rem. [M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1965).]

[6]Rosenfeld et al., Ref. 4.

[7]K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters 16, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966).

[8]Riazuddin and Fayyazuddin, Ref. 7; V. S. Mathur, L. K. Pandit, and R. E. Marshak, Phys. Rev. Letters 16, 947 (1966).

[9]The fact that the symmetry limit is reached earlier in this case than in the previous cases of the chiral symmetry groups within the framework of the pole dominances is indicative of the fact that the SU(3) is a better symmetry of nature.

MICROCAUSALITY AND THE REPRESENTATIONS OF SELF-CONJUGATE BOSONS

Gordon N. Fleming and E. Kazes
Department of Physics, The Pennsylvania State University, University Park, Pennsylvania
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In a recent Letter Carruthers[1] has demonstrated a most interesting connection between the possible representations of the isospin symmetry group SU(2) possessed by self-conjugate spinless bosons and microcausality. The theorem, which was derived in the framework of canonical quantization for free fields, states that self-conjugate spinless bosons comprising spinorial (even number of dimensions) representations of the isospin group are the quanta of fields which do not commute for spacelike separation. Carruthers explicitly calculates the nonvanishing commutator.

We have found a generalization of Carruthers's result in which the dependence of the argument on canonical quantization and the restriction to free fields is removed. Furthermore, our results can be stated in a manner immediately applicable to arbitrary internal symmetry groups and at the end of this Letter we shall indicate how the generalization to higher (integral) mechanical spin can be made.

Our proof depends on the following assumptions:

(i) The set of scalar fields $\varphi_\alpha{}^+(x)$, $\alpha=1, \cdots, N$, transform among themselves according to an irreducible, unitary representation of some group G,

$$U(g)^{-1}\varphi_\alpha{}^+(x)U(g)=D_{\alpha\beta}(g)\varphi_\beta{}^+(x), \tag{1}$$

where $U(g)$ is the unitary operator in the quantum mechanical state space which effects the group transformation on the states.

(ii) The vacuum is invariant under $U(g)$,

$$U(g)|0\rangle=|0\rangle. \tag{2}$$

(iii) The fields $\varphi_\alpha(x)$ are self-conjugate, i.e., there exists some c-number function $K_{\alpha\beta}(x-x')$, such that

$$\varphi_\alpha{}^+(x)=\int d^4x' K_{\alpha\beta}(x-x')\varphi_\beta(x'). \tag{3}$$

(iv) The fields commute for spacelike separation,

$$[\varphi_\alpha(x), \varphi_\beta(y)]=0 \tag{4}$$

for $(x-y)^2<0$.

The appearance of the same space-time four-vector on either side of (1) stamps the group transformations as referring to internal degrees of freedom. The invariance of the vacuum, (2), is, following Coleman,[2] tantamount to the assumption that the group G is a symmetry group. Equation (3) is much weaker than the usual statements of self-conjugation in which $K_{\alpha\beta}(x-x')$ has the form

$$K_{\alpha\beta}(x-x')=\eta_\alpha\delta_{\alpha\tilde{\beta}}\delta^4(x-x'), \tag{5}$$

where $\beta\to\tilde{\beta}$ is a one-to-one mapping of the β's onto themselves and $|\eta_\alpha|=1$. Whether the weaker condition (3) has any practical advantage over (5) is difficult to say.

Theorem. – The assumptions (i)-(iv) demand that the representation of G provided by the $D(g)$ in (1) is equivalent to the representation provided by the complex conjugated $D^*(g)$, and that the transformation from the $D^*(g)$ to the $D(g)$ must be via a symmetric, unitary matrix. Following Wigner's[3] terminology the allowable representations are "potentially real," i.e., it is possible to find a basis in which all the D's are real.

Proof. – Consider

$$\langle 0|\varphi_\alpha(x)\varphi_\beta(y)|0\rangle\equiv C_{\alpha\beta}(x-y). \tag{6}$$

This is a Lorentz-invariant function of $x-y$ and for spacelike separation must be a function of $(x-y)^2$. From (4),

$$C_{\alpha\beta}((x-y)^2)=C_{\beta\alpha}((x-y)^2) \tag{7}$$

for $(x-y)^2<0$ so that C is symmetric in the internal indices.

