the absorption constant for combined resonance is expected to increase with increasing field.[2] The experimental results indicate, however, that between 50 and 100 kG the combined resonance strength is only weakly dependent on magnetic field. Preliminary measurements of the absorption strength as a function of the angle between the applied field and the crystalline axes indicate that anisotropy is weak or absent. A search was made for absorption at the frequencies of the other two combined resonance transitions, but neither was found. From the data, an upper bound on the absorption constants of these transitions has been obtained as follows: $2\nu_C+\nu_S$, 0.054 cm^{-1}; $2\nu_C-\nu_S$, 0.38 cm^{-1}. (The latter transition requires population of the $L=0$, spin-down level.)

Both the observed isotropy of the intensities of the ν_S [12] and $\nu_C+\nu_S$ transitions, and the absence in the spectra of the $2\nu_C+\nu_S$ and $2\nu_C-\nu_S$ transitions, indicate that the nonparabolicity of the bands,[2] rather than the absence of inversion symmetry, is responsible for most of the combined resonance intensity. Combination resonance should therefore be observable in crystals possessing inversion symmetry and having nonparabolic bands, such as PbTe, as well as in other III-V semiconductors.

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[1]E. I. Rashba and V. I. Sheka, Fiz. Tverd. Tela 3, 1735 (1961) [translation: Soviet Phys.—Solid State 3, 1257 (1961)].

[2]V. I. Sheka, Fiz. Tverd. Tela 6, 3099 (1964) [translation: Soviet Phys.—Solid State 6, 2470 (1965)].

[3]E. I. Rashba and V. I. Sheka, Fiz. Tverd. Tela 3, 1863 (1961) [translation: Soviet Phys.—Solid State 3, 1357 (1961)].

[4]R. L. Bell and K. T. Rogers, Phys. Rev. 152, 746 (1966).

[5]C. R. Pidgeon and R. N. Brown, Phys. Rev. 146, 575 (1966).

[6]R. Bowers and Y. Yafet, Phys. Rev. 115, 1165 (1959).

[7]H. Y. Fan and M. Marfaing, private communication.

[8]Perkin Elmer part No. 186-0081 (silver bromide substrate).

[9]R. Kaplan, J. Phys. Soc. Japan Suppl. 21, 249 (1966).

[10]H. Hasegawa and R. E. Howard, J. Phys. Chem. Solids 21, 179 (1961).

[11]C. R. Pidgeon, D. L. Mitchell, and R. N. Brown, Phys. Rev. 154, 737 (1967).

[12]R. L. Bell, Phys. Rev. Letters 9, 52 (1962).

ULTRASONIC ATTENUATION IN NORMAL AND SUPERCONDUCTING LEAD; ELECTRONIC DAMPING OF DISLOCATIONS*

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In a recent publication[1] it was reported that a maximum is found in the ultrasonic attenuation, as a function of wave amplitude, in normal and superconducting lead at 4.2°K and the normal state at 8°K. At a given temperature the maximum occurs at higher amplitudes in the normal state. These experiments were interpreted in terms of the difference in the damping of dislocation motion due to conduction electrons in a metal.[2]

This Letter reports on some further experiments and gives a more detailed analysis of the results. Specifically, the shift in amplitude at maximum ultrasonic attenuation in normal and superconducting lead was used to determine the parameter B associated with damping of dislocations by conducting electrons in the normal state. The analysis does not involve any ad hoc assumptions concerning inaccurately known features of the dislocation network in crystals, and the result is believed to be the first of its kind.

The experimental conditions and techniques used were described in Ref. 1. Figures 1 and 2 show, respectively, the results obtained at frequencies of 10 and 20 Mc/sec. The features relevant to the present consideration are the following: (a) At 10 Mc/sec the amplitude corresponding to maximum attenuation in the superconducting state at 4.2°K is 5 dB lower than

