

INELASTIC LIGHT SCATTERING FROM LANDAU-LEVEL ELECTRONS
 IN SEMICONDUCTORS

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Inelastic scattering of $10.6\text{-}\mu$ CO_2 laser radiation by mobile electrons in a magnetic field has been observed in $n\text{-InSb}$. Spectra of the scattered light reveal three distinct lines corresponding to electron spin flip and to transitions with $\Delta l = 1$ and $\Delta l = 2$, where l is the Landau-level quantum number. Observations are compared with theoretical predictions and electronic parameters obtainable from the spectra are discussed.

It has recently been proposed that inelastic light scattering from mobile carriers in Landau levels of a semiconductor might be observable.¹ Using $10.6\text{-}\mu$ (ω_0) radiation from a CO_2 laser, we have observed such scattering by electrons in $n\text{-InSb}$ in magnetic fields up to 53 kOe. For a given magnetic field, the scattered light exhibits peaks at three distinct frequencies: $\omega_0 - \mu_B g_{\text{eff}} B$, $\omega_0 - 2\omega_c$, and $\omega_0 - \omega_c$, where $\mu_B g_{\text{eff}} B$ is the frequency spacing between spin sublevels and $\omega_c = eB/m^*c$ is the Landau-level spacing (cyclotron frequency). The first two of these processes have been considered theoretically by several authors.¹⁻³ However, the third process, $\Delta l = 1$, although of similar physical origin to the $\Delta l = 2$ transition, has not been predicted (l is the Landau-level quantum number). Though the spin-flip process is strongest, the cross sections of the observed processes are all roughly of the order $100\sigma_T$, where σ_T is the Thomson cross section for free electrons ($\sim 6 \times 10^{-25} \text{ cm}^2$). These large cross sections arise from virtual interband electronic transitions and from the strong coupling between valence and conduction bands (i.e., the nonparabolicity of the conduction band). The small effective mass, m^* , and large g_{eff} for electrons in $n\text{-InSb}$ allowed us to tune the wavelength of scattered light from 10.6 to 15μ by varying the magnetic field.

The linearly polarized $10.6\text{-}\mu$ output from the Q -switched CO_2 laser⁴ occurred in 160-nsec pulses having peak power of $\sim 25 \text{ kW}$ and a repetition rate of 120 sec^{-1} . The $n\text{-InSb}$ samples, in shapes of bars $1 \times 2 \times 10 \text{ mm}^3$, were mounted inside the superconducting solenoid such that their long axis was parallel to the magnetic-field direction as well as to the direction of the incident light. In focusing the beam, care was taken to minimize nonlinear effects such as multiphoton pair production in the InSb .⁵

The sample was placed at the focus of a parabolic mirror to collect light scattered at right angles to the incident $10.6\text{-}\mu$ beam (see inset in Fig. 1). With this arrangement, scattered light was collected from a solid angle of 0.15 sr . The samples were shaped with 45° rooftops to prevent forward $10.6\text{-}\mu$ radiation from entering the detection system. The scattered light was analyzed with a 25-cm focal length grating spectrometer, and a Ge:Cu photoconductor (4.2°K) was used as a detector.

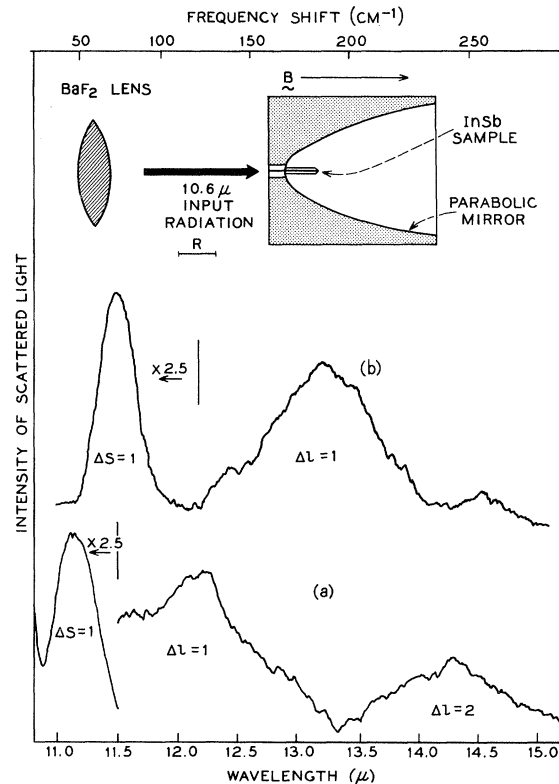


FIG. 1. Spectrum of light scattered at right angles from $n\text{-InSb}$ ($n_e = 5 \times 10^{16} \text{ cm}^{-3}$). (a) Magnetic field of 26.2 kOe. (b) Magnetic field of 36.7 kOe. Notice the scale change for the intensity of the spin-flip scattered light.

