THERMAL CONDUCTIVITY OF PURE He³ AND OF DILUTE SOLUTIONS OF He³ IN He⁴ AT LOW TEMPERATURES*

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The thermal conductivity κ of pure He³ and of two dilute solutions of He³ in He⁴ has been measured from 30 down to 5 mdeg K or below. For pure He³, κT increases with increasing temperature. For the dilute solutions at low enough temperatures, κ is consistent with the T^{-1} temperature dependence of a normal Fermi liquid, but the magnitudes of κT do not agree with values computed from an effective potential based on spin-diffusion coefficient measurements.

We have measured at saturated vapor pressure the thermal conductivity both of pure liquid He³ and of the same two dilute solutions of He³ in He⁴ for which measurements of specific heat, spin-diffusion coefficient, and magnetic susceptibility have already been reported.[1] The results relate to the question of the anomalous behavior of pure He³ at low temperatures[2,3] and to the effective interactions between He³ quasiparticles in the dilute solutions.[4] Measurements reported here extend downward from 30 mdeg K to, in the case of pure He³ and the 5.0% solution, about 3 mdeg K, and to about 5 mdeg K for the 1.3% solution.

The experimental cell shown in Fig. 1 is situated at the lower end of a demagnetization cryostat which has been used for other experiments in this laboratory.[5] The cell was designed to minimize the effect of thermal boundary resistance between solids and helium. The cerium magnesium nitrate (CMN) and helium in the upper part of the cell have a much larger heat capacity than that of their counterparts in the appendix. The magnetic temperatures of the upper and lower CMN are measured separately, though not simultaneously, using separate coil sets and a 17-cps mutual inductance bridge. It has been assumed that the magnetic temperature of the upper CMN is very nearly the same as the Kelvin temperature.[6] The CMN is powdered, with grain size less than 37 μ. When the heater is off, the magnetic temperature of the lower CMN is calibrated against that of the upper CMN. When the heater is on, a dynamic equilibrium is reached from which the temperature difference ΔT between the two salts can be obtained. According-

ing to the theory of this device, the temperature drop across the helium is equal to this ΔT even when the thermal boundary resistance
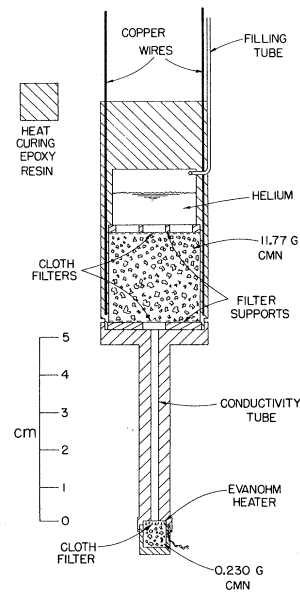

FIG. 1. Thermal conductivity cell. Internal parts are drawn to scale. The upper CMN powder weighs 11.77 g and is packed to 54% of crystalline density. The lower CMN is 0.230 g and is packed to 68% of crystalline density. Both CMN powders are packed in the shape of right circular cylinders with diameter equal to height. The volume of helium contained below the half-height point on the conductivity tube is approximately 0.16 cm³. The effective ratio of length to area of the helium is 147.5 cm⁻¹, of which about 2% is estimated to result from helium not in the conductivity tube. The Evanohm heater is woven into the cotton-cloth filter covering the lower CMN.

between the CMN and the helium is appreciable, provided this resistance is inversely proportional to the mass of the CMN. A study of the thermal transients on applying and removing heater power allowed boundary resistances to be estimated. It is likely that their effect on the results is less than about 10%. Also, the conductivities were found to be independent of the value of $\Delta T/T$, typically 5 to 10%, used to make the measurements. Some error may be introduced by heat shunted through the epoxy surrounding the helium column, though this effect is expected to be not more than 5 to 10% at 30 mdeg K and to be lower at lower temperatures.[7] The heat shunted through the lead-coated manganin leads to the Evanohm heater is negligible.

The method of cell construction and all cryogenic techniques are similar to those previously used in this laboratory.[5,8] In the case of pure He^3, 6.5 cm^3 of liquid was cooled. For the dilute solutions, 8.5 cm^3 of liquid was cooled.

