the present problem shows that if H_N is taken as the sum of terms for the nuclear rotation, vibration, distortion, etc., the separate terms may have energy shifts which are linear in the a_k, but their sum has zero linear dependence on the a_k. To the extent that the quadratic terms are not important, the observed change in the $2^+ \to 0^+$ transition energy in the presence of the muon is due to the difference in the bound muon energy, $\langle H_\mu + H_{\text{int}}\rangle$, for the 0^+ and 2^+ nuclear states. The energy difference is due to the difference of the nuclear charge distribution for the two states.

In a slight generalization of the expression given in Chinn and Wilets's paper, the total energy of the system when the nucleus is in the rotational state I in a muonic atom is given by[5]

$$E_I = E_0 + I(I+1)\{A(\beta_0) + A'(\beta_0)(\beta-\beta_0) + \tfrac{1}{2}A''(\beta_0)(\beta-\beta_0)^2\} + \tfrac{1}{2}C(\beta-\beta_0)^2 + \tfrac{1}{2}D(\beta^2-\beta_0^2). \quad (1)$$

Here the rotational energy constant $A(\beta) = \hbar^2/2\mathscr{I}(\beta)$ is expressed in a Taylor series about β_0, the deformation of the ground state in the absence of the $1s$ muon, with terms to the second order retained; the change in the deformation energy of the nucleus is given by the term $\frac{1}{2}C(\beta-\beta_0)^2$; the change in the interaction energy between the $1s$ muon and the nucleus due to the change of deformation $(\beta-\beta_0)$ is given by the last term, $\frac{1}{2}D(\beta^2-\beta_0^2)$, where D is calculated[6] by making a Taylor expansion for $\langle H_\mu\rangle_{1s}$ about $\beta = \beta_0$. From Eq. (1), one obtains the value of β which gives minimum energy:

$$\delta\beta = \beta - \beta_0 = -\frac{I(I+1)A' + D\beta_0}{C + I(I+1)A'' + D}. \quad (2)$$

Evaluation of the contributions of the various terms in Eq. (2) for Sm^{152} shows that the muon-nucleus interaction term D is much smaller than the terms describing the intrinsic change of deformation of the state I. If we neglect the contribution of the D term, it then follows that the change in deformation, $\delta\beta$, of the nucleus between the rotational state, I, and the ground state, $I = 0$, in the presence of the $1s$ muon is equal to the corresponding change in the deformation without the muon, $(\delta\beta)_{\text{no }\mu}$, i.e.,

$$\delta\beta \simeq -\frac{I(I+1)A'}{C}\left[1 - \frac{I(I+1)A''}{C}\right] = (\delta\beta)_{\text{no }\mu}.$$

The quantity $(\delta\beta)_{\text{no }\mu}$ is related to the isomer shift.

Using Eqs. (1) and (2), one obtains the shift in the energy of the nuclear ($2^+ \to 0^+$) transition. Keeping second-order terms, the result is

$$\delta E = \frac{A'I(I+1)D\beta_0}{C} \times \left[1 - \frac{I(I+1)A'}{2C\beta_0} - \frac{I(I+1)A''}{C} - \frac{D}{C} - \frac{A''D\beta_0}{2CA'}\right]. \quad (3)$$

The first three terms are linear in the muon-nuclear interaction parameter D and are called "isomer shift" terms. The last two terms, which are quadratic in D, are called "polarization terms," and are small correction terms in the case of Sm^{152}.

Experimental considerations. (a) Nuclear rotational gamma ray. – The value of $\beta_0 D$ is largest in the region of heavy deformed nuclei; however, the low-energy rotational gamma rays are highly internally converted, which complicates their observation. A favorable case is Sm^{152}, which has the first rotational level at 121.8 keV and $\beta_0 = 0.30$,[7] so that using $R_0 = 1.2A^{1/3}$ F gives $D = 1340$ keV. Furthermore, the low rotational levels of Sm^{152} can be directly fed from the beta decay of Eu^{152}, which can be easily activated and has an ample half-life of 1.2 yr.

