the sum rule (28). Neglecting the term m_π^2/m_f^2, we get from Eq. (31)

$$\frac{g_{\sigma\pi\pi}^{\;\;2} m_\rho^{\;2}}{(m_\sigma^{\;2}-m_\pi^{\;2})^2}+\tfrac{2}{3}m_\rho^{\;2}g_{f\pi\pi}^{\;\;2}-\gamma_{\rho\pi\pi}^{\;\;2}=0. \qquad (32)$$

Now $\gamma_{\rho\pi\pi}/4\pi=2.5$ and $g_{f\pi\pi}^2/4\pi\approx 0.02/m_\pi^2$ (calculated from $\Gamma_f=0.72m_\pi$ and $m_f^2=80.0m_\pi^2$) and we see that the contribution of σ is essential to satisfy the sum rule (32), although the experimental evidence for σ is doubtful.

It may be seen that we have derived the sum rule (16) under the assumptions of conservation of electromagnetic current, CVC, unsubtracted dispersion relations, and the commutation relation (9). In our approach and that of Harari and Pagels, all other assumptions are common, except CVC and the last assumption. CVC is well established experimentally, therefore as far as the derivation of sum rule (16) is concerned, the current algebra, the quark model, and Regge-pole theory lead to the same result.

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SUPERCONVERGENT SUM RULES FOR PHOTOPRODUCTION*

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Assuming that the Regge trajectories $\alpha_{10}(0)$, $\alpha_{10^*}(0)$, and $\alpha_{27}(0)$ are less than 0, we derive sum rules for photoproduction. Comparison with the experiment is given.

De Alfaro et al.[1] have recently derived a class of "superconvergent" sum rules for strong-interaction scattering amplitudes on the basis of analyticity and reasonable arguments about the high-energy behavior. The subject has attracted considerable attention ever since.[2] For the purpose of verifying the relations and the assumptions involved, a number of authors[3] have considered the case of meson-baryon scattering and analyzed the one superconvergent sum rule assuming the high-energy behavior given by the Regge-pole model. It seems to us of importance to verify these assumptions in other processes as well. We have analyzed the superconvergence relations for the process of photoproduction of mesons from baryons. In this case, we obtain more than one nontrivial sum rule so that the question of mutual consistency can also be examined. In deriving these sum rules the basic assumption made is that the Regge trajectories $\alpha(t)$ have $\alpha_{27}(0)$, $\alpha_{10}(0)$, and $\alpha_{10^*}(0)$ less than 0. If the sum rules are valid, we may regard this as strong evidence for the correctness of our assumption.

Let k, q, p_1, and p_2 be the four-momenta of the photon, the meson, the initial baryon, and the final baryon, respectively. We decompose the T matrix in terms of the four invariant amplitudes, A, B, C, and D.[4] They are functions of the invariants

$$\nu=-\frac{(p_1+p_2)\cdot k}{2M}=-\frac{(p_1+p_2)\cdot q}{2M}$$

and $t=-(p-p')^2$, where M is the baryon mass. In the Regge-pole model, the invariant amplitudes $A,\cdots,D$ all behave[5] like $\nu^{\alpha(t)-1}$ as $\nu\to\infty$, where $\alpha(t)$ refers to the dominant Regge trajectory in the t channel, $\gamma+\bar{\pi}\to N+\bar{N}$. Since, there is no experimental evidence for the existence of any low-lying mesons with $I=\frac{3}{2}$ or 2, we may assume that[3]

$$\alpha_{10}(0),\ \alpha_{10^*}(0),\ \alpha_{27}(0)<0.$$

We are thus led to consider the following five nontrivial (i.e., those which are not trivially satisfied due to the crossing properties of A,

$\cdots, D)$ sum rules.

