where B is the damping constant, f_m is the maximum binding force between pinning points and a dislocation, $A=\pi\rho b^2$ is the effective mass of a dislocation per unit length, $\omega_0=(\pi/L_c)(C/A)^{1/2}$ is the resonant frequency of a dislocation of loop length L_c, and C is the line tension of a dislocation. At 4.2°K, $B_n/B_s\approx 18$; one can therefore write

$$\frac{\Gamma_n'}{\Gamma_s'}\simeq\left[1+\left\{\frac{B\omega}{A(\omega_0^{\,2}-\omega^2)}\right\}^2\right]^{1/2}, \tag{2}$$

since B_s can be neglected in comparison with B_n, and the other parameters in Eq. (1) are not expected to change on going from the normal to the superconducting state. From the data for the amplitude shift at 10 and 20 Mc/sec, one obtains

$$\left.\frac{\Gamma_n'}{\Gamma_s'}\right|_{10\ \mathrm{Mc/sec}}\simeq 1.78\quad(\text{i.e., }\sim 5\ \mathrm{dB}), \tag{3}$$

$$\left.\frac{\Gamma_n'}{\Gamma_s'}\right|_{20\ \mathrm{Mc/sec}}\simeq 3.55\quad(\text{i.e., }\sim 11\ \mathrm{dB}). \tag{4}$$

Relations (3) and (4) can be solved for the two unknowns ω_0 and B_n, thus yielding a value for B_n which is independent of any ad hoc assumptions about the dislocation density, loop length, or any other inaccurately known parameters. One thus obtains $B\simeq 8.6\times10^{-5}$ dyn sec cm^{-2} and $\omega_0\simeq 2\pi\times 47.5$ Mc/sec. This value of B agrees with the result of a calculation by Holstein.[3] An independent experimental study of ω_0 (on the same specimen) yielded a value in good agreement with the one obtained above.

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*This research is supported in part by Research and Technology Division, U. S. Air Force Systems Command, U. S. Air Force.

[1]A. Hikata, R. A. Johnson, and C. Elbaum, Phys. Rev. Letters 17, 526 (1966).

[2]W. P. Mason, Phys. Rev. 143, 229 (1966).

[3]Calculation due to T. Holstein quoted by B. R. Tittman and H. E. Bömmel, Phys. Rev. 151, 178 (1966).

[4]J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).

DIAGONAL COHERENT-STATE REPRESENTATION OF QUANTUM OPERATORS

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(Received 16 March 1967)

We derive a simple expression for the diagonal coherent-state representation of quantum operators, and discuss some of its applications.

It was first observed by Sudarshan[1] that it is possible to express the density operator $\hat{\rho}$ of an arbitrary statistical state of a one-dimensional harmonic oscillator in the "diagonal" form[2]

$$\hat{\rho}=\int\varphi(v)\,|v\rangle\langle v|\,d^2v. \tag{1}$$

Here $|v\rangle$ is the normalized eigenstate[3] of the annihilation operator $\hat{a}$ with the (complex) eigenvalue v:

$$\hat{a}\,|v\rangle=v\,|v\rangle, \tag{2}$$

$$|v\rangle=\exp(v\hat{a}^\dagger-v^*\hat{a})\,|0\rangle, \tag{3}$$

and $\langle v|$ is the Hermitian adjoint of $|v\rangle$. The states $|v\rangle$ are also called the coherent states. The weight factor $\varphi(v)$ is in general not a well-behaved function and can be interpreted only in the sense of generalized function theory (see, for example, Mehta and Sudarshan,[4] Klauder, McKenna, and Currie,[5] and Klauder[6]). However, in most cases of practical interest it is possible to find a well-behaved function $\varphi(v)$ which satisfies the relation (1). On the other hand, Sudarshan's original explicit expression for φ is a formal series expansion involving derivatives of Dirac's delta function and as such it is hard to use; it is therefore desirable to give some other explicit expression for φ which will yield a well-behaved function whenever possible. In an earlier publication[4] a relation between the normally ordered and antinormally ordered characteristic functions was established which can be used to evaluate φ (see Refs. 4 and 5). However, this method is again not very simple. In this paper we wish to present a simple explicit expression for $\varphi(v)$.

