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CURRENT ALGEBRA AND RADIATIVE DECAYS OF MESONS*

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(Received 20 March 1967)

Some time back Fubini[1] derived a new method of obtaining sum rules of interest in strong-interaction physics. In this note we apply Fubini's method to electromagnetic and weak processes. In particular we obtain sum rules for radiative decays of mesons. Making contact with strong interaction via Goldberger-Treiman-type relations and ρ dominance, we also get sum rules for strong decays of mesons.

(i) We begin by considering the matrix elements

$$M_{\nu\mu} = i\int d^4x e^{-ik\cdot x}\theta(x_0)\langle 0|[j_\nu^{\mathrm{el}}(x), j_{\mu 5}^{-}(0)]|\pi^+, p\rangle$$

which are related to the process $\pi^+ \to l^+ + \nu + \gamma$. Here we have two independent four-momenta p and k and one invariant $\nu = -p\cdot k$. Using the commutation relation

$$\delta(x_0)[j_0^{\mathrm{el}}(x), j_{\mu 5}^{-}(0)] = -\delta^4(x) j_{\mu 5}^{-}(x)$$

and the current conservation, we get

$$ik_\nu M_{\nu\mu} = -i\langle 0|j_{\mu 5}^{-}|\pi^+\rangle = -f_\pi p_\mu. \tag{1}$$

The most general form of the matrix elements $M_{\nu\mu}$ is given by

$$M_{\nu\mu} = i[H_1(\nu)p_\nu p_\mu + H_2(\nu)p_\nu k_\mu + H_3(\nu)k_\nu P_\mu + H_4(\nu)k_\nu k_\mu + H_5(\nu)\delta_{\nu\mu}].$$

We separate the Born term and write Eq. (1) as

$$ik_\nu M_{\nu\mu}(\mathrm{Born}) + ik_\nu \tilde{M}_{\nu\mu} = -f_\pi p_\mu, \tag{2}$$

where

$$M_{\nu\mu}(\mathrm{Born}) = -if_\pi(2p-k)_\nu(p-k)_\mu/2\nu.$$

Hence from Eq. (2), we obtain

$$-f_\pi + \nu\tilde{H}_1(\nu) = -f_\pi,$$
$$f_\pi - \tilde{H}_5(\nu) + \nu\tilde{H}_2(\nu) = 0, \tag{3}$$

First we note that at $\nu = 0$, we get $\tilde{H}_5(0) = f_\pi$. From Eq. (3), we get $\tilde{H}_1(\nu) = 0$ at $\nu \neq 0$ and the sum rule

$$\frac{1}{2\pi i}\int \tilde{h}_2(\nu')d\nu' = f_\pi, \tag{4}$$

where $\tilde{h}_2(\nu)$ is the coefficient of $p_\nu k_\mu$ to be picked out from

$$\tilde{m}_{\nu\mu}=i(2\pi)^4\sum_s[\delta^4(k-s)\langle 0|j_\nu^{\text{el}}|s\rangle\langle s|j_{\mu 5}^-|\pi^+\rangle$$
$$-\delta^4(p-k-s)\langle 0|j_{\mu 5}^-|s\rangle\langle s|j_\nu^{\text{el}}|\pi^+\rangle]. \quad (5)$$

The sum rule (4) can also be obtained as follows: From Eq. (1), we get

$$H_5(\nu)=\nu H_2(\nu). \quad (6)$$

If we write an unsubtracted dispersion relation for $H_2(\nu)$,

$$H_2(\nu)=\frac{f_\pi}{\nu}+\frac{1}{\pi}\int\frac{\operatorname{Im}H_2(\nu')}{\nu'-\nu},$$

and assume that $H_1(\nu)\to 0$ as $\nu\to\infty$, we immediately get the sum rule (4). In view of Eq. (6), the sum rule (4) can also be written as

$$\frac{1}{2\pi i}\int\frac{1}{\nu'}h_5(\nu')d\nu'=f_\pi. \quad (7)$$

From Eq. (6), we see that only axial-vector meson of negative G parity (A_1) can contribute to sum rule (4) or (7). Hence we get

$$\frac{g_{A_1}f_{A_1\pi\gamma}}{m_{A_1}^2-m_\pi^2}=f_\pi, \quad (8)$$

where the coupling constants g_{A_1} and $f_{A_1\pi\gamma}$ are defined by

$$\langle 0|j_{\mu 5}^-|A_1^+,p\rangle=g_{A_1}\eta_\mu,$$

$$\langle A_1^+,p|j_\nu^{\text{el}}|\pi^+,q\rangle=if_{A_1\pi\gamma}\left[\delta_{\nu\rho}+\frac{q_\rho(p+q)_\nu}{p^2-q^2}\right]\eta_\rho.$$

We also note that the contribution of A_1 to $\tilde{h}_1(\nu)$ is indeed 0.