Figures 1(a) and 1(b) show typical spectrometer traces of light scattered from n -InSb with $n_e = 5 \times 10^{16} \text{ cm}^{-3}$ at $\sim 30^\circ\text{K}$, taken at 26.2 and 36.7 kOe, respectively. In Fig. 1(a) three peaks are seen corresponding to the $\Delta l = 0, \Delta s = 1$, $\Delta l = 1, \Delta s = 0$, and $\Delta l = 2, \Delta s = 0$ processes, respectively (s is the electron-spin quantum number). Figure 1(b) shows the spin-flip and the $\Delta l = 1$ lines which are now shifted towards longer wavelengths. The small peak at 14.5μ , not observed in all samples, corresponds to $\Delta l = 1, \Delta s = 1$. Taking into account the difference in linewidths for the different lines, the spin-flip transition is seen to be the strongest, and the $\Delta l = 1$ and $\Delta l = 2$ lines are progressively weaker. However, the relative sizes of the various cross sections were found to change with changing carrier concentrations. In the $3 \times 10^{16}\text{-cm}^{-3}$ InSb sample, the spin-flip transition was about five times stronger than, but the $\Delta l = 1$ and 2 transitions were weaker by a factor of ~ 10 than, the respective transitions in the $5 \times 10^{16}\text{-cm}^{-3}$ sample. In the $1 \times 10^{16}\text{-cm}^{-3}$ sample, the spin-flip line was as strong as that in the $5 \times 10^{16}\text{-cm}^{-3}$ sample, but the $\Delta l = 1$ and 2 transitions could not be observed. These variations appear to be connected with the position of the Fermi surface with respect to the Landau levels involved in the transitions.

Figure 2 shows the positions of the various

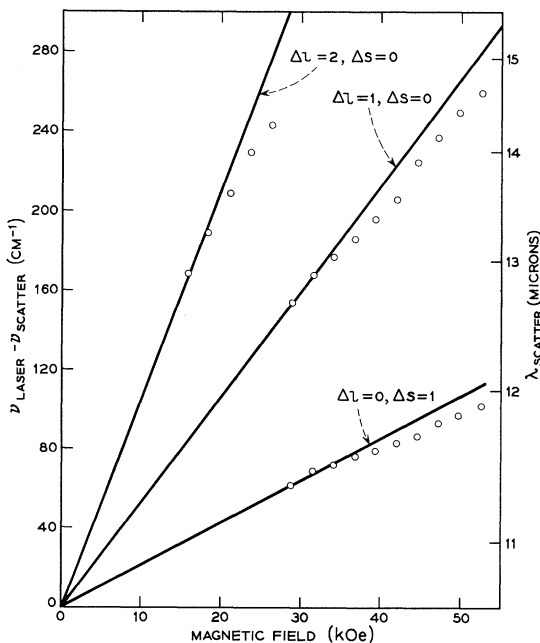


FIG. 2. Magnetic field dependence of wavelength of scattered light for the spin-flip, $\Delta l = 1$, and $\Delta l = 2$ processes, taken in n -InSb ($n_e = 5 \times 10^{16} \text{ cm}^{-3}$).

spectral lines as a function of the magnetic field for $n_e = 5 \times 10^{16} \text{ cm}^{-3}$. The maximum frequency shift ($\Delta\nu$) occurs for the $\Delta l = 2, \Delta s = 0$ process. For a field of ~ 50 kOe the scattered light for $\Delta l = 2$ should have a wavelength of 20μ . However, wavelengths longer than $\sim 15 \mu$ were not observed. Possible reasons for this will be discussed later. For carrier concentration in the range of 10^{16} cm^{-3} , with the magnetic field used here, several Landau levels lie beneath the Fermi surface.⁶ Thus $\Delta\nu$ for the $\Delta l = 1$ or $\Delta l = 2$ transitions provides a measure of m^* averaged over several Landau-level transitions and over electron momenta. The departure from a straight line for the plot of $\Delta\nu$ vs B for $\Delta l = 1$ and 2 processes measures the energy dependence of m^* (i.e., nonparabolicity of the conduction band). For these two processes, m^* varies from $0.017m$ for $\Delta\nu = 160 \text{ cm}^{-1}$ to $0.020m$ for $\Delta\nu = 250 \text{ cm}^{-1}$. In addition, the $\Delta\nu$ for the spin-flip process measures the energy dependence of g_{eff} for conduction electrons. The observed g_{eff} decreased from 45.8 at 26 kOe to 41.9 at 52 kOe. These values of g_{eff} can be compared with the lower field and lower carrier concentration values obtained by Bemski.⁷ Using Zawadzky's⁸ expression relating g_{eff} to m^* , we obtain $m^* = 0.0165m$ at 26 kOe, and $m^* = 0.018m$ at 52 kOe from the spin-flip scattering results. These m^* variations with B are in reasonable agreement with earlier measurements.⁹ It is interesting to note that in all the samples studied, no striking effects were observed in any of the scattering processes when $\Delta\nu$ equaled the plasma frequency.