(b) Energy comparison. – To observe reliably such a small energy shift of the nuclear gamma ray with and without the influence of a muon, we compared the nuclear gamma ray from Sm^{152} which is in coincidence with a stopped muon directly and simultaneously with the gamma ray emitted from a radioactive Eu^{152} source in the same detecting system and under exactly the same conditions. Pulses corresponding to the two γ rays were stored separately in designated parts of the computer according to the tagging signals: The nuclear gamma rays from the muonic atoms were gated by the muon stopping signals (as described in a previous paper[8]); the nuclear gamma rays from a radioactive Eu^{152} source were gated by the coincidence of the cascade nuclear gamma rays $(4^+ \xrightarrow{\gamma_2} 2^+ \xrightarrow{\gamma_1} 0^+)$ with the aid of a large NaI detector. Shifts between the two spectra due to beam structure were found to be less than 0.5 channel, corresponding to 0.08 keV. For stabilization of the whole electronic system, an electronic pulser[9] stable to 20 parts per million was developed and incorporated into our on-line computer (PDP-8) system. The over-all stability was tested to be better than 50 parts per million for a period of several days. The positions of the lines were determined by finding the best least-squares fit to a Gaussian after background subtraction. In the case of the muonic spectrum, three Gaussians were fitted simultaneously.

Results and discussion. – The experiment was carried out at the Nevis Cyclotron Laboratory. Our results are shown in Fig. 1. The $(2^+ \rightarrow 0^+)$ nuclear gamma ray from the Sm^{152} muonic atom appears in the upper half of the figure while that from the decay

$$Eu^{152} \rightarrow Sm^{152*} + \beta^- + \bar{\nu}$$
$$\llcorner\!\rightarrow Sm^{152} + \gamma$$

is shown in the lower half. The shift in the energy is

$$\delta E = +1.03 \pm 0.15 \text{ keV},$$

the gamma ray from the muonic atom being the more energetic. An earlier measurement, using the less accurate techniques described in Ref. 8, gave a result of 0.8 ± 0.2 keV, in good

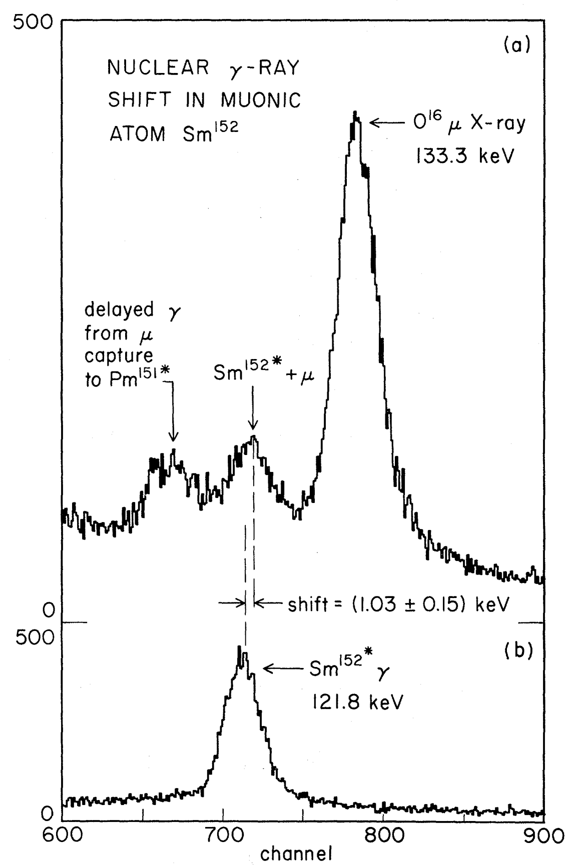

FIG. 1. The shift in the energy of the first rotational nuclear gamma ray of Sm^{152} due to the 1s muon. (a) De-excitation gamma ray $(2^+ \rightarrow 0^+)$ in the presence of the muon. The 133.3-keV line is the K_α muonic x ray of O^{16} in our sample of Sm_2O_3. (b) Normal $2^+ \rightarrow 0^+$ gamma ray resulting from the decay

$$Eu^{152} \rightarrow Sm^{152*} + \beta^- + \bar{\nu}.$$
$$\llcorner\!\rightarrow Sm^{152} + \gamma$$

