a consequence of the factor $Y(T)$, to increase faster than T^2 above 0.05°K; this feature is observed experimentally. When the additional attenuation due to phonon-phonon scattering is added to that due to He^3 viscosity, the falloff of the attenuation beyond 0.15°K becomes less pronounced than in Fig. 1.

The discrepancies between the present theory and the experiments are not crucial for two reasons. First, in this calculation, terms of relative order x have been neglected; thus one expects Eq. (9) for a 5% solution to be accurate at most to 5 or 10%. Second, the experiments measured only attenuation differences at each frequency. The absolute normalization of the data, i.e., the zero of attenuation at each frequency, was inferred indirectly and the uncertainties in this procedure are a possible source of discrepancy.

It is a pleasure to thank Professor David Pines, Professor John Wheatley, and Mr. Charles Ebner for several useful discussions concerning this calculation, and to express my gratitude to Dr. Bernard Abraham for an enlightening discussion about the experiments.

*Research supported in part by the National Science Foundation Grant No. NSF GP 4937.

*Alfred P. Sloan Research Fellow.

[1]J. Bardeen, G. Baym, and D. Pines, Phys. Rev. Letters 17, 372 (1966).

[2]B. M. Abraham, Y. Eckstein, J. Ketterson, and J. Vignos, Phys. Rev. Letters 17, 1254 (1966).

[3]This estimate is that given by H. London, G. R. Clarke, and E. Mendoza [Phys. Rev. 128, 1992 (1962)] from the data of D. J. Sandiford and H. A. Fairbank, in Proceedings of the Seventh International Conference on Low-Temperature Physics, Toronto, 1960, edited by G. M. Graham and A. C. Hollis Hallett (University of Toronto Press, Toronto, Canada, 1961), p. 641, evaluated for a thermal phonon.

[4]C. Ebner, private communication.

[5]R. Roach, dissertation, University of Illinois, 1966 (unpublished).

[6]J. Bardeen, G. Baym, and D. Pines, to be published.

[7]G. Baym, Phys. Rev. Letters 17, 952 (1966); ϵ_0 is given by $(\partial E/\partial n_3)_{n_4}$, where E is the ground-state energy per unit volume of the mixture minus the fermion exchange energy.

[8]Note how this result differs from the usual energy shift $\vec{p}\cdot\vec{v}_s$ for a phonon [I. M. Khalatnikov, Introduction to the Theory of Superfluidity (W. A. Benjamin, Inc., New York, 1965)]. Indeed, if the He^3 did not interact with the He^4, then $\delta m = 0$ and the coupling (4) would vanish.

[9]The shift in the first-sound velocity is in accord with that calculated by Khalatnikov, Ref. 8, Eq. (24-73).

[10]S. Eckstein, Phys. Rev. Letters 17, 1257 (1966). Among the shortcomings of this paper are that it fails to account correctly for the coupling (4) of the He^3 to the superfluid velocity arising from Galilean invariance, or for effects of the He^3 on the superfluid acceleration. As a consequence the magnitude of the attenuation that one would derive from that paper differs substantially from (9).

ULTRASONIC ATTENUATION IN A PURE TYPE-II SUPERCONDUCTOR NEAR H_{c2}

Reynold Kagiwada, Moisés Levy, and Isadore Rudnick
Department of Physics, University of California, Los Angeles, California

and

Harriet Kagiwada
The RAND Corporation, Santa Monica, California

and

Kazumi Maki*
Department of Physics, University of California, San Diego, La Jolla, California

(Received 19 December 1966)

One of us[1,2] has recently proposed a new theory of ultrasonic attenuation in the mixed state of a pure type-II superconductor in a high magnetic field. The purpose of this Letter is to present recent experimental results on longitudinal and transverse wave propagation in two pure single crystals of niobium and to compare these results with the theoretical predictions. The crystals were oriented in the [111] direction and in the [100] direction and had resistivity ratios of $\rho_{300}/\rho_{4.2} \approx 150$ and ≈ 300, respectively.

