in reasonable agreement with the experimental value[10] of 4.6 MeV. It is not surprising that the approximation of keeping only the lowest lying resonances in the spectral functions leads to a reasonable value of the π^+-π^0 mass difference, since the $\Delta I = 2$ part of the effective electromagnetic interaction relevant in this case is well described[11] by low q^2 values.

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‡On leave from Tata Institute of Fundamental Research, Bombay, India.

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[6]We assume that the Schwinger terms arising in the equal-time commutation relations are c numbers.

[7]In contrast to Weinberg's notation (Ref. 1) our currents satisfy the usual commutation relations.

[8]Note that because of the sum rule (7) we have $g_\rho^2 = g_A^2$, which plays a crucial role in providing the convergence of the integral over g^2 in (9).

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SYMMETRY, SUPERCONVERGENCE, AND SUM RULES FOR SPECTRAL FUNCTIONS*

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We discuss the convergence and the superconvergence properties of the invariant amplitudes which occur in suitable combinations of the propagator functions of the vector and the axial-vector currents, based on the use of symmetry arguments for the asymptotic behavior. The sum rules so obtained for the spectral functions are in good agreement with experiment.

In this note we discuss a general way to obtain sum rules for the spectral functions of vector and axial-vector currents. Two of the sum rules we derive have been recently obtained by Weinberg[1] under the more restrictive assumption that the pion is massless, so that the axial-vector current is divergenceless. Our method exploits the possibility of superconvergence of amplitudes occurring in suitable combinations of the vector and axial-vector two-point functions. We show that the case for convergence or superconvergence can be made on the basis of group-symmetry arguments alone. This provides some interesting insight into the relation between the concepts of superconvergence and group symmetry. Finally, we show that the sum rules we so obtain are well saturated by suitable low-lying particle states.

We start by considering the vacuum expectation values of the time-ordered product of two vector and two axial-vector currents, whose Fourier transforms are given, respectively, by

$$\Delta_{\mu\nu}^{V}(q) = \int d^4x\, e^{-iq\cdot x}\langle 0|T\{V_{\mu j}^{i}(x)V_{\nu i}^{j}(0)\}|0\rangle, \tag{1}$$

$$\Delta_{\mu\nu}^{A}(q) = \int d^4x\, e^{-iq\cdot x}\langle 0|T\{A_{\mu j}^{i}(x)A_{\nu i}^{j}(0)\}|0\rangle, \tag{2}$$

where i and j are the SU(2) indices ($i, j = 1, 2$). If SU(2)⊗SU(2) were an exact symmetry, we would have $\Delta_{\mu\nu}^{V}(q) = \Delta_{\mu\nu}^{A}(q)$ for all values of q. However, in nature this symmetry seems to be broken, but nevertheless one expects that in the asymptotic limit $q \to \infty$, SU(2)⊗SU(2) would be an exact symmetry. Thus it seems reasonable to assume that

$$\lim_{q\to\infty}[\Delta_{\mu\nu}{}^{V}(q)-\Delta_{\mu\nu}{}^{A}(q)]=0. \tag{3}$$

Writing the general structure

$$\Delta_{\mu\nu}{}^{V}(q)-\Delta_{\mu\nu}{}^{A}(q) =F(q^2)\delta_{\mu\nu}+G(q^2)q_\mu q_\nu+H\delta_{\mu4}\delta_{\nu4}, \tag{4}$$

