# MAGNETIC CONTRIBUTION TO THE PROTON-NEUTRON MASS DIFFERENCE*

John M. Cornwall† and Sharashchandra H. Patil
Department of Physics, University of California, Los Angeles, California
(Received 3 April 1967)

In this note we point out that there can be a quite appreciable magnetic $n$-$p$ mass difference if proper account is taken of the Roper resonance[1] at 1400 MeV, with exactly the same quantum numbers as the nucleon. We calculate $M_n - M_p \simeq 2.4$ MeV without the corrections for Coulomb self-energy.

The calculation is based on the assumption that (a) unsubtracted dispersion relations hold for the nucleon proper self-energy part, similar to those used by Barger and Kazes,[2,3] (b) the intermediate states with the hadrons having $I=\frac{1}{2}$ are dominated (in the sense of dispersion theory) by the $N\gamma$ state, and (c) the magnetic $NN\gamma$ proper vertex functions are dominated by the Roper resonance. Assumption (a) is closely connected to the hypothesis that the nucleon is composite, as expressed by the vanishing of the wave-function renormalization constant $Z_2$. Let us illustrate this by considering the electromagnetic mass differences in a multiplet of scalar particles. The imaginary part of the electromagnetic proper self-energy (keeping only intermediate states with one scalar and one photon) is

$$\mathrm{Im}\Pi(s) = (\alpha/4s)(s-M^2)\Gamma_\mu{}^*(s)\Gamma^\mu(s), \tag{1}$$

where $\alpha$ is the fine-structure constant, and $\Gamma_\mu(s)$ is the proper scalar-scalar-photon vertex with the photon (of momentum $p-p'$) and one scalar (of momentum $p'$) on the mass shell, while the other scalar has mass $p^2=s$. We have

$$\Gamma_\mu(s) = (p+p')_\mu F_1(s) + (p-p')_\mu F_2(s),$$

and thus

$$\mathrm{Im}\Pi(s) = (\alpha/2s)[(s+M^2)|F_1|^2 + (s-M^2)\mathrm{Re}F_1F_2{}^*]. \tag{2}$$

In lowest order of perturbation theory, $F_1=1$, $F_2=0$, and the self-energy is quadratically divergent. To all orders of the strong interactions, $F_2$ presumably goes like $s^{-1}$ (since it is an induced form factor). For $F_1$, we use Ward's identity to conclude

$$(s-M^2)F_1(s) = D^{-1}(s) \underset{s\to\infty}{\longrightarrow} Z_2(s-M^2), \tag{3}$$

where $D^{-1}$ is the inverse propagator. Thus, if $Z_2=0$, we expect $F_1(s)$ to converge rapidly (possibly like $s^{-1}$ or $s^{-1}\ln s$) at infinite $s$, which allows an unsubtracted dispersion relation for $\Pi(s)$ and makes the self-mass finite, even in the presence of electromagnetism. A similar investigation for the $NN\gamma$ vertex would be considerably more complicated, since the vertex with all particles off the mass shell has six form factors, and several of these appear in Ward's identity.

For computing the $n$-$p$ mass difference, we use the spin-$\frac{1}{2}$ analog of Eq. (1). The electromagnetic self-energy contribution can be written as

$$\Pi(s) = A(s) + \not{p}B(s), \tag{4}$$

and the mass shift is just

$$\delta M_{n,p} = A_{n,p}(M^2) + MB_{n,p}(M^2). \tag{5}$$

With unsubtracted dispersion relations for $A$ and $B$,

$$\delta M = \frac{\mathcal{P}}{\pi}\int_{M^2}^{\infty}\frac{ds'}{s'-M^2}[\mathrm{Im}A(s') + M\,\mathrm{Im}B(s')] \tag{6}$$

($\mathcal{P}$ means principal part). The proper $NN\gamma$ vertex functions which appear in $\mathrm{Im}A, B$ can be expressed in terms of the matrix element $\langle 0|J_N(0)|N\gamma\rangle$, where $J_N(0)$ is the nucleon current. (Note that intermediate states like $N_{33}{}^*\gamma$ contribute equally to the proton and neutron self-energies, and hence cancel out in $M_n-M_p$.) Very little is known experimentally about the $NN\gamma$ vertex with one fermion off the mass shell. But phase-shift analysis of $\pi N$ scattering in the $P_{11}$ channel indicates a resonance with $IJ^P = \frac{1}{2}\frac{1}{2}^+$, at a mass of 1400 MeV, with a width of 200 MeV, called the Roper resonance. Let us assume that this dominates the Sachs magnetic form factor $F_M(s)$:

$$F_M{}^{n,p}(s) = \mu_{n,p}\left[\frac{M^2-M_R{}^2-iM_R\Gamma_R}{s-M_R{}^2-iM_R\Gamma_R}\right] \tag{7}$$

Here $\mu$ is the total magnetic moment in units of e/2M, and $R$ refers to the Roper resonance.