<u>10</u>:

$$\int_0^\infty d\nu[-(9/40)\,\mathrm{Im}C^{(27)}(\nu,t)+\tfrac{1}{4}\,\mathrm{Im}C^{(10)}(\nu,t)+\tfrac{1}{4}\,\mathrm{Im}C^{(10*)}(\nu,t)-\tfrac{2}{5}\mathrm{Im}C^{(8ss)}(\nu,t)$$
$$-(1/\sqrt{5})\,\mathrm{Im}C^{(8sa)}(\nu,t)+(1/\sqrt{5})\,\mathrm{Im}C^{(8as)}(\nu,t)+\tfrac{1}{8}\mathrm{Im}C^{(1)}(\nu,t)]=0; \quad (1)$$

<u>10</u>*:

$$\int_0^\infty d\nu[-(9/40)\,\mathrm{Im}C^{(27)}(\nu,t)+\tfrac{1}{4}\,\mathrm{Im}C^{(10)}(\nu,t)+\tfrac{1}{4}\,\mathrm{Im}C^{(10*)}(\nu,t)-\tfrac{2}{5}\,\mathrm{Im}C^{(8ss)}(\nu,t)$$
$$+(1/\sqrt{5})\,\mathrm{Im}C^{(8sa)}(\nu,t)-(1/\sqrt{5})\,\mathrm{Im}C^{(8as)}(\nu,t)+\tfrac{1}{8}\,\mathrm{Im}C^{(1)}(\nu,t)]=0; \quad (2)$$

<u>27</u>:

$$\int_0^\infty d\nu[(7/40)\,\mathrm{Im}A^{(27)}(\nu,t)-\tfrac{1}{12}\,\mathrm{Im}A^{(10)}(\nu,t)-\tfrac{1}{12}\,\mathrm{Im}A^{(10*)}(\nu,t)$$
$$+\tfrac{1}{5}\,\mathrm{Im}A^{(8ss)}(\nu,t)-\tfrac{1}{3}\,\mathrm{Im}A^{(8aa)}(\nu,t)+\tfrac{1}{8}\,\mathrm{Im}A^{(1)}(\nu,t)]=0. \quad (3)$$

The amplitudes $A^{(R)}$, etc., are the SU(3) eigenamplitudes in the S channel $\gamma+N\rightarrow\pi+N$. The coefficients of $A^{(R)}$, etc., are elements of the relevant SU(3) crossing matrix.[6]

The sum rules for B and D can be obtained by substituting B and D for A in (3).

Equivalent to the relations (1) and (2) are the following sum rules:

<u>10</u> + <u>10</u>*:

$$\int_0^\infty d\nu[-(9/20)\,\mathrm{Im}C^{(27)}(\nu,t)+\tfrac{1}{2}\,\mathrm{Im}C^{(10)}(\nu,t)+\tfrac{1}{2}\,\mathrm{Im}C^{(10*)}(\nu,t)-\tfrac{4}{5}\,\mathrm{Im}C^{(8ss)}(\nu,t)+\tfrac{1}{4}\,\mathrm{Im}C^{(1)}(\nu,t)]=0; \quad (4)$$

<u>10</u> − <u>10</u>*:

$$\int_0^\infty d\nu[-(2/\sqrt{5})\,\mathrm{Im}C^{(8sa)}(\nu,t)+(2/\sqrt{5})\,\mathrm{Im}C^{(8as)}(\nu,t)]=0. \quad (5)$$

We approximate the integrals by keeping $N(938)$, $N_{33}^*(1236)$, and $N^{**}(1518)$ only. The contributions due to higher intermediate states are expected to contribute much less to photoproduction than to scattering.[7,8]

The matrix elements involving N_{33}^* are defined as follows:

$$\langle N_\gamma^*(p')|J_\pi^{(\alpha)}(0)|N_\beta(p)\rangle=\left(\frac{MM^*}{p_0p_0'}\right)^{1/2}\left(-i\sqrt{3}\,\frac{\lambda}{m_\pi}\right)\begin{pmatrix}8&8&10\\ \alpha&\beta&\gamma\end{pmatrix}(p'-p)_\lambda\bar{u}_\lambda(p')u(p) \quad (6)$$

and

$$\epsilon_\lambda\langle N_\gamma^*(p')|V_\lambda^{(\alpha)}(0)|N_\beta(p)\rangle=\left(\frac{MM^*}{p_0p_0'}\right)^{1/2}\begin{pmatrix}8&8&10\\ \alpha&\beta&\gamma\end{pmatrix}i\sqrt{3}\,\frac{eC}{m_\pi}\bar{u}_\mu(p')\gamma_\nu\gamma_5u(p)F_{\mu\nu}(p'-p), \quad (7a)$$