Let us multiply both sides of Eq. (1) by $\langle-\alpha|$ on the left and $|\alpha\rangle e^{|\alpha|^2}$ on the right, and also use the scalar-product relation[3]

$$\langle\beta|\alpha\rangle=\exp\{-\tfrac{1}{2}|\beta|^2-\tfrac{1}{2}|\alpha|^2+\beta^*\alpha\}. \tag{4}$$

We then obtain

$$\langle-\alpha|\hat{\rho}|\alpha\rangle e^{|\alpha|^2}=\int\varphi(v)e^{-|v|^2}e^{v^*\alpha-v\alpha^*}d^2v. \tag{5}$$

Since $v^*\alpha-v\alpha^*$ is a purely imaginary quantity, Eq. (5) is simply a double-Fourier-transform relation. On applying the Fourier-inversion formula to Eq. (5), we obtain the following expressions for $\varphi(v)$:

$$\varphi(v)=(1/\pi^2)e^{|v|^2}\times\int\langle-\alpha|\hat{\rho}|\alpha\rangle e^{|\alpha|^2}e^{\alpha^*v-\alpha v^*}d^2\alpha. \tag{6}$$

The Fourier inversion leading from Eq. (5) to (6) is justified whenever $\langle-\alpha|\hat{\rho}|\alpha\rangle e^{|\alpha|^2}$ is square integrable. In such cases $\varphi(v)e^{-|v|^2}$ is also square integrable. However, in the general case such a Fourier inversion can be interpreted only in the sense of generalized function theory.

In deriving relation (6), we have not used any specific property of the density operator and hence the method can be employed for finding a diagonal representation of any arbitrary operator. We give below some examples to illustrate the usefulness of the relation (6):

Example 1. Diagonal representation of the operator $\exp(-\lambda\hat{a}^\dagger\hat{a}+\mu\hat{a}^\dagger+\nu\hat{a})$.—We can write

$$\exp(-\lambda\hat{a}^\dagger\hat{a}+\mu\hat{a}^\dagger+\nu\hat{a})=\exp\left\{-\lambda\left(\hat{a}^\dagger-\frac{\nu}{\lambda}\right)\left(\hat{a}-\frac{\mu}{\lambda}\right)+\frac{\mu}{\lambda}\right\}. \tag{7}$$

Making use of the commutation relation between the operators $(\hat{a}-\mu/\lambda)$ and $(\hat{a}^\dagger-\nu/\lambda)$, we can derive the normally ordered expression[7] for the operator on the right-hand side of (7), viz.,

$$\exp\left\{-\lambda\left(\hat{a}^\dagger-\frac{\nu}{\lambda}\right)\left(\hat{a}-\frac{\mu}{\lambda}\right)\right\}=:\exp\left\{-(1-e^{-\lambda})\left(\hat{a}^\dagger-\frac{\nu}{\lambda}\right)\left(\hat{a}-\frac{\mu}{\lambda}\right)\right\}:, \tag{8}$$

where $:\cdots:$ denotes the normal ordering operation. From (8) we find

$$\left\langle-\alpha\left|\exp\left\{-\lambda\left(\hat{a}^\dagger-\frac{\nu}{\lambda}\right)\left(\hat{a}-\frac{\mu}{\lambda}\right)\right\}\right|\alpha\right\rangle=\exp\left\{(1-e^{-\lambda})\left(\alpha^*+\frac{\nu}{\lambda}\right)\left(\alpha-\frac{\mu}{\lambda}\right)-2\left|\alpha\right|^2\right\}. \tag{9}$$

Using Eqs. (7) and (9) in (6), we then obtain the following expression for the diagonal representation $\varphi(v)$ of the operator $\exp\{-\lambda\hat{a}^\dagger\hat{a}+\mu a^\dagger+\nu\hat{a}\}$:

$$\varphi(v)=\frac{1}{\pi}\exp\left\{\lambda+\frac{\mu\nu}{\lambda}-(e^{\lambda}-1)\left(v^*-\frac{\nu}{\lambda}\right)\left(v-\frac{\mu}{\lambda}\right)\right\}. \tag{10}$$

The operator $\lambda\hat{a}^\dagger\hat{a}+\mu\hat{a}^\dagger+\mu^*a$ occurs in many problems of physical interest as an interacting Hamiltonian. The above expression [Eq. (10)] can therefore be used to get the diagonal representation of the density operator for such an interacting system in thermal equilibrium.

Example 2. Diagonal representation of the operator $\hat{a}^{\dagger m}\hat{a}^n$.—If we substitute $\hat{\rho}=\hat{a}^{\dagger m}\hat{a}^n$ in Eq. (6), we obtain

$$\varphi(v)=\frac{1}{\pi^2}e^{|v|^2}\int(-\alpha^*)^m\alpha^n e^{-|\alpha|^2}e^{\alpha^*v-\alpha v^*}d^2\alpha=\frac{1}{\pi}e^{|v|^2}\left(-\frac{\partial}{\partial v}\right)^m\left(-\frac{\partial}{\partial v^*}\right)^n e^{-|v|^2}. \tag{11}$$