In the usual way, we find

$$\operatorname{Im}A_{n,p}(s)+M\operatorname{Im}B_{n,p}(s)$$
$$=-(\alpha/16Ms^2)(s-M^2)^3|F_M^{n,p}(s)|^2. \quad (8)$$

Of course, to Eq. (8) we should add the contribution from electric form factors, which is of the opposite sign. But we do not expect electric form factors to be dominated by the Roper resonance, essentially because the $RN\gamma$ vertex is purely magnetic when all the particles are on the mass shell; the usual polology prescription then says that the residue of the $R$ pole in the $NN\gamma$ vertex is just this on-mass-shell $RN\gamma$ vertex. Alternatively, one may use Ward's identity for the electric form factors to relate these to the inverse propagator; we do not expect a Roper pole to dominate the nucleon proper self-energy. When the nucleon is off the mass shell, there are other form factors besides the two Sachs form factors, but we will ignore them for lack of better knowledge.

The rest of the calculation proceeds in a standard way. For the width $\Gamma_R(s)$ in Eq. (7) we use a simple expression reflecting the $p$-wave nature of the resonance,

$$\Gamma_R(s)=\beta M_R(s-M^2)^3 s^{-2}, \quad (9)$$

where $\beta=8.4\times10^{-3}$ for a mass-shell width of 200 MeV. Using Eqs. (6)-(9), we find

$$M_n-M_p\simeq 2.4 \text{ MeV}. \quad (10)$$

The mass difference depends on the product of the isoscalar and isovector magnetic moments, i.e., on $(\mu_p+\mu_n)(\mu_p-\mu_n)$. The value in Eq. (10) is somewhat greater than the experimental value of 1.3 MeV. It is difficult to estimate the corrections which must be applied for the Coulomb energy and other terms neglected. Coulomb energy will decrease this value, but may be of the order 1 MeV or less,[4] which would bring $M_n-M_p$ as calculated here to $\simeq 1.4$ MeV, which is very close to the experimental value.[5]

We do not want to oversimplify the extremely difficult problem of electromagnetic mass differences. There is not space here to discuss all the possibly relevant factors (strange particles, baryon resonances, feedback, etc.) which make the $n$-$p$ mass difference truly complicated. The reader should consult Barton and Dare[6] for a discussion of these points.

In certain respects, the emphasis in our work here is virtually orthogonal to the recent work of other authors[6-9] who emphasize the Coulomb self-energy and the possibility of reversing its sign, and ignore the magnetic self-energy. These authors ignored the magnetic energies because the isoscalar anomalous magnetic moment $\mu_p+\mu_n-1$ is very small. But the total isoscalar moment $\mu_p+\mu_n$ is comparable with the electric charge, and the isovector moment $\mu_p-\mu_n$ is rather large compared with 1. If the Roper resonance further enhances the magnetic contribution, as this calculation shows, then this must be considered an important part of the $n$-$p$ mass difference. We are, in effect, replacing our ignorance of the potentials, which act to bind the composite nucleon, with our knowledge of their effects – in this case, two $p$-wave $I=\frac{1}{2}$ bound states separated by some 500 MeV.

We shall discuss these points in a lengthier article, as well as give applications to other electromagnetic mass differences.

We thank Professor L. A. P. Balász for valuable discussions.

---

*Work supported in part by the National Science Foundation.

†Alfred P. Sloan Foundation Fellow.

[1]For the original references, see A. H. Rosenfeld et al., Rev. Mod. Phys. 39, 1 (1967).

[2]V. Barger and E. Kazes, Phys. Rev. 124, 279 (1961).

[3]K. Nishijima, Phys. Rev. Letters 12, 39 (1964).

[4]We can estimate the Coulomb energy using the Sachs electric form factor and the dispersion relations. However, with constant form factor, Eq. (6) diverges strongly. With a cutoff at $2M$ and constant electric form factor, we get Coulomb energy $\simeq 1$ MeV.

[5]If instead of using dispersion relations in $s$, we use dispersion relations in $W=\sqrt{s}$, we get a magnetic mass difference of 1.9 MeV, which is slightly smaller than 2.4 MeV.

[6]G. Barton and D. Dare, Phys. Rev. 150, 1220 (1966).

[7]R. F. Dashen and S. C. Frautschi, Phys. Rev. 135, B1190 (1964).

[8]H. R. Pagels, Phys. Rev. 144, 1261 (1966).

[9]H. M. Fried and T. N. Truong, Phys. Rev. Letters 16, 557 (1966).