The theory is developed for circumstances where the mean free path of the electron is much larger than the coherence length, $l/\xi_0 \gg 1$, and where the upper critical field $H_{c2} \gg H_{c2} - H_0$ (H_0 is the applied external magnetic field). In the case where the wave vector ql is parallel to the external field, the ratio of the longitudinal wave attenuation in the superconducting state α_s^L to that in the normal conducting state α_n^L is given by

$$\frac{\alpha_s^L}{\alpha_n^L}=1-\frac{\Delta}{2kT}\int_{-\infty}^{\infty}\Phi_1\left(\frac{\alpha}{\mathcal{E}},y\right)\cosh^{-2}\left(\frac{\alpha}{2kT}\right)\frac{d\alpha}{\mathcal{E}}, \tag{1}$$

where

$$\Phi_1(x,y)=\pi^{1/2}I_0^{-1}\int_0^1 dz\,\frac{1-3z^2}{(1-z^2)^{1/2}}\exp\left(-\frac{x^2}{1-z^2}\right)\{[1+(yz)^2]^{-1}(1-y^{-1}\arctan y)^{-1}-3/y^2\}. \tag{2}$$

The attenuation ratio of the transverse wave is

$$\frac{\alpha_s^T}{\alpha_n^T}=g(y)\left\{1-\frac{\Delta}{2kT}\int_{-\infty}^{\infty}\Phi_2\left(\frac{\alpha}{\mathcal{E}},y\right)\cosh^{-2}\left(\frac{\alpha}{2kT}\right)\frac{d\alpha}{\mathcal{E}}\right\}, \tag{3}$$

where

$$\Phi_2(x,y)=\frac{3y^2}{2\pi^{1/2}(1-g)}\int_0^1\frac{dz}{(1-z^2)^{1/2}}\exp\left(-\frac{x^2}{1-z^2}\right)\frac{z^2(1-z^2)}{1+(yz)^2}, \tag{4}$$

and where

$$y=ql,$$

$$g(y)=(3/2y^2)[(y^2+1)y^{-1}\arctan y-1],$$

$$I_0=\frac{y^{-1}\arctan y}{1-y^{-1}\arctan y}-\frac{3}{y^2},$$

$$\mathcal{E}=(V_F/\sqrt{2})[(e/c)\hbar H_{c2}(T)]^{1/2},$$

$$\Delta^2=-[2/N(0)]M[H_{c2}(T)-\tfrac{1}{2}TdH_{c2}(T)/dT],$$

$$-4\pi M=\frac{H_{c2}(T)-H_0}{\beta[2\kappa_2^2(T)-1]},\quad \beta=1.16,$$

where V_F is the Fermi velocity, $N(0)$ is the density of states, M is the magnetization, κ_2 is the second Ginzburg-Landau parameter, and the Pippard function $g(ql)$ is close to unity in the present case. Thus the theory predicts that the quantities $(1-\alpha_s^L/\alpha_n^L)$ and $(g-\alpha_s^T/\alpha_n^T)$ are proportional to order parameter Δ $\{\Delta=[\langle\Delta(r)^2\rangle_{av}]^{1/2}\}$ and hence to $[H_{c2}(T)-H_0]^{1/2}$.

Figures 1(a) and 1(b) show the quantities $(1-\alpha_s^L/\alpha_n^L)$ and $(g-\alpha_s^T/\alpha_n^T)$ obtained from ultrasonic attenuation data plotted as functions of $[H_{c2}(T)-H_0]^{1/2}$ for fixed helium-bath temperatures. Note the linearity of both the transverse and longitudinal attenuation over a fairly large range. From figures such as these, the slopes $(1-\alpha_s^L/\alpha_n^L)/[H_{c2}(T)-H_0]^{1/2}$ and $(g-\alpha_s^T/\alpha_n^T)/[H_{c2}(T)-H_0]^{1/2}$ were obtained. These are plotted against reduced temperature, as shown by the x's in Figs. 2(a) and 2(b).