The results of the measurements are shown in Fig. 2. For the case of pure He^3, the thermal conductivity increases somewhat less rapidly with decreasing temperature than the T^{-1} dependence predicted by the Fermi-liquid theory.[9] Between 4 and 20 mdeg K the data fit well the dependence $\kappa = 3600 \times (10 \text{ mdeg K}/T)^{0.92}$ erg/sec cm °K, where κ is the thermal conductivity. The points below 4 mdeg K should not be weighted heavily because of possible thermometry and thermal equilibrium errors. Such a small deviation from the prediction of the Fermi-liquid theory may be difficult to prove experimentally. The results are, however, in qualitative agreement with the theory of Rice.[10] In the same temperature and pressure region the spin-diffusion coefficient[5] varies nearly as T^{-2}, as expected for a normal Fermi liquid,[11] while the ratio C/T, where C is the specific heat, continues to increase weakly with decreasing temperature.[2,3] So far as the magnitude of the thermal conductivity is concerned, the present results fall about 15% below those of Anderson, Salinger, and Wheatley[12] near 30 mdeg K, although at higher temperatures the results of the latter authors are over 10% greater than those obtained by Anderson, Connolly, Vilches, and Wheatley.[13] We believe that a careful study of the properties of pure He^3 at constant pressure over an extended temperature range is in order.
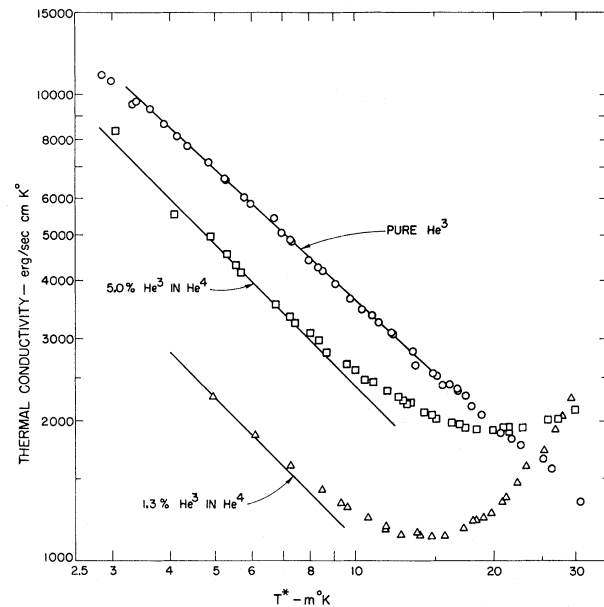
The thermal conductivity for the dilute solu-

FIG. 2 Thermal conductivity of pure He^3 and of two dilute solutions of He^3 in He^4.

tions shows a minimum. This behavior can be explained by assuming a He^3 quasiparticle conductivity proportional to T^{-1} in parallel with a He^4 phonon conductivity. By taking $(\kappa T)_{1.32\%} = 11$ erg/sec cm and $(\kappa T)_{5.02\%} = 24$ erg/sec cm for the quasiparticle conductivities, we find the phonon conductivity to be the same for both solutions, within experimental scatter, up to 20 mdeg K. When analyzed in terms of the relationship $\kappa_{ph} = \frac{1}{3} C_{ph} s \lambda$, where C_{ph} is the phonon specific heat per unit volume and s is the phonon velocity, the phonon conductivity κ_{ph} corresponds to a temperature-dependent phonon mean free path, λ, somewhat larger than the diameter of the conductivity tube at the lowest temperatures. It is difficult to assess the systematic errors in the above values of κT, though errors as large as 10 to 20% are probably not out of the question. Errors in the relative conductivity are probably less than 10%. We also measured an effective thermal conductivity at higher temperatures. However, systematic errors whose relative magnitude increase with increasing temperature, because of both the shunting effect of the epoxy walls and the thermal resistance within the salts, render these data qualitative. Measurements above 30 mdeg K would be useful in studying the He^3-phonon interaction.

The present results for κ together with the

ones for the spin-diffusion coefficient D obtained earlier in this laboratory[1] constitute a stringent test of the assumptions of the theory of Bardeen, Baym, and Pines (BBP) for the effective interaction between He^3 quasiparticles in dilute solutions.[4] For purposes of comparing experiment with theory, it is convenient to express the experimental results in terms of the following dimensionless quantities:

$$J_D = \frac{1}{DT^2}\frac{v_F^2}{3}\frac{\chi_{ideal}}{\chi}\frac{8\pi\hbar^7}{m^{*3}k_B^2}\left(\frac{n_4}{m_4 s_4^2}\right)^2,\qquad(1)$$

and

$$J_K = \frac{1}{\kappa T}\frac{v_F^2}{3}\frac{C_v}{T}\frac{6\pi\hbar^7}{m^{*3}k_B^2}\left(\frac{n_4}{m_4 s_4^2}\right)^2,\qquad(2)$$

where v_F is the Fermi velocity; χ_{ideal} is the magnetic susceptibility of an ideal Fermi gas at $T = 0$ having the appropriate number density and a particle mass equal to m^*, the effective mass of the He^3 quasiparticles; χ is the measured susceptibility; k_B is Boltzmann's constant; m_4 is the mass of the He^4 atom; s_4 is the first sound velocity in pure He^4; n_4 is the number density of atoms in pure He^4; and C_v is the specific heat per unit volume of the solution.

Using the present results and those of Ref. 1, we obtain for the two concentrations studied the experimental values of J_D and J_K presented in Table I. Owing to possible systematic errors which are difficult to assess, we believe that the ratio J_D/J_K for a given concentration may be in error by as much as 20 to 30%. However, the ratios $J_D(5.0\%)/J_D(1.3\%)$ and $J_K(5.0\%)/J_K(1.3\%)$ are probably correct to 10%. We have the interesting result that J_K increases while J_D decreases as the He^3

concentration changes from 1.3 to 5.0%, a behavior which is indeed predicted by the BBP theory.