where H is the contribution due to the Schwinger terms, we observe that Eq. (3) implies (at least) that $F(q^2)$ satisfies a dispersion relation without subtractions, $G(q^2)$ must be superconvergent, and finally the Schwinger terms in the vector and axial-vector cases must be identical. The identity of the two Schwinger terms has also been noted by Weinberg under somewhat more restrictive assumptions. To make stronger statements of convergence or superconvergence on the invariant amplitudes $F(q^2)$ and $G(q^2)$, one has to know how fast $\Delta_{\mu\nu}{}^{V}(q)-\Delta_{\mu\nu}{}^{A}(q)$ goes to 0 asymptotically. Stated otherwise, one has to know beyond what value of q will the $SU(2)\otimes SU(2)$ symmetry become "almost" exact. If $SU(2)\otimes SU(2)$ is not such a bad symmetry, one would expect symmetry to "set in" at somewhat lower values of q, so that $F(q^2)$ and $G(q^2)$ would satisfy stronger requirements of convergence than the ones implied by Eq. (3). These requirements would then lead to sum rules which are a step closer to the results of the exact $SU(2)\otimes SU(2)$ symmetry. Clearly, this argument can be extended indefinitely until the infinite hierarchy of sum rules would lead to the solution identical with the symmetry result, i.e., $F(q^2)=G(q^2)=0$ for all q^2 and $H=0$. For practical purposes, to investigate the effects of broken symmetry, it is easy to see at which stage of superconvergence to stop, since with the assumption of the dominance of the sum rules by a few low-mass states, one would already start encountering the symmetry result beyond a few steps.

Using Källén-Lehmann representation we can express $F(q^2)$ and $G(q^2)$ in the following forms:

$$F(q^2)=-i\int_0^\infty\frac{\rho_V(m^2)-\rho_A(m^2)}{q^2+m^2-i\epsilon}dm^2, \tag{5}$$

$$G(q^2) =-i\left[\int_0^\infty\frac{\rho_V{}'(m^2)-\rho_A{}'(m^2)}{m^2(q^2+m^2-i\epsilon)}dm^2-\frac{F_\pi{}^2}{q^2+m_\pi{}^2-i\epsilon}\right], \tag{6}$$

where ρ and ρ' are the corresponding spectral functions[2] and F_π is the pion-decay amplitude defined by

$$\langle 0|A_{\mu2}{}^1(0)|\pi^+(q)\rangle=iF_\pi q_\mu/(2q_0V)^{1/2}. \tag{7}$$

In Eq. (6) we have explicitly separated out the contribution due to the pion pole. The demands of superconvergence[3] of $G(q^2)$ and $F(q^2)$ lead to the following sum rules of Weinberg[1] which contain information of broken symmetry:

$$\int_0^\infty\frac{\rho_V{}'(m^2)-\rho_A{}'(m^2)}{m^2}dm^2=F_\pi{}^2, \tag{8}$$

$$\int_0^\infty[\rho_V(m^2)-\rho_A(m^2)]dm^2=0. \tag{9}$$

If we dominate the spectral functions by keeping only the ρ and A_1 poles, we obtain Weinberg's results.

Since our derivation is not restricted by the assumption of a zero-mass particle or divergenceless currents, it seems natural that one can extend these ideas to currents of a larger group, such as the chiral $SU(3)\otimes SU(3)$. Following the same procedure as before, one obtains sum rules similar to (8) and (9) involving spectral functions of strangeness-changing vector and axial-vector currents. If we now assume that the spectral functions of the vector current can be dominated by the K^* pole, and that there exists a strangeness-carrying 1^+ resonance denoted by Q, which dominates the axial-vector spectral functions, we obtain the following result[4]:

$$\frac{G_{K^*}{}^2}{M_{K^*}{}^2}\left[1-\frac{M_{K^*}{}^2}{M_Q{}^2}\right]=F_K{}^2, \tag{10}$$

where G_{K^*} and F_K are defined by the following matrix elements:

$$\langle 0|V_{\mu3}{}^1(0)|K^{*+}(q)\rangle=G_{K^*}E_\mu/(2q_0V)^{1/2} \tag{11}$$

and

$$\langle 0|A_{\mu3}{}^1(0)|K^+(q)\rangle=iF_Kq_\mu/(2q_0V)^{1/2}. \tag{12}$$

One can estimate G_{K^*} by dominating the K_{l3} form factor f_+ by the K^* pole, so that

$$G_{K^*}=-f_+(0)M_{K^*}^2/G_{K^*K\pi}, \tag{13}$$

where $G_{K^*K\pi}$ is the $K^*K\pi$ coupling constant and hence related to the K^* width. Using[5] $f_+(0)=-1/\sqrt{2}$, $F_K=1.04m_\pi$ from K_{l2} decays, and the known K^* width, we obtain

$$M_Q/M_{K^*}\simeq 1.47, \tag{14}$$

which gives $M_Q\simeq 1311$ MeV remarkably close to the mass of the $K\pi\pi$ resonance observed at 1313 ± 8 MeV.[6]