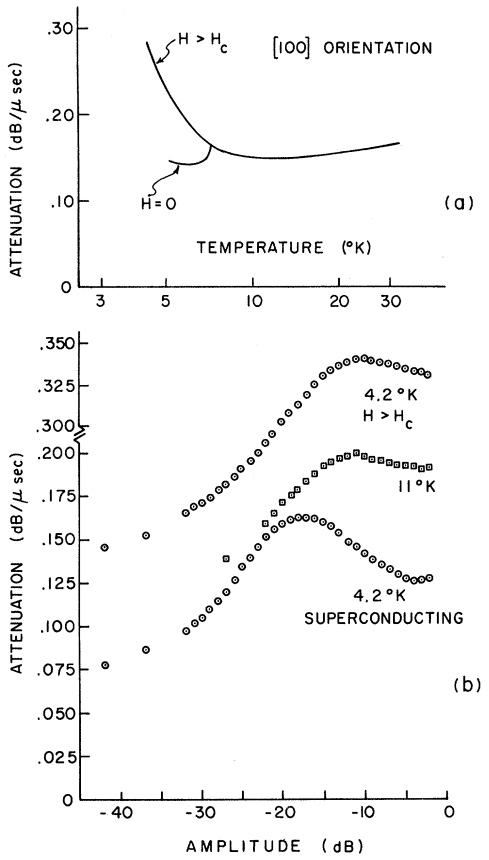

FIG. 1. (a) Ultrasonic attenuation at 10 Mc/sec as a function of temperature. $H=0$ and $H>H_c$ indicate, respectively, the values measured in the superconducting state and in the normal state. (b) Attenuation as a function of the amplitude of the 10-Mc/sec ultrasonic wave measured at 4.2°K superconducting state, 4.2°K normal state, and at 11°K.

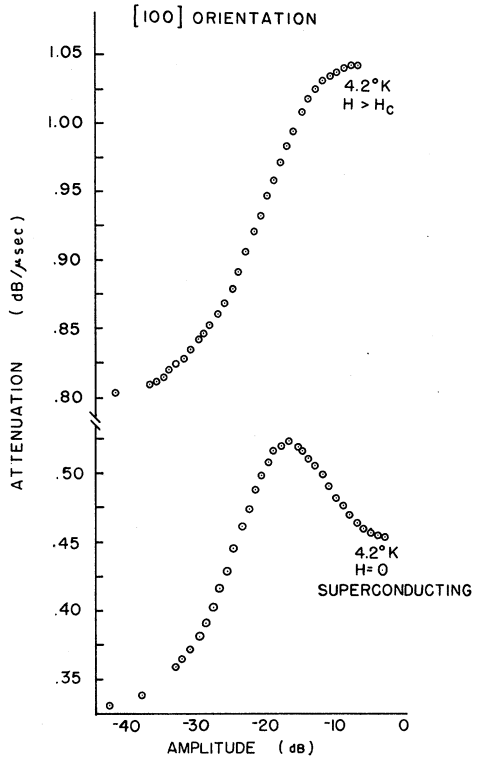

FIG. 2. Attenuation as a function of the amplitude of the 20-Mc/sec ultrasonic wave measured at 4.2°K superconducting state and 4.2°K normal state.

the corresponding amplitude in the normal state at both 4.2°K and at 11°K; (b) in the range $8°K \lesssim T \lesssim 14°K$ the attenuation has the lowest value observed at any temperature in the normal state; and (c) at 20 Mc/sec the amplitudes corresponding to maximum attenuation in the two states at 4.2°K differ by 11 dB.

These results are interpreted as follows: At low temperatures ($T \lesssim 15°K$) the major contribution to dislocation damping comes from interactions with conduction electrons (at higher temperatures phonon damping is important and eventually becomes the dominant contribution). In the normal state these interactions are expected to be temperature independent, because they are essentially independent of the electron mean free path, while the atomic displacements around a dislocation are also insensitive to temperature.[3] This point is substantiated experimentally by the observation that the amplitude for maximum attenuation (at 10 Mc/sec) is shifted by the same amount from the superconducting state at 4.2°K to the normal state at 4.2 and 11°K.

In the superconducting state the interaction is expected to have a temperature dependence given by the BCS[4] ratio of normal to superconducting electrons, in analogy with the ratio of ultrasonic attenuation, i.e.,

$$B_s(T)/B_n = 2f(\Delta(T)),$$

where $B_S(T)$ is the temperature-dependent dislocation damping parameter in the superconducting state, B_n is the temperature-independent damping parameter in the normal state, and $f(\Delta(T))$ is the Fermi function of the superconducting energy gap. As indicated in Ref. 1, the oscillatory stress required to break away a dislocation from a pinning point is given by