The quantities J_D and J_K have the significance of normalized averages of quasiparticle-quasiparticle scattering probabilities. For spin diffusion one has approximately[11]

$$J_D = \frac{\hbar}{2\pi}\left(\frac{n_4}{m_4 s_4^2}\right)^2\left\langle W_D(\theta,\varphi)\frac{\sin^2(\tfrac{1}{2}\theta)(1-\cos\varphi)}{\cos\tfrac{1}{2}\theta}\right\rangle_{av},\qquad(3)$$

and for the thermal conductivity one has approximately[9]

$$J_K = \frac{\hbar}{2\pi}\left(\frac{n_4}{m_4 s_4^2}\right)^2\left\langle W_K(\theta,\varphi)\frac{\sin^2(\tfrac{1}{2}\theta)}{\cos\tfrac{1}{2}\theta}\right\rangle_{av}.\qquad(4)$$

In these equations $W_D(\theta,\varphi)$ and $W_K(\theta,\varphi)$ are, for spin diffusion and thermal conductivity, respectively, the appropriate probabilities for the mutual scattering of two He^3 quasiparticles whose momenta make an angle θ with one another, and whose initial and final planes of scattering make an angle φ with one another.

In the work of BBP it is assumed that the effective interaction between quasiparticles depends only on their separation. The scattering probabilities are computed assuming that the Born approximation is sufficiently accurate, and they are expressed in terms of the momentum transform V_k of the quasiparticle interaction. The average strength of the interaction is given by V_0, which can conveniently be expressed in terms of a dimensionless parameter f defined by the equation $V_0 = -f(m_4 s_4^2/n_4)$. Baym[14] has shown that $f \cong \alpha^2$, where α is the quantity which appears in the equation for the mean molar volume of the solution $v(x) = v_{04}(1 + \alpha x)$. Here v_{04} is the molar volume of pure He^4 and x is the He^3 concentration. According to the measurements of Kerr[15] at higher tem-

Table I. Experimental values for the quantities J_D and J_K defined by Eqs. (1) and (2) for the two dilute solutions. In addition to the experimental values of J_D and J_K, the same quantities derived from the original BBP cosine potential,[a] which was based on the experimental spin-diffusion data, are given under "Cosine fit," and the same quantities derived from the power law given in the text are given under "Power-law fit."

	He^3 concentration (%)	Experimental	Cosine fit	Power-law fit
$10^2 J_D$	1.32	0.26	0.25	0.26
$10^2 J_D$	5.02	0.100	0.097	0.135
$10^2 J_K$	1.32	0.38	0.24	0.40
$10^2 J_K$	5.02	0.59	0.27	0.45

[a]Ref. 4.

peratures, α equals 0.28, yielding a value of 0.078 for f.

The form $V_k = V_0 \cos(\beta k/\hbar)$, with $f = 0.075$ and $\beta = 3.16 \times 10^{-8}$ cm, originally chosen by BBP primarily to fit the spin-diffusion coefficient results, gives rather poor quantitative agreement with the present thermal conductivity results, as may be seen by referring to Table I. We have tried a number of different forms of V_k in an attempt to obtain a better fit to the experimental results for J_D and J_K within the framework of the assumptions of the BBP theory, but so far we have been unable to obtain good quantitative agreement. Typical of our most successful attempts are the results presented in Table I, obtained by using a power law for V_k given by $V_0[1 - 0.544(\beta k/\hbar)^2 + 0.0562 \times (\beta k/\hbar)^4]$ with $f = 0.10$ and $\beta = 3.78 \times 10^{-8}$ cm. This potential, similar to the cosine, is also 0 for $\beta k/\hbar = \frac{1}{2}\pi$, but it has a maximum for $\beta k/\hbar = 2.2$.

In view of the quantitative disagreement found here, we believe that there is a clear need for further theoretical and experimental work on the dilute solutions. On the side of theory, the assumptions made by BBP should be re-examined carefully. On the side of experiment, we feel that the present transport coefficient measurements are inadequate to determine the effective interactions, and that a series of measurements at a variety of concentrations, down to as low a concentration as possible, of particularly the spin-diffusion coefficient and the ultrasonic attenuation[16] would be useful.

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PHASE TRANSFORMATIONS ON SOLID SURFACES

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Phase transformations with varying temperature are common in solids. The possibility of phase transformations on crystal surfaces appears not to have been given theoretical consideration although the binding energy of misfitting monolayers has been examined,[1] and it is known that minimum potential configuration of the (100) surface of alkali halides is not flat and that the anions and cations do not lie in the same plane.[2] It has been thought that a thermodynamically stable crystal surface must always exhibit the structure of the bulk solid. The purpose of this Letter is to show that phase transformations can occur on sur-