where $F_{\mu\nu}(k)=k_\mu\epsilon_\nu-k_\nu\epsilon_\mu$ while J_π and V_λ are the pionic and electromagnetic currents, respectively. Our definitions of the coupling constants λ and C are the same as those in Gourdin and Salin.[7,8] We have $\lambda=1.81$ and $C=0.345$. [Of course, there exists another independent gauge-invariant $N^*N\gamma$ coupling, namely

$$2\sqrt{3}(eC_4/m_\pi^2)p_\mu'\bar{u}_\nu(p')\gamma_5u(p)F_{\mu\nu}(p'-p)\cdots. \quad (7b)$$

Experimentally,[7] the coupling constant $C_4=-0.0043$. Thus its contribution to the photoproduction amplitude is negligible.[9]] To write down the corresponding matrix elements of N^{**}, we assume that it is a member of an SU(3) octet. We have

$$\langle N_\gamma^{**}(p')|J_\pi^{(\alpha)}(0)|N_\beta(p)\rangle=-\left(\frac{MM^{**}}{p_0p_0'}\right)^{1/2}\frac{2\lambda'}{m_\pi}i\left[-\sqrt{3}\begin{pmatrix}8&8&8^a\\ \alpha&\beta&\gamma\end{pmatrix}f'+(\tfrac{5}{3})^{1/2}\begin{pmatrix}8&8&8^s\\ \alpha&\beta&\gamma\end{pmatrix}(1-f')\right]\bar{u}_\lambda(p')\gamma_5u(p)(p'-p)_\lambda, (8)$$

and

$$\epsilon_\lambda \langle N_\gamma^{**}(p')|V_\lambda^{(\alpha)}(0)|N_\beta(p)\rangle = \left(\frac{MM^{**}}{p_0'p_0}\right)^{1/2}(-\sqrt{3})\begin{pmatrix} 8 & 8 & 8_a \\ \alpha & \beta & \gamma \end{pmatrix} \frac{eD}{m_\pi^2}\bar{u}_\mu(p')p_\nu' u(p)F_{\mu\nu}(p'-p). \tag{9}$$

As suggested by Fubini from the analysis of photoproduction sum rules,[8] we have kept only the F coupling of N^{**} with the electromagnetic current. Again the coupling constants are given by Gourdin and Salin[7,8] as $\lambda' = 1.97$ and $D = 0.0177$. The contribution of the other independent gauge-invariant $N^{**}N_\gamma$ coupling to the photoproduction amplitude is negligible.[7]

If we substitute in (1)-(5) contributions from N and N^* only, we arrive at the following relations: From C with $\underline{10}-\underline{10}^*$ exchange,

$$\frac{1-f}{f} = \frac{F_2^{(d)}(0)}{F_2^{(f)}(0)}. \tag{10}$$

From C with $\underline{10}+\underline{10}^*$ exchange,

$$eg(1-f)F_2^{d}(0) = eC\lambda\frac{3}{16}\left(\frac{M}{M^*}\right)\left[\left(\frac{M}{m_\pi}\right)^2\left(\frac{M}{M^*}-1\right)+\left(\frac{M^*}{m_\pi}\right)^2\left(3+\frac{M}{M^*}\right)-\frac{M}{M^*}\right]. \tag{11}$$

From $\underline{27}$ exchange,

$$A:\quad egf = eC\lambda\frac{1}{48}\left\{-\left[2\frac{M}{M^*}+\left(\frac{M}{M^*}\right)^2\right]+\left(\frac{M}{m_\pi}\right)\left(\frac{M-M^*}{m_\pi}\right)\left(1+\frac{M}{M^*}\right)^2\right\}, \tag{12}$$