Our results so far are based purely on the asymptotic behavior of suitable combinations of matrix elements obtained on the basis of broken chiral-symmetry groups. The constraints imposed by the SU(3) symmetry alone also lead to some interesting results. Defining

$$\Delta_{\mu\nu}^{\pi}(q) = \int d^4x\, e^{-iq\cdot x}\langle 0|T\{V_{\mu 2}^{\ 1}(x)V_{\nu 1}^{\ 2}(0)\}|0\rangle, \tag{15}$$

$$\Delta_{\mu\nu}^{K}(q) = \int d^4x\, e^{-iq\cdot x}\langle 0|T\{V_{\mu 3}^{\ 1}(x)V_{\nu 1}^{\ 3}(0)\}|0\rangle, \tag{16}$$

and assuming that the SU(3) symmetry becomes exact in the limit $q\to\infty$, we have

$$\lim_{q\to\infty}[\Delta_{\mu\nu}^{\pi}(q)-\Delta_{\mu\nu}^{K}(q)]=0. \tag{17}$$

Proceeding as before, we obtain the following sum rule:

$$\int_0^\infty \frac{\rho^{(\pi)}(m^2)-\rho^{(K)}(m^2)}{m^2}dm^2=0. \tag{18}$$

We assume that the spectral functions are peaked at the ρ and the K^* masses, i.e.,

$$\rho^{(\pi)}(m^2)\simeq G_\rho^{\ 2}\delta(m^2-M_\rho^{\ 2}),$$

$$\rho^{(K)}(m^2)\simeq G_{K^*}^{\ 2}\delta(m^2-M_{K^*}^{\ 2}), \tag{19}$$

where G_{K^*} has been defined by Eq. (11) and G_ρ is analogously defined. Using the current-algebra result[7] $G_\rho^2=2M_\rho^2F_\pi^2$ and Eq. (13), we obtain from the sum rule (18) the result

$$G_{K^*K\pi}^{\ 2}=f_+^{\ 2}(0)M_{K^*}^{\ 2}/2F_\pi^{\ 2}, \tag{20}$$

which leads to a width of the K^*,

$$\Gamma(K^*)\simeq 46\text{ MeV}, \tag{21}$$

to be compared with the experimental value[6] of 50 ± 1.4 MeV. It should be noted that the previous calculations of $\Gamma(K^*)$ based on current algebra[8] make use of the kaon partially conserved axial-vector current, which has not been employed in the present derivation. We would also like to point out that stronger conditions of convergence on

$$\lim_{q\to\infty}[\Delta_{\mu\nu}^{\pi}(q)-\Delta_{\mu\nu}^{K}(q)]$$

than the one used here already lead to symmetry results in the pole approximation considered.[9]

One can apply the same technique to the case of axial-vector currents of the π and the K types. In this case it is easy to see that the resulting sum rule is not independent but is already contained in the previous sum rules.

The technique of studying the behavior of superconvergence of some suitable combination of matrix elements on the basis of symmetry arguments is clearly of more general applicability than the cases considered here. An application of this method to scattering amplitudes and form factors will be treated elsewhere.

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†On leave from Tata Institute of Fundamental Research, Bombay, India.

‡On leave from The Center of Advanced Studies in Physics and Astrophysics, Delhi, India.

[1]S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

[2]For a conserved vector current, $\rho_V=\rho_V'$ and for axial-vector spectral functions, $\rho_A=\rho_A'$, if, for example, partial conservation of axial-vector current is assumed. However, for generality we do not assume $\rho=\rho'$, particularly in view of our later considerations of the strangeness-changing currents.