But

$$\begin{aligned} C_{\alpha\beta}((x-y)^2) &= \langle 0|\varphi_\alpha(x)\varphi_\beta(y)|0\rangle = \langle 0|U(g)^{-1}\varphi_\alpha(x)U(g)U(g)^{-1}\varphi_\beta(y)U(g)|0\rangle \\ &= \langle 0|D_{\alpha\gamma}{}^*(g)\varphi_\gamma(x)D_{\beta\delta}{}^*(g)\varphi_\delta(y)|0\rangle \\ &= D_{\alpha\gamma}{}^*(g)C_{\gamma\delta}((x-y)^2)D_{\beta\delta}{}^*(g). \end{aligned} \tag{8}$$

Suppressing the space-time variables and noting that the D's are unitary, this may be written as

$$CD(g)=D^*(g)C. \tag{9}$$

But since

$$C^*CD(g)=C^*D^*(g)C=D(g)C^*C \tag{10}$$

it follows that

$$C^*C=\lambda I, \tag{11}$$

and the symmetry of C makes C proportional to a unitary matrix and λ is positive semidefinite. If $\lambda\neq 0$ then C^{-1} exists and

$$D(g)=C^{-1}D^*(g)C; \tag{12}$$

the D's form a "potentially real" representation.

If $\lambda=0$ then $C=0$ and applying the Hall-Wightman[4] theorem to the $C_{\alpha\beta}((x-y)^2)$ we conclude that

$$C_{\alpha\beta}((x-y)^2)=0 \tag{13}$$

for any $x-y$, timelike and null as well as spacelike. But from (iii)

$$\begin{aligned} \|\varphi_\alpha{}^+(f)|0\rangle\|^2 &= \langle 0|\varphi_\alpha(f)\varphi_\alpha{}^+(f)|0\rangle \\ &= \int d^4x d^4y\, f(x)f^*(y)\langle 0|\varphi_\alpha(x)\varphi_\alpha{}^+(y)|0\rangle \\ &= \int d^4x d^4y\, f(x)f^*(y)\int d^4y'\, K_{\alpha\beta}(y-y')\langle 0|\varphi_\alpha(x)\varphi_\beta(y')|0\rangle \\ &= 0. \end{aligned} \tag{14}$$

Clearly one can also obtain $\|\varphi_\alpha(f)|0\rangle\|=0$. Hence a nontrivial self-conjugate scalar field theory permits only "potentially real" representations of internal symmetry groups, i.e., those representations with symmetric C's.

(a) For SU(2) the integral-spin representations require C antisymmetric.[3] Hence the latter are excluded.

(b) For SU(3), those representations $D(\lambda_1,\lambda_2)$ with $\lambda_1\neq\lambda_2$ have complex characters and are excluded.[5] The remaining representations with $\lambda_1=\lambda_2$ have a symmetric C, and are therefore allowed.

(c) For a particle with integral mechanical spin $\varphi_{\alpha,\mu}(x)$ (where the second subscript refers to its space-time properties), it suffices to require that

$$[\varphi_{\alpha,\mu}(x),\varphi_{\beta,\mu}(y)]=0 \tag{15}$$

for spacelike separation. Now, for spacelike separation,

$$\langle 0|\varphi_{\alpha,\mu}(x)\varphi_{\beta,\mu}(y)|0\rangle$$

is symmetric[6] under the interchange of x and y and hence must further be symmetric in the exchange of α and β in order to satisfy microcausality. Hence if the representation of the internal symmetry group is not equivalent to its complex conjugate via a symmetric matrix then we must have $C=0$ again. The Hall-Wightman theorem can again be invoked as for (13) and the conclusions (a) and (b) above still apply. Note that the appropriate generalization of (3) must not mix mechanical spin indices.

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[1]P. Carruthers, Phys. Rev. Letters 18, 353 (1967).
[2]S. Coleman, J. Math. Phys. 7, 787 (1967).
[3]E. P. Wigner, Group Theory and Its Applications to Quantum Mechanics of Atomic Spectra (Academic Press, Inc., New York, 1959), pp. 285-289.
[4]D. Hall and A. S. Wightman, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 31, No. 5 (1957).
[5]R. E. Behrends, J. Dreitlein, C. Fronsdal, and B. W. Lee, Rev. Mod. Phys. 34, 1 (1962); see also P. Carruthers, Introduction to Unitary Symmetry (John Wiley & Sons, Inc., New York, 1966).
[6]N. Burgoyne, Nuovo Cimento 8, 607 (1958).

ERRATUM

FACTORIZATION OF HELICITY AMPLITUDES AT HIGH ENERGIES. G. C. Fox and Elliot Leader [Phys. Rev. Letters 18, 628 (1967)].

Equation (11) should read

$$f_{cd;ab}^{(s)} \propto (-t)^{\frac{1}{2}(|\Lambda_{ac}| + |\Lambda_{bd}|)}.$$