$$\Gamma' \cong \frac{\pi f_m}{4bL_c}\left[1+\left\{\frac{B\omega}{A(\omega_0^2-\omega^2)}\right\}^2\right]^{1/2}, \qquad (1)$$

where B is the damping constant, f_m is the maximum binding force between pinning points and a dislocation, $A=\pi\rho b^2$ is the effective mass of a dislocation per unit length, $\omega_0=(\pi/L_c)(C/A)^{1/2}$ is the resonant frequency of a dislocation of loop length L_c, and C is the line tension of a dislocation. At 4.2°K, $B_n/B_s\approx 18$; one can therefore write

$$\frac{\Gamma_n'}{\Gamma_s'}\simeq\left[1+\left\{\frac{B\omega}{A(\omega_0^2-\omega^2)}\right\}^2\right]^{1/2}, \qquad (2)$$

since B_s can be neglected in comparison with B_n, and the other parameters in Eq. (1) are not expected to change on going from the normal to the superconducting state. From the data for the amplitude shift at 10 and 20 Mc/sec, one obtains

$$\left.\frac{\Gamma_n'}{\Gamma_s'}\right|_{10\ \mathrm{Mc/sec}}\simeq 1.78 \ \text{(i.e., ~5 dB)}, \qquad (3)$$

$$\left.\frac{\Gamma_n'}{\Gamma_s'}\right|_{20\ \mathrm{Mc/sec}}\simeq 3.55 \ \text{(i.e., ~11 dB)}. \qquad (4)$$

Relations (3) and (4) can be solved for the two unknowns ω_0 and B_n, thus yielding a value for B_n which is independent of any ad hoc assumptions about the dislocation density, loop length, or any other inaccurately known parameters. One thus obtains $B\simeq 8.6\times10^{-5}$ dyn sec cm^{-2} and $\omega_0\simeq 2\pi\times 47.5$ Mc/sec. This value of B agrees with the result of a calculation by Holstein.[3] An independent experimental study of ω_0 (on the same specimen) yielded a value in good agreement with the one obtained above.

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[1]A. Hikata, R. A. Johnson, and C. Elbaum, Phys. Rev. Letters 17, 526 (1966).
[2]W. P. Mason, Phys. Rev. 143, 229 (1966).
[3]Calculation due to T. Holstein quoted by B. R. Tittman and H. E. Bömmel, Phys. Rev. 151, 178 (1966).
[4]J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).

DIAGONAL COHERENT-STATE REPRESENTATION OF QUANTUM OPERATORS

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We derive a simple expression for the diagonal coherent-state representation of quantum operators, and discuss some of its applications.

It was first observed by Sudarshan[1] that it is possible to express the density operator $\hat{\rho}$ of an arbitrary statistical state of a one-dimensional harmonic oscillator in the "diagonal" form[2]

$$\hat{\rho}=\int\varphi(v)\,|v\rangle\langle v|\,d^2v. \qquad (1)$$

Here $|v\rangle$ is the normalized eigenstate[3] of the annihilation operator $\hat{a}$ with the (complex) eigenvalue v:

$$\hat{a}\,|v\rangle=v\,|v\rangle, \qquad (2)$$

$$|v\rangle=\exp(v\hat{a}^\dagger-v^*\hat{a})\,|0\rangle, \qquad (3)$$

and $\langle v|$ is the Hermitian adjoint of $|v\rangle$. The states $|v\rangle$ are also called the coherent states. The weight factor $\varphi(v)$ is in general not a well-behaved function and can be interpreted only in the sense of generalized function theory (see, for example, Mehta and Sudarshan,[4] Klauder, McKenna, and Currie,[5] and Klauder[6]). However, in most cases of practical interest it is possible to find a well-behaved function $\varphi(v)$ which satisfies the relation (1). On the other hand, Sudarshan's original explicit expression for φ is a formal series expansion involving derivatives of Dirac's delta function and as such it is hard to use; it is therefore desirable to give some other explicit expression for φ which will yield a well-behaved function whenever possible. In an earlier publication[4] a relation between the normally ordered and antinormally ordered characteristic functions was established which can be used to evaluate φ (see Refs. 4 and 5). However, this method is again not very simple. In this paper we wish to present a simple explicit expression for $\varphi(v)$.

Let us multiply both sides of Eq. (1) by $\langle-\alpha|$ on the left and $|\alpha\rangle e^{|\alpha|^2}$ on the right, and also