The solid-line curves shown in Figs. 2(a) and 2(b) were obtained from computations performed on an IBM 7094 computer. The integrals in Eqs. (1)-(4) were evaluated by a numerical quadrature method. We determined the function $H_{c2}(T)$ experimentally by finding the field at which the normal and superconducting ultrasonic attenuation curves intersected and obtaining a least-squares fit to a Gor'kov-type polyno-

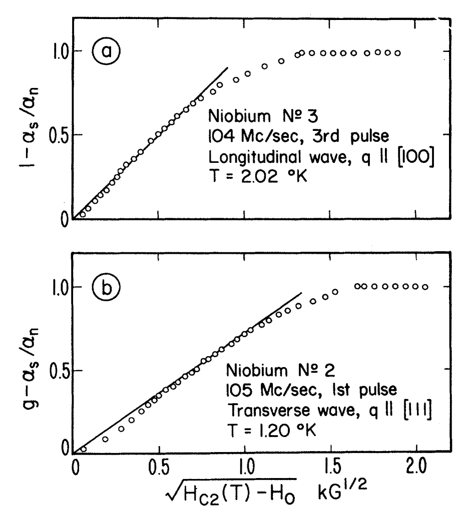

FIG. 1. Ultrasonic attenuation data showing the field dependence for (a) longitudinal wave propagation, and (b) transverse wave propagation.

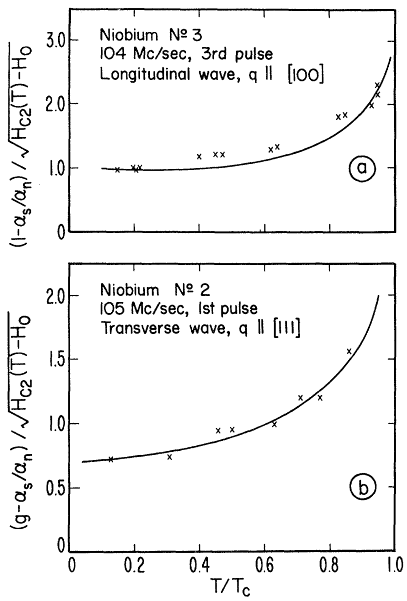

FIG. 2. Theoretical and experimental slopes versus reduced temperature for (a) longitudinal wave propagation, and (b) transverse wave propagation.

mial,[3] which yielded a value of $H_{c2}(0) = 3.906$ kG. This value compares favorably with the values 3.914 and 4.040 kG obtained by McConville and Serin[4] and Finnemore, Stromberg and Swenson,[5] respectively. The magnetization was obtained from the temperature dependence κ_2 which was experimentally determined by McConville and Serin.[4]

When the experimentally determined values of V_F, $N(0)$, and ql were first used in determining the theoretical value of the attenuation ratio, the theoretical values in Fig. 2(a) were found to be half as great as the corresponding experimental data. A computational program designed to find the appropriate values required to get a good fit resulted in the curve shown in Fig. 2(a) when the following values were used: $V_F = 3\times10^7$ cm/sec, $ql = 0.19$, and $N(0) = 1.5\times10^{34}$ states/cm^3 erg. Of these only the last value differs from the best experimentally available values. The value of $N(0)$ which is estimated from the measured energy gap is 6.1×10^{34} states/cm^3 erg and that estimated from specific heat data[4,6,7] is 5.6×10^{34} states/cm^3 erg. The transverse measurements analyzed subsequently are shown in Fig. 2(b), and it is interesting that precisely the same values of V_F and $N(0)$ were used to obtain the curve shown [at this frequency, for this sample $ql = 0.30$, so that $g(ql) \approx 1$].

We wish to thank Professor P. Pincus, Dr. A. Griffin, Dr. E. Guyon, and Dr. T. G. Berlincourt for stimulating conversations, and Mr. J. Fraser for help in some of the measurements. We are grateful to Professor E. H. Gregory for the loan of one of the samples.

---

*On leave of absence from the Research Institute for Mathematical Sciences, Kyoto University, Kyoto, Japan.

[1]K. Maki, in Proceedings of the Tenth International Conference on Low Temperature Physics, 1966 (to be published), Paper No. S15.

[2]K. Maki, Phys. Rev. (to be published).

[3]L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. 37, 835 (1959) [translation: Soviet Phys.—JETP 10, 593 (1960)],

[4]T. McConville and B. Serin, Phys. Rev. 140, A1169 (1965).

[5]D. K. Finnemore, T. F. Stromberg, and C. A. Swenson, Phys. Rev. 149, 231 (1966).

[6]H. A. Leupold and H. A. Boorse, Phys. Rev. 134, A1322 (1964).

[7]B. J. C. van der Hoeven and P. H. Keesom, Phys. Rev. 134, A1320 (1964).