[3]The cancellation of the Schwinger terms does not give any extra information other than that contained in Eq. (8).

[4]We have ignored the contribution of the scalar κ, because its existence is doubtful (A. H. Rosenfeld et al., University of California Radiation Laboratory Report No. UCRL-8030). Even if it exists, its contribution will be negligible if the width is small.

[5]We have used the SU(3) value of $f_+(0)$, which is quite reliable as a consequence of the Ademollo-Gatto theo-

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[6]Rosenfeld et al., Ref. 4.

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[9]The fact that the symmetry limit is reached earlier in this case than in the previous cases of the chiral symmetry groups within the framework of the pole dominances is indicative of the fact that the SU(3) is a better symmetry of nature.

MICROCAUSALITY AND THE REPRESENTATIONS OF SELF-CONJUGATE BOSONS

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In a recent Letter Carruthers[1] has demonstrated a most interesting connection between the possible representations of the isospin symmetry group SU(2) possessed by self-conjugate spinless bosons and microcausality. The theorem, which was derived in the framework of canonical quantization for free fields, states that self-conjugate spinless bosons comprising spinorial (even number of dimensions) representations of the isospin group are the quanta of fields which do not commute for spacelike separation. Carruthers explicitly calculates the nonvanishing commutator.

We have found a generalization of Carruthers's result in which the dependence of the argument on canonical quantization and the restriction to free fields is removed. Furthermore, our results can be stated in a manner immediately applicable to arbitrary internal symmetry groups and at the end of this Letter we shall indicate how the generalization to higher (integral) mechanical spin can be made.

Our proof depends on the following assumptions:

(i) The set of scalar fields $\varphi_\alpha^{+}(x)$, $\alpha=1, \cdots, N$, transform among themselves according to an irreducible, unitary representation of some group G,

$$U(g)^{-1}\varphi_\alpha^{+}(x)U(g)=D_{\alpha\beta}(g)\varphi_\beta^{+}(x), \qquad (1)$$

where $U(g)$ is the unitary operator in the quantum mechanical state space which effects the group transformation on the states.

(ii) The vacuum is invariant under $U(g)$,

$$U(g)|0\rangle=|0\rangle. \qquad (2)$$

(iii) The fields $\varphi_\alpha(x)$ are self-conjugate, i.e., there exists some c-number function $K_{\alpha\beta}(x-x')$, such that

$$\varphi_\alpha^{+}(x)=\int d^4x' K_{\alpha\beta}(x-x')\varphi_\beta(x'). \qquad (3)$$

(iv) The fields commute for spacelike separation,

$$[\varphi_\alpha(x), \varphi_\beta(y)]=0 \qquad (4)$$

for $(x-y)^2<0$.

The appearance of the same space-time four-vector on either side of (1) stamps the group transformations as referring to internal degrees of freedom. The invariance of the vacuum, (2), is, following Coleman,[2] tantamount to the assumption that the group G is a symmetry group. Equation (3) is much weaker than the usual statements of self-conjugation in which $K_{\alpha\beta}(x-x')$ has the form

$$K_{\alpha\beta}(x-x')=\eta_\alpha\delta_{\alpha\tilde{\beta}}\delta^4(x-x'), \qquad (5)$$

where $\beta\to\tilde{\beta}$ is a one-to-one mapping of the β's onto themselves and $|\eta_\alpha|=1$. Whether the weaker condition (3) has any practical advantage over (5) is difficult to say.

Theorem. – The assumptions (i)-(iv) demand that the representation of G provided by the $D(g)$ in (1) is equivalent to the representation provided by the complex conjugated $D^*(g)$, and that the transformation from the $D^*(g)$ to the $D(g)$ must be via a symmetric, unitary matrix. Following Wigner's[3] terminology the allowable representations are "potentially real," i.e., it is possible to find a basis in which all the D's are real.

Proof. – Consider

$$\langle 0|\varphi_\alpha(x)\varphi_\beta(y)|0\rangle\equiv C_{\alpha\beta}(x-y). \qquad (6)$$