$$B:\quad egf = \frac{1}{16}eC\lambda, \tag{13}$$

$$D:\quad -\frac{1-f}{3}geF_2^{(d)}(0)+gefF_2^{(f)}(0)$$
$$= \frac{1}{24}eC\lambda\left(\frac{M}{M^*}\right)\left[\left(\frac{M}{m_\pi}\right)^2\left(1-\frac{M}{M^*}\right)+\left(\frac{M^*}{m_\pi}\right)^2\left(3+5\frac{M}{M^*}\right)+\frac{M}{M^*}\right], \tag{14}$$

where $F_2^{(f)}(0) = \mu_p' + \frac{1}{2}\mu_n'$, $F_2^{(d)}(0) = -\frac{3}{2}\mu_n'$, and $g^2/4\pi = 14.5$. $(1-f)/f$ is the D/F ratio for the $\pi N\bar{N}$ vertex. μ_p' and μ_n' are the anomalous magnetic moments of the proton and neutron, respectively, with the values $\mu_p' = 1.793$ and $\mu_n' = -1.913$.

We now turn to the numerical analysis of these relations. To start with, we look at the relations obtained from $\underline{10}$ and $\underline{10}^*$ exchange. The relation (10) is interesting in that it expresses the universality between the strong and electromagnetic D/F ratio. We note also that it is independent of the N_{33}^* contribution. Of course, the relation is modified if we include contributions from another octet containing, for instance, the $N^{**}(1518)$. Using the values $F_2^{(f)}(0) = 0.837$ and $F_2^{(d)}(0) = 2.87$, we find from (10) that $f = 0.226$. The inclusion of N^{**} contribution raises this number as we shall see later.

The relation (11) connects the couplings of N and N^*. Numerically we find that $f = 0.35$. This number is fairly close to $f = 0.40$ corresponding to the D/F ratio of $\frac{3}{2}$ given by SU(6).

The numerical analysis of (12), (13), and (14) corresponding to $\underline{27}$ exchange gives rather inconsistent results. In particular, we find that[9] with $\underline{27}$ exchange,

A gives $f = -0.044$,

B gives $f = +0.003$,

D gives $f = +0.974$. (15)

We shall now briefly discuss the effect of N^{**} contribution on these results. The relation (11) remains unchanged while (10) becomes

$$efF_2^{(d)}(0) = e(1-f)F_2^{(f)}(0) - \frac{e\lambda' D(1-f')}{gm_\pi^3}\frac{1}{6}\left(\frac{M}{M^{**}}\right)\left[(M^2-M^{**2})(5M^{**}+M)+m_\pi^2(2M^{**}-M)\right]. \tag{16}$$

If we assume that the D/F ratio for $\pi N^{**}\bar{N}^{**}$ is the same as for the $\pi N\bar{N}$ vertex (i.e., $f=f'$), we obtain $f=0.36$, which is remarkably close to the value obtained from (11). The two sum rules due to $\underline{10}+\underline{10}^*$ and $\underline{10}-\underline{10}^*$ exchange, therefore, seem to be consistent. With the assumption $f=f'$, the modification introduced by the N^{**} contribution in the values of f from the sum rules for A, B, and D is less than 7, 1, and 4%, respectively.

For a sum rule to be useful, we should be able to approximate it by a small number of low-lying states. Experience with photoproduction shows that the contributions from N and N^* are the most dominant ones.[7-9] Thus, even if the higher intermediate states give a fair amount of contribution in the πN scattering case, they may yet be negligible in the case of photoproduction since higher intermediate states excite the high-l multipoles whose contributions are expected to be damped. We have explicitly seen that the inclusion of the N^{**} contribution does not appreciably change the results in (15). In view of this situation, we feel that if we accept the assumption $\alpha_{27}(0)<0$ to be correct as is suggested by the analysis of the meson-baryon case,[3] then the $\underline{27}$ exchange in the t channel is somehow forbidden in the photoproduction case. There is, of course, the possibility that the asymptotic behavior of the amplitudes A, B, and D is not as simple as the one given by Regge-pole model. This is an open question. We have already noted that as far as the $\underline{10}$ and $\underline{10}^*$ exchange is concerned, the two sum rules seem to be consistent and that the value of the parameter f lies within the generally accepted range. This may indicate correctness of the assumptions $\alpha_{10}(0)<0$ and $\alpha_{10^*}(0)<0$ and usefulness of the superconvergent sum rules.

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