QUANTUM LIMIT GALVANOMAGNETIC PHENOMENA IN n-InSb

O. Beckman,* E. Hanamura,† and L. J. Neuringer
National Magnet Laboratory,‡ Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 20 March 1967)

The binding energy of magnetically induced bound states has been measured in n-InSb subjected to intense magnetic fields and found to be in agreement with theoretical predictions. Observations of quantum-limit scattering and impurity conduction are discussed.

We have measured the transverse magnetoresistance and Hall coefficient of n-type InSb in the quantum limit, i.e., $\omega_c\tau \gg 1$ and $\hbar\omega_c \geqslant \zeta$, where $\omega_c = eH/mc$ is the cyclotron frequency and ζ is the chemical potential. We have observed several quantum effects at magnetic field strengths much greater than those at which the usual Shubnikov-de Haas oscillatory phenomena[1,2] occur. In particular we report (1) evidence for the formation of bound states caused by the presence of an intense magnetic field; (2) an $H^{1/3}$ magnetic-field dependence of the binding energy ϵ_b of these bound states; (3) excellent agreement of both the magnetic-field dependence and magnitude of the experimentally determined binding energy with the theoretical calculation of ϵ_b by Keyes[3]; (4) the first observation, in the quantum limit, of a magnetic-field and temperature dependence of the transverse magnetoresistance which is very close to that predicted by theory[4,5] for the case of ionized impurity scattering by long-range forces.

Previous experimental investigations[1,6,7] of the Hall coefficient of lightly doped samples have indicated that as the magnetic field strength is increased, the carrier concentration in the conduction band decreases. In the work of Frederikse and Hosler[1] and of Keyes and Sladek[7] this decrease was attributed qualitatively to the freeze-out of carriers from the conduction band into the shallow bound states formed by the magnetic field. Sladek analyzed his Hall coefficient data, measured to 28 kG, and obtained the ionization energy of the bound states. He found that the theoretical[8] binding energy exceeded the ionization energy by about 1-Ry unit at all fields. The scatter in the values of the ionization energies deduced by Sladek from his experiments was such as to preclude a precise determination of their magnetic-field dependence. In the present experiments, because of the extended range of available magnetic field strengths, it was possible to investigate more heavily doped specimens. This circumstance proves to be favorable for studying the magnetic freeze-out phenomenon. In Sladek's experiments, carried out on lightly doped specimens, the analysis was made complicated because it was necessary to consider two-band conduction (free carrier, impurity). However, in a more heavily doped sample the binding energy is deduced more directly because there exists a large field interval over which the Hall coefficient is determined primarily by the freeze-out process.

The transverse magnetoresistance and Hall coefficient were measured with the conventional four-probe technique on bar-shaped samples which were spark machined from single crystals of n-InSb. The samples studied had carrier concentrations in the exhaustion range of 2.6×10^{15} cm^{-3}, 3.7×10^{15} cm^{-3}, and 1.1×10^{16} cm^{-3}. The measurements were performed between 1.3 and 4.2°K in a water-cooled solenoid capable of generating static magnetic fields up to 150 kG. The accuracy with which the magnetic field was measured is better than 2%. In all of our experiments care was taken to limit the electric field strength across the sample to values well below those which cause impact ionization[7,9] and nonlinear effects.[10]

In Fig. 1 the magnetoresistance data, plotted as $\log[\rho(H)/\rho(0)]$ vs $\log H$, are shown at several temperatures in the liquid-helium range. Proceeding in the direction of increasing magnetic field strength several prominent features are noticed. The $n=0$ Landau level coincides with the Fermi level at 8 kG for sample No. 315 and at 22 kG for sample No. 116. The accompanying peak in the magnetoresistance is clearly seen in the case of sample No. 116. When the magnetic field is increased above 30 kG, the magnetoresistance of sample No. 116 obeys a power-law dependence on H, namely, $\rho(H) \sim H^{3.3}$ both at 4.2 and 1.3°K in the interval 30-90 kG. The magnetoresistance is thus practically independent of temperature in this magnetic-field range. A similar dependence of the resistivity upon temperature and magnetic field is exhibited by sample No. 315 in the interval ~12-25 kG. Our experimental-