(ii) We now consider the process $\pi^+\to\pi^0+l^++\nu+\gamma$. The matrix elements related to this process are given by

$$M_{\nu\mu}=i\int d^4xe^{-ik\cdot x}\theta(x_0)$$
$$\times\langle\pi^0,p'|[j_\nu^{\text{el}}(x),j_\mu^-(0)]|\pi^+,p\rangle.$$

Again using current conservation and the commutation relation

$$\delta(x_0)[j_0^{\text{el}}(x),j_\mu^-(0)]=-\delta^4(x)j_\mu^-(x), \quad (9)$$

we get

$$ik_\nu\cdot M_{\nu\mu}=-i\langle\pi^0|j_\mu^-|\pi^+\rangle=(i/\sqrt{2})F(t)(p+p')_\mu. \quad (10)$$

Here the standard notation

$$P=\tfrac{1}{2}(p+p'),\quad q=p'-p+k,$$
$$\nu=-P\cdot k,\quad \nu_B=\tfrac{1}{2}k\cdot q,\quad t=-(p-p')^2$$

is used. Since we are dealing with real photons, $k^2=0$ and $t=\nu_B-q^2$. The matrix elements $M_{\nu\mu}$ can be written in the most general form as

$$M_{\nu\mu}=[A_1P_\nu P_\mu+A_2P_\nu k_\mu+A_3P_\nu q_\mu+A_4k_\nu P_\mu$$
$$+A_5q_\nu P_\mu+A_6k_\nu k_\mu+A_7k_\nu q_\mu$$
$$+A_8q_\nu k_\mu+A_9q_\nu q_\mu+A_{10}\delta_{\nu\mu}].$$

We take $\nu_B=0$ so that $t=-q^2$ and A_i are functions of ν and t only.

Separating the Born term, we write Eq. (10) as

$$ik_\nu M_{\nu\mu}(\text{Born})+ik_\nu\tilde{M}_{\nu\mu}=(2i/\sqrt{2})F(t)P_\mu, \quad (11)$$

where

$$M_{\nu\mu}(\text{Born})=-\sqrt{2}[2F(t)P_\mu-F(t)k_\mu][(2P-q)_\nu]/2\nu.$$

Therefore, from Eq. (11), we obtain

$$2\sqrt{2}F(t)-\nu\tilde{A}_1(\nu,t)=2\sqrt{2}F(t)$$
$$-\sqrt{2}F(t)-\nu\tilde{A}_2(\nu,t)+\tilde{A}_{10}(\nu,t)=0. \quad (12)$$

We note that at $\nu=0$, $\tilde{A}_{10}(0,t)=\sqrt{2}F(t)$. From Eq. (12), we get $\tilde{A}_1(\nu,t)=0$ at $\nu\neq 0$ and the sum rule[2]

$$\frac{1}{2\pi i}\int\tilde{a}_2(\nu',t)d\nu'=\sqrt{2}F(t), \quad (13)$$

where $\tilde{a}_2(\nu,t)$ is the coefficient of $P_\nu k_\mu$ to be picked out from

$$\tilde{m}_{\nu\mu}(\nu,t)$$
$$=i(2\pi)^4\sum_s[\delta^4(p'+k-s)\langle\pi^0|j_\nu^{\text{el}}|s\rangle\langle s|j_\mu^-|\pi^+\rangle$$
$$-\delta^4(p-k-s)\langle\pi^0|j_\mu^-|s\rangle\langle s|j_\nu^{\text{el}}|\pi^+\rangle]. \quad (14)$$

The sum rule (13) can also be obtained as follows: From Eq. (10), we get at $\nu_B=0$

$$A_{10}(\nu,t)=\nu A_2(\nu,t). \quad (15)$$

If we write an unsubtracted dispersion relation for $A_2(\nu,t)$ for fixed t,

$$A_2(\nu,t)=\frac{\sqrt{2}F(t)}{\nu}+\frac{1}{\pi}\int\frac{\operatorname{Im}A_2(\nu',t)}{\nu'-\nu}d\nu',$$

and assume that $A_{10}(\nu, t) \to 0$ as $\nu \to \infty$, we get the sum rule (13). On the other hand the sum rule (13) implies that $A_{10}(\nu, t) \to 0$ as $\nu \to \infty$. In view of the relation (15), we can write Eq. (13) as

$$\frac{1}{2\pi i}\int \frac{a_{10}(\nu', t)}{\nu'} d\nu' = \sqrt{2}\, F(t)$$

which at $t = 0$ gives

$$\frac{1}{2\pi i}\int \frac{a_{10}(\nu')}{\nu'} d\nu' = \sqrt{2}. \tag{16}$$

This sum rule is closely related to the sum rule very recently derived by Pagels[3] and Harari.[4] This can be seen as follows: We are interested in evaluating the matrix elements $\langle s | j_\mu^- | \pi^+\rangle$ to order e only; therefore, we can use conserved vector current (CVC) for j_μ^- and regard s and π^+ as eigenstates of I, so that we get

$$\langle s | j_\mu^- | \pi^+\rangle = \langle s | [I^-, j_\mu^{\text{el}}] | \pi^+\rangle = \sqrt{2}\langle s | j_\mu^{\text{el}} | \pi^0\rangle \tag{17}$$

as $\langle s | I^-$ must be 0. We can write the optical theorem:

$$e^2\langle \epsilon_\nu m_{\nu\mu}(\nu, 0)\epsilon_\mu\rangle = \sqrt{2}\, 2ie^2\langle \epsilon_\nu \operatorname{Im} M_{\nu\mu}(\nu, 0)\epsilon_\mu\rangle$$
$$= \sqrt{2}\, 2i 2\nu\sigma_{\text{tot}}(\nu),$$

where we have used Eq. (17). But $\langle \epsilon_\nu m_{\nu\mu}(\nu, 0)\epsilon_\mu\rangle = a_{10}(\nu)$. Hence in view of Eq. (14), we can write the sum rule (16) as

$$\alpha = \frac{1}{2\pi^2}\int [\sigma_{\gamma\pi^0}(\nu') - \sigma_{\gamma\pi^+}(\nu')] d\nu'. \tag{18}$$

We prefer to use the sum rule in the form given in Eq. (13) or (16). If we confine ourselves to single-particle intermediate states, then only vector, axial-vector, and tensor mesons can contribute to sum rule (13) or (16). Furthermore, because of C invariance, the above states must be of negative G parity. Hence we get from Eq. (13) or (16)

$$\sqrt{2} = -\frac{f_{A_1} f_{A_1\pi\gamma}}{m_{A_1}^2 - m_\pi^2} + \tfrac14 f_\omega f_{\omega\pi\gamma}(m_\omega^2 - m_\pi^2) + \tfrac12 f_{A_2} f_{A_2\pi\gamma}\frac{(m_{A_2}^2 - m_\pi^2)^3}{m_{A_2}^2}, \tag{19}$$

where we have neglected the contribution of φ meson. We also note that the contribution of these states to $\tilde{a}_1$ is indeed 0. The coupling constants f_{A_1}, f_ω, etc. are defined by

$$\langle \pi^0, q | j_\mu^- | p, A_1^+\rangle = i[f_{A_1}\epsilon_\mu + f_2(\eta\cdot q)(p+q)_\mu + f_3(\eta\cdot q)(p-q)_\mu];$$

$$\langle \pi^{0,+}, q | j_\nu^{\text{el},-} | p, \omega\rangle = (f_{\omega\pi\gamma}, f_\omega)\epsilon_{\nu\alpha\beta\gamma} q_\alpha p_\beta \eta_\gamma;$$

$$\langle \pi^{+,0}, q | j_\nu^{\text{el},-} | p, A_2^+\rangle = (f_{A_2\pi\gamma}, f_{A_2})\epsilon_{\nu\alpha\beta\gamma} p_\beta (q-k)_\gamma (q-k)_\rho A_{\rho\alpha};$$

where $k = p - q$.

The parameters of A_1 are least known experimentally. But we can eliminate these parameters completely: Consider

$$M_{\mu\nu} = i\int d^4x\, e^{-iq\cdot x}\theta(x_0)\langle 0 | [j_{\mu 5}^3(x), j_\nu^-(0)] | A_1^+\rangle.$$

Using partially conserved axial-vector current $[\partial_\mu j_{\mu 5}^3 = -(f_\pi/\sqrt{2})m_\pi^2\varphi^3]$ and the commutation relation

$$\delta(x_0)[j_{05}^3(x), j_\nu^-(0)] = -\delta^4(x) j_\nu^-(x),$$

we get

$$-i\langle 0 | j_{\nu 5}^- | A_1^+\rangle = \frac{f_\pi}{\sqrt{2}}\langle \pi^0 | j_\nu^- | A_1^+\rangle,$$

which gives $g_{A_1} = -(f_\pi/\sqrt{2}) f_{A_1}$. Using this relation with our Eq. (8), we get

$$-f_{A_1} f_{A_1\pi\gamma}/(m_{A_1}^2 - m_\pi^2) = \sqrt{2}. \tag{20}$$

Hence we get from Eq. (19)

$$0 = \tfrac14 f_\omega f_{\omega\pi\gamma}(m_\omega^2 - m_\pi^2) + \tfrac12 f_{A_2} f_{A_2\pi\gamma}(m_{A_2}^2 - m_\pi^2)^3/m_{A_2}^2. \tag{21}$$

Now CVC gives $f_\omega = \sqrt{2} f_{\omega\pi\gamma}$ and $f_{A_2} = -\sqrt{2} f_{A_2\pi\gamma}$, so that we get from Eq. (21), neglecting m_π^2/m_ω^2 and $m_\pi^2/m_{A_2}^2$,

$$f_{\omega\pi\gamma}^2 m_\omega^2 = 2 f_{A_2\pi\gamma}^2 m_{A_2}^4, \tag{22}$$

which gives

$$\frac{\Gamma(A_2^+ \to \pi^+ + \gamma)}{\Gamma(\omega \to \pi^0 + \gamma)} = \frac{3m_{A_2}}{5m_\omega} \approx 1. \tag{23}$$

Experimentally only $\Gamma(\omega \to \pi^0 + \gamma)$ is known and its value[4] is 1.2 ± 0.3, whereas the value of $\Gamma(A_2^+ \to \pi^+ + \gamma)$ calculated from the