One additional remark is in order. It can be shown[9] that if we replace v and v^* in the expression for φ by $\hat{a}$ and $\hat{a}^\dagger$, respectively, and rearrange the factors such that all the powers of $\hat{a}$ occur to the left of all the powers of $\hat{a}^\dagger$, then we obtain the antinormally ordered expression for $(1/\pi)\hat{\rho}$. Written explicitly,

$$\hat{\rho}=\hat{\rho}^{(A)}=\pi\text{“}\varphi(\hat{a},\hat{a}^\dagger)\text{”}, \tag{12}$$

where "$\cdots$" denotes the antinormal-ordering operation. Relation (6) together with (12) can therefore be used to evaluate the antinormally ordered expression for the operator $\hat{\rho}$. An analogous relation holds for normal ordering: The phase-space distribution function for antinormal ordered rule of association[4] $(1/\pi)\langle v|\hat{\rho}|v\rangle$ is related to the normally ordered expression for $\hat{\rho}$ by the relation

$$\hat{\rho}=\hat{\rho}^{(N)}=\pi:\varphi_A(\hat{a},\hat{a}^\dagger):, \tag{13}$$

where $\varphi_A(\hat{a},\hat{a}^\dagger)$ is the function obtained by replacing v and v^* in $(1/\pi)\langle v|\hat{\rho}|v\rangle$ by $\hat{a}$ and $\hat{a}^\dagger$, respectively, and $:\cdots:$ denotes the normal ordering operation.

In the preceding discussion we considered only the one-dimensional harmonic oscillator. However, the problem can obviously be generalized to systems having more than one degree of freedom.

*Research supported by the U. S. Air Force Office of Scientific Research, Office of Aerospace Research.

[1]E. C. G. Sudarshan, Phys. Rev. Letters 10, 277 (1963).

[2]All symbols with a circumflex, e.g., $\hat{\rho}$, $\hat{a}$, etc., denote operators.

[3]J. R. Klauder, Ann. Phys. (N.Y.) 11, 123 (1960); R. J. Glauber, Phys. Rev. 131, 2766 (1963).

[4]C. L. Mehta and E. C. G. Sudarshan, Phys. Rev. 138, B274 (1965).

[5]J. R. Klauder, J. McKenna, and D. G. Currie, J. Math. Phys. 6, 733 (1965).

[6]J. R. Klauder, Phys. Rev. Letters 16, 534 (1966).

[7]The corresponding expression for the case $\mu=\nu=0$ has been derived by many authors [see, for example, W. H. Louisell, Radiation and Noise in Quantum Electronics (McGraw-Hill Book Company, Inc., New York, 1965), p. 116; L. Mandel, Phys. Rev. 144, 1077 (1966)]. A much simpler way of deriving the relation is to use the general relation (13) given later on in this paper. In this case we have

$$\varphi_A(v,v^*)=\frac{1}{\pi}\langle v|e^{-\lambda\hat{a}^\dagger\hat{a}}|v\rangle=\frac{1}{\pi}\sum_n e^{-\lambda n}|\langle v|n\rangle|^2$$

$$=\frac{1}{\pi}\exp\{-(1-e^{-\lambda})v^*v\}.$$

Hence

$$e^{-\lambda\hat{a}^\dagger\hat{a}}=:\exp\{-(1-e^{-\lambda})\hat{a}^\dagger\hat{a}\}:.$$

[8]Louisell, Ref. 7; Mandel, Ref. 7.

[9]See, for example, M. Lax and W. H. Louisell, J. Quant. Electron. QE3, 47 (1967).

RESONANCE DECAYS FROM O(3, 1) DYNAMICS. A REGULARITY IN THE PARTIAL DECAY WIDTHS*

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(Received 13 February 1967)

We have evaluated in closed form the scalar form factors in an irreducible representation of the noncompact group O(3, 1) containing antiparticles. In particular, we have calculated the decay of baryon resonances into the ground state and compared it with experiment.

In this note we report the calculation of the decay rates of baryon resonances of arbitrary spin into another baryon and a meson, and point out a remarkable regularity of the partial decay widths of baryons as a function of spin.

The basis of the calculation is the ordering of the observed baryon resonances into unitary irreducible representations of the dynamical group O(3, 1), a group isomorphic to the homogeneous Lorentz group, extended by parity, and containing antiparticles. The unitary irreducible representations are characterized by two numbers, a lowest spin j_0 that takes integer or half-integer values and a continuous imaginary number $j_1=i\nu$. The state will be labeled by $|J,J_z\rangle$, where $J=j_0,j_0+1,j_0+2,\cdots$. After the extension by parity, the requirement of the existence of a four-vector current operator Γ_μ, which allows the particles to couple to the electromagnetic field, restricts the physically interesting representations of the group to the following types[1]:

Representation	States	Scalar, ps, vector, $\cdots$, vertex
$j_0=0,j_1=\frac{1}{2}$	no doubling of states	S,V
$j_0=\frac{1}{2},j_1=0$	no doubling of states	S,V
$j_0=\frac{1}{2},j_1=i\nu$	doubling	$S,P\ V,A$

The first two representations have been considered in a previous paper in the calculation of form factors and transition probabilities.[2] The doubling of states in the third representation will be associated with antiparticles because Γ_0 for these states has the opposite sign compared with the original states, just as in Dirac theory, and clearly occurs only for fermions. We denote the states of the two parts of the Hilbert spaces of the doubled representation by $|1\rangle$ and $|2\rangle$, and the parity of the low-