This spectrum was taken simultaneously with the muonic spectrum (a).

agreement with the present value within experimental uncertainty.

(a) For Sm^{152}, the contribution of the polarization terms in Eq. (3) towards the observed shift is only a few tens of eV and can therefore be ignored. Thus, we find that the difference in the deformation parameter β between the two nuclear states, with or without the muon present is

$$\delta\beta \simeq (2.3 \pm 0.5) \times 10^{-3}.$$

It is not necessary to interpret the shift in terms of the change in deformation of a uniformly charged ellipsoid; more generally, one can speak in terms of the difference in the mean-square charge radius of the 0^+ and 2^+ states. The value for this change is

$$\frac{\delta\langle R^2\rangle}{\langle R^2\rangle} = (5.8 \pm 0.7) \times 10^{-4},$$

which is to be compared with the result of Yeboah-Amankwah, Grodzins, and Frankel,[10] who obtained the value $(+10 \pm 3) \times 10^{-4}$ for the same quantity using Mössbauer techniques.

(b) Substituting Eq. (2) into Eq. (1) shows that the A' and C are related to the rotation-vibration correction term B[11]:

$$B = -A'^2/2C.$$

Combining this with the measured energy shift, $E = +1.03$ keV, we obtain

$$A' = -8.86 \times 10^2 \text{ keV},$$
$$A'' = +2.18 \times 10^4 \text{ keV},$$

and

$$C = +1.97 \times 10^6 \text{ keV}.$$

A' is a parameter of considerable interest. From the definitions

$$A' \equiv \partial A(\beta)/\partial\beta \text{ and } A \equiv \hbar^2/2\mathscr{I},$$

it follows that

$$A'(\beta) = -A(\beta)\frac{1}{\mathscr{I}}\left(\frac{\partial \mathscr{I}}{\partial \beta}\right)$$

for small changes of β about β_0. Our measurements give

$$\left(\frac{1}{\mathscr{I}}\frac{\partial \mathscr{I}}{\partial \beta}\right)_{\beta=0.3} \simeq 40.$$

Another way to estimate this quantity is by a theoretical and experimental study of the systematic variation of $\mathscr{I}(\beta)$ vs β for different nuclei.[12] The value obtained is

$$\left(\frac{1}{\mathscr{I}}\frac{\partial \mathscr{I}}{\partial \beta}\right)_{\beta=0.3} = 3.2.$$

That the quantity $(1/\mathscr{I})(\partial\mathscr{I}/\partial\beta)$ as obtained in our experiment for the particular nucleus Sm^{152} differs by an order of magnitude from the value obtained from a plot for nuclei throughout the range $150 < A < 190$ is perhaps not surprising. Sm^{152} is situated in the transition region where the onset of large deformation suddenly occurs. It is, therefore, an interesting nucleus in which to search for departures from the strong-coupling, adiabatic description of rotational spectra. Measurements in the past have indicated that the experimental ratios of transition probabilities for interband transitions, and the deviation from the $I(I+1)$ rule Sm^{152} may be explained by including the mixing of ground-state, beta-band, and gamma-band wave functions caused by the "soft rotor" characteristics of this nucleus.[13] The determination of the quantity A' further suggests the desirability of pursuing theoretical studies in these directions.