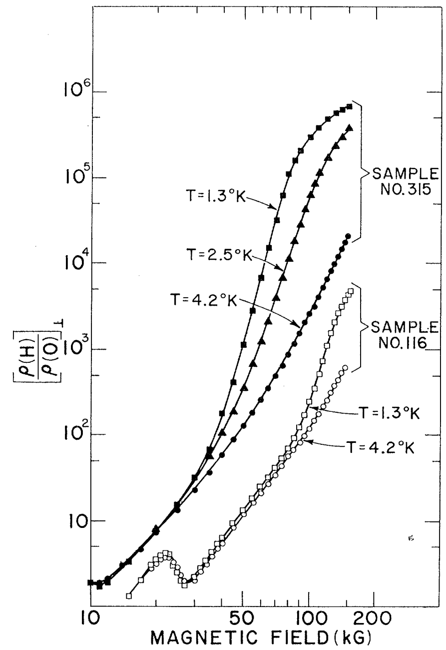

FIG. 1. Transverse magnetoresistance of n-InSb at liquid-helium temperatures in the quantum limit. The carrier concentrations in the exhaustion range are 2.6×10^{15} cm^{-3} and 1.1×10^{16} cm^{-3} for samples Nos. 315 and 116, respectively.

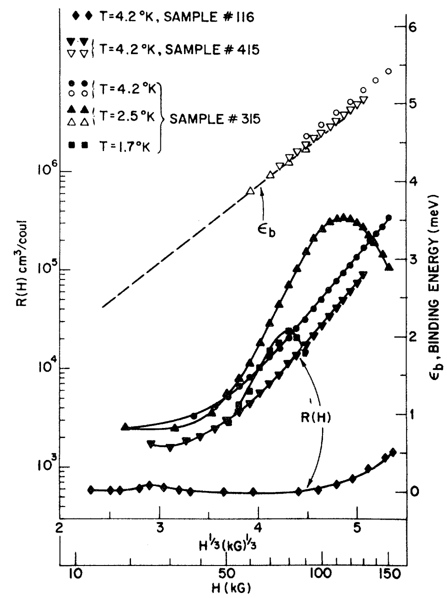

FIG. 2. Magnetic freeze-out in n-InSb in the quantum limit. Lower curves: magnetic-field dependence of the Hall coefficient. Samples Nos. 315 and 116 are the same as in Fig. 1. The carrier concentration of sample No. 415 in the exhaustion range is 3.7×10^{15} cm^{-3}. Upper curve: The dashed line is the theoretical variation of the binding energy of the ground state, computed with $m^*=0.014\,m_0$ and $\kappa_0=16.6$. The points represent the magnetic-field dependence of the binding energy deduced from the Hall data.

ly observed temperature and magnetic-field variation of the resistivity is in close agreement with the one expected theoretically. In particular, for a degenerate distribution of electrons in the quantum limit when ionized impurity scattering by long-range forces is operative, theory[4,5] predicts that $\rho(H) \sim H^3T^0$.

As the magnetic field becomes more intense, it starts to affect the equilibrium population of the free charge carriers. Bound states are formed due to the presence of the magnetic field and the binding energy of these states increases as the field progressively becomes greater. As a result, freeze-out of charge carriers into the impurity bound states occurs. This phenomenon is clearly evident in the case of sample No. 315 above ~40 kG. In this sample the marked increase of the magnetoresistance with increasing magnetic field at fixed temperature furnishes evidence for the presence of ever deepening bound states which remove carriers from the conduction band. The increase of magnetoresistance with $1/T$ at fixed magnetic field is also evidence of the existence of bound states.

Finally, at the very highest fields the slope of the curve of $\rho(H)$ vs H for sample No. 315 decreases markedly. It is significant that this change of slope occurs at the same magnetic field as the Hall-coefficient maximum in Fig. 2. The cause of this behavior may lie in the onset of impurity conduction, once most of the charge carriers are frozen out into impurity sites. The data are suggestive of the existence of a low-mobility conduction process (operating in addition to that of the free carriers) which contributes to the normal conductivity but not to the Hall coefficient.