An appreciable variation in linewidth was observed for the spin-flip transition as the carrier concentration was varied: $\sim 5 \text{ cm}^{-1}$ for $n_e = 1 \times 10^{16} \text{ cm}^{-3}$ to $\sim 30 \text{ cm}^{-1}$ for $6 \times 10^{16} \text{ cm}^{-3}$. Linewidths for the $\Delta l = 1$ and 2 transitions were considerably larger ranging from ~ 60 to $\sim 100 \text{ cm}^{-1}$ in the above samples. Contributions to these linewidths include the effects of averaging over various Landau levels and over electron momenta mentioned above, as well as intrinsic electron lifetimes.

The sample geometry employed here is necessary for observation of the spin-flip and the $\Delta l = 1$ scattering since the incident $10.6\text{-}\mu$ radiation is linearly polarized perpendicular to \vec{B} . Theory predicts that the light scattered by spin-flip and $\Delta l = 1$ processes is linearly polarized along \vec{B} , while the $\Delta l = 2$ scattered

light is linearly polarized normal to \vec{B} . Thus, the free-carrier absorption of the scattered light at a given wavelength is nearly the same for all three processes. For the spin-flip process in a 3×10^{16} -cm $^{-3}$ sample a scattered power of $\sim 8 \times 10^{-5}$ W was observed for an input power of ~ 10 kW. This corresponds to an observed cross section of $\sim 10^{-23}$ cm 2 sr $^{-1}$, which is in order-of-magnitude agreement with theoretical predictions.^{2,3} As mentioned earlier the $\Delta l = 1$ and 2 transitions could be barely detected from the 3×10^{16} -cm $^{-3}$ sample. In the $n_e = 5 \times 10^{16}$ -cm $^{-3}$ sample, where $\Delta l = 1$ and 2 transitions were observed, both the processes were of comparable strength, having a cross section of $\sim 10^{-24}$ cm 2 sr $^{-1}$. The $\Delta l = 1$ process, previously untreated, can arise from the $(p_z A_z)(p^+ A^-)$ part of the $(\vec{p} \cdot \vec{A})^2$ term¹⁰ of the interaction Hamiltonian employed by Wolff.¹ (z is the direction of the applied magnetic field and that of the incident 10.6- μ radiation.) The remaining part, $(p^+ A^-)^2$, of the $(\vec{p} \cdot \vec{A})^2$ term is responsible for the $\Delta l = 2$ transition. For samples with $n_e \approx 10^{16}$ cm $^{-3}$, it can be shown that the $\Delta l = 2$ and the $\Delta l = 1$ processes should have about equal cross sections. However, the $\Delta l = 1$ scattering cross section should increase with n_e since it involves the averaged z component of electron momentum.

The magnetic field dependence of the observed scattered intensity does not agree with the theoretical predictions. The observed intensity of the spin-flip process increases with magnetic field (by about a factor of 2 between 26 and 52 kOe), whereas Yafet² has calculated a nearly field-independent cross section. Moreover, the B^2 dependence in the cross section of the $\Delta l = 2$ process predicted by Wolff¹ has not been observed. Instead, at 26 kOe the $\Delta l = 2$ scattered light was about 2 to 3 times weaker than that at 15 kOe. In addition, increasing the field from 26 to 30 kOe resulted in total disappearance of the $\Delta l = 2$ line. This behavior cannot be attributed solely to the increased free-carrier absorption at longer wavelengths. The position of the Landau levels with respect to the Fermi surface may be important when interpreting the above results, because several pairs of Landau levels are involved in the scattering processes, and relative populations of Landau levels vary with magnetic field.⁶ Additional effects on scattering cross sections may arise from polaron interactions¹¹ for magnetic fields of 30 to 40 kOe.