In conclusion, we suggest that the measurement of the shift of the nuclear gamma ray in muonic atoms may provide two interesting types of information.

(1) A method of measuring $\delta\langle R^2\rangle/\langle R^2\rangle$ for the rotational states of deformed nuclei which does not depend on an uncertain knowledge of electron densities at the nucleus. The ratio of the isomer shifts for two or more isotopes of an element can be measured very precisely by Mössbauer techniques.[14] Measurement of the isomer shift for one of these isotopes in muonic atoms would therefore permit the normalization of the Mössbauer results and the accurate determination of the isomer shifts in the other isotopes.

(2) A value of $(1/\mathscr{I})(\partial\mathscr{I}/\partial\beta)$ for a definite nucleus, which allows a direct comparison of model calculations.

It should be pointed out that the effect directly measured here should also be observable in the hyperfine structure of the muonic K x rays. The energies of transitions originating from a given $2p$ level and going to the $1s_{1/2}(I=0)$ and $1s_{1/2}(I=2)$ states should differ by $E(2^+ \to 0^+) + \delta E$. Present determinations of these energy splittings are approaching sufficient accuracy to make this measurement.

We wish to thank Dr. G. L. Rogosa of the U. S. Atomic Energy Commission for providing us with the enriched Sm^{152} target. It is also a great pleasure to thank Professor L. Wilets for his many enlightening discussions. We also owe a great debt to Professor V. Telegdi for his stimulating question which generated interesting discussions and furthered the understanding of this problem.

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†Work partially supported by U. S. Atomic Energy Commission and U. S. Office of Naval Research.

*Present address: Physikalisches Institut, University of Heidelberg, Heidelberg, Germany.

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I	E_I (keV)
0	0
2^+	121.8 ± 0.1
4^+	366.3 ± 0.1
6^+	710.6 ± 0.1

Assuming $E_I=AI(I+1)+B[I(I+1)]^2+\gamma[I(I+1)]^3$, we obtain $A=21.4$ keV, $B=-0.20$ keV, and $\gamma=0.002$ keV.

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OBSERVATION OF CENTRIFUGAL STRETCHING IN Sm^{152}

D. Yeboah-Amankwah and L. Grodzins
Laboratory for Nuclear Science* and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts

and

Richard B. Frankel
National Magnet Laboratory,† Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 27 March 1967)

An isomer shift has been observed in the Mössbauer study of the 121.8-keV ($2^+\rightarrow 0^+$) transition in Sm^{152}. A value of $(\Delta\langle R^2\rangle/\langle R^2\rangle)_{2-0}=(10\pm 3)\times 10^{-4}$ is deduced, based on isotope shift data for Sm^{152} and Sm^{154}. This value disagrees with present theoretical prediction. In particular, the influence of the β band in Sm^{152} can only account for about 20 % of the observed stretching.

In a previous Letter[1] we reported the observation that in the deformed nucleus W^{182}, the mean square radius of the first 2^+ state was greater than that of the 0^+ ground state. The observation was based on an unambiguous measurement of the isomer shift (I.S.) in the Mössbauer spectrum between two valence states of W. A meaningful estimate of $\Delta\langle R^2\rangle/\langle R^2\rangle$ for W^{182} was, however, impossible due to the large uncertainties in estimating the electronic contribution $\Delta|\Psi(0)|^2$ to the I.S. Comparison with nuclear models was further complicated by the sparsity of relevant nuclear information for W^{182}.

We report here the observation of the I.S. for the $2^+\rightarrow 0^+$ transition in Sm^{152}. In this case, $\Delta\langle R^2\rangle/\langle R^2\rangle$ can be reliably estimated from experimental data. Moreover, the rotational and vibrational bands of this pivotal nucleus have been studied in sufficient detail so that a value of $\Delta\langle R^2\rangle/\langle R^2\rangle$ provides a meaningful test of nuclear models. We find that the value of $\Delta\langle R^2\rangle/$