The charge-carrier freeze-out phenomenon becomes most apparent upon examination of the magnetic-field dependence of the Hall coefficient. In Fig. 2, the Hall-coefficient data, plotted as $\log R(H)$ vs $H^{1/3}$, are shown at several temperatures in the liquid-helium range. Charge carrier freeze-out is initiated at a magnetic field of about 43, 49, and 110 kG for samples Nos. 315, 415, and 116, respectively, since at these fields $R(H)$ starts to increase with increasing H. In the case of sample No. 116 only the onset of carrier freeze-out is seen, but the phenomenon is clearly evident for samples Nos. 315 and 415. It is apparent from Fig. 2 that in the carrier freeze-out domain, the Hall coefficient of the latter two samples varies as $R(H) \sim \exp(bH^{1/3}/kT)$, where b is a constant, k is Boltzmann's constant, and T is the temperature. The binding energy of the magnetically induced bound state is simply $\epsilon_b = bH^{1/3}$.

Comparison of the curves in Fig. 2 for the different impurity concentrations shows that the more heavily doped the specimen, the greater is the magnetic field required to initiate carrier freeze-out. The dependence of this threshold magnetic field upon impurity concentration can be estimated as follows: The radius of the cyclotron orbit in the plane perpendicular to the magnetic field varies as $r_c \sim H^{-1/2}$. To obtain the spread of the electronic wave function in the z direction, we follow Elliot and Loudon.[3] For bound states the binding energy varies as $E_z \sim \hbar/2m^*\alpha^2$ and the wave function varies as $\psi \sim \exp(-|z|/\alpha)$, where m^* is the effective mass of the electron. Since the magnetic-field dependence of the binding energy is $\epsilon_b \sim H^{1/3}$, we find that $\alpha \sim H^{-1/6}$. The freeze-out process occurs when the electronic wave function becomes localized within the volume occupied by a single donor impurity, i.e., $\alpha r_c^{\,2} \cong N_{\text{imp}}^{-1}$. According to this estimate the threshold field for freeze-out H_0 varies with donor impurity concentration as $H_0 \sim N_{\text{imp}}^{6/7}$. The data were compared with this estimate by choosing, as an operational definition, H_0 to be the field at which one-half of the charge carriers are frozen out. The agreement was found to be satisfactory.

In the upper half of Fig. 2 is shown a comparison between the experimentally determined binding energies and the theoretical curve given by Keyes.[3] In calculating the theoretical curve the value of the effective mass was taken as $m^* = 0.014m_0$ and the static dielectric constant as $\kappa_0 = 16.6$. This value of the static dielectric constant is in good agreement with that measured for pure n-Insb at microwave frequencies by Perrin, Perrin, and Mercouroff.[11]

In summary it has been shown that the galvanomagnetic properties of n-InSb can be effectively investigated in the quantum limit with the use of very intense magnetic fields. This material is unique in that it is possible to study in a single sample, as a function of increasing magnetic field, Shubnikov–de Haas oscillations, quantum-limit scattering processes, carrier freeze-out, and impurity conduction.

---

*Present address: Department of Physics, Uppsala University, Uppsala, Sweden.

†Address after March, 1967: Institute of Solid State Physics, Tokyo University, Azabu, Tokyo, Japan.

‡Work supported by the U. S. Air Force Office of Scientific Research.

[1]H. P. R. Frederikse and W. R. Holser, Phys. Rev. 108, 1136 (1957).

[2]S. S. Shalyt and A. L. Éfros, Fiz. Tverd. Tela 4, 1233 (1962) [translation: Soviet Phys.—Solid State 4, 903 (1962)].

[3]R. W. Keyes, Westinghouse Research Report, 1958 (unpublished). See, however, R. J. Elliot and R. Loudon, J. Phys. Chem. Solids 15, 196 (1960), Fig. 6.

[4]E. N. Adams and T. D. Holstein, J. Phys. Chem. Solids 10, 254 (1959).

[5]R. Kubo, S. J. Miyake, and N. Hashitsume, in Solid State Physics, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1965), Vol. 17, p. 269.

[6]R. W. Keyes and R. J. Sladek, J. Phys. Chem. Solids 1, 143 (1956).

[7]R. J. Sladek, J. Phys. Chem. Solids 5, 157 (1958).

[8]Y. Yafet, R. W. Keyes, and E. N. Adams, J. Phys. Chem. Solids 1, 137 (1956).

[9]R. H. Putley, in Proceedings of the Seventh International Conference on Physics of Semiconductors, Paris, 1964 (Dunod, Paris, 1964), p. 443.

[10]K. F. Komatsubara and E. Yamada, Phys. Rev. 144, 702 (1966).

[11]N. Perrin, B. Perrin, and W. Mercouroff, Symposium on Plasma Effects in Solids, Paris, 1964 (Dunod, Paris, 1965), p. 37.