The effect observed here provides a versatile method of studying electronic parameters in semiconductors, such as m^* , g_{eff} , τ , etc. Unlike resonance or absorption experiments, where similar information can be obtained, transitions with widely different frequencies can be conveniently studied since there is no requirement that the frequency of the incident light match that of the transition of interest. Further, our measurements are not complicated by the details of the valence-band structure, the position of the plasma resonance, or the absorption due to optical phonons. A particular advantage of the 10.6- μ radiation over shorter wavelengths in studying scattering from mobile carriers is the near absence of Raman scattering from the optical phonons due to the ω_0^4 dependence¹² of the latter process.

Because the frequency of the scattered light can be varied over a wide range, the possibility of producing tunable stimulated Landau-level scattering is of considerable interest. Wolff¹³ has investigated the threshold for the $\Delta l = 2$ process, and our measurements of cross sections and linewidths should be useful in predicting more accurate values for the threshold. However, the larger cross section and the narrower linewidth observed for the spin-flip process suggests that it should have a considerably lower threshold for laser oscillation.

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⁹See, for example, O. Madelung, Physics of III-V Compounds (John Wiley & Sons, Inc., New York, 1964), p. 92; see also C. R. Pidgeon and R. N. Brown, Phys. Rev. **146**, 575 (1966).

¹⁰An additional contribution to the $\Delta l = 1$ process may arise from the lack of inversion symmetry in InSb. However, this effect will not be discussed here.

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¹²See, for example, M. Born and K. Huang, Dynamical

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PARITY MIXING IN NUCLEAR HARTREE-FOCK CALCULATIONS*

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It is well known that although the nuclear Hamiltonian may possess a certain symmetry, the Hartree-Fock (HF) Hamiltonian will, in general, not commute with the operator corresponding to that symmetry. Thus, restrictions on the variational single-particle wave functions should not be inferred from properties of the actual many-body wave function. Specifically, it is an assumption, a priori unjustifiable, to restrict the HF wave functions to be eigenstates of the parity operator. Such an assumption is especially dubious in the light of the fact that the tensor force can give no contribution (at least the direct term) if parity is not mixed in the trial wave functions.¹ Previous investigations²⁻⁴ have shown that agreement with the observed spin-orbit splittings and magnetic moments could be obtained from the tensor force if parity mixing were allowed. However, in the HF calculations presently being performed with realistic forces, the lowest energy solution is always found to have single-particle wave functions of good parity. (The details of the HF procedure and the results of these calculations can be found in Baranger.⁵)

Since the forces being used in these calculations (the Tabakin⁶ and Yale-Shakin⁷ forces) are rather complicated, a simplified force was constructed and a detailed investigation of parity mixing was carried out. The force consisted of the central and tensor parts of the Hamada-Johnston⁸ force with the infinite hard cores replaced by finite cores of heights 150 and 0 MeV, respectively. The radius of the soft core was 0.67 F. This model force, though unrealistic, contains some of the features of the realistic potential of Bressel⁹ and is well suited to a study of the relation between parity mixing and the strength of the tensor force. The nuclei considered were the closed-shell nuclei He⁴, O¹⁶, and Ca⁴⁰, and

the deformed nuclei Be⁸, C¹², Ne²⁰, and Si²⁸. The trial wave functions were taken to be linear combinations of harmonic oscillator wave functions:

$$|\lambda\rangle = \sum C |nljm\tau_z\rangle$$

with the summation on n , l , and j . Thus, though radial variations, deformations, and parity mixing were allowed, it was assumed that the HF solutions are axially symmetric and eigenstates of τ_z . The variational space included the $1s$, $1p$, $2s-1d$, and $2p-1f$ shells. The orbits of all A nucleons were treated variationally.

It is worth going into some detail concerning the initial choice of the variational parameters (C). If the total Hamiltonian possesses a symmetry and the C 's are suitably chosen for the first iteration, then the HF solutions may also have that symmetry. It was found in our other calculations that even if the C 's for $+m$ states and $-m$ states were initially unrelated, the minimal solution was invariant under time reversal, i.e., if

$$|\lambda_+\rangle = \sum_{nlj} C_{nlj}^{m\tau_z} |nljm\tau_z\rangle,$$

and

$$|\lambda_-\rangle = \sum_{nlj} C_{nlj}^{-m\tau_z} |nlj-m\tau_z\rangle,$$

are occupied states, then

$$T|\lambda_+\rangle = \sum_{nlj} (C_{nlj}^{m\tau_z})^* (-1)^{j+l-m} |nlj-m\tau_z\rangle = |\lambda_-\rangle,$$

or

$$(C_{nlj}^{m\tau_z})^* (-1)^{j+l-m} = C_{nlj}^{-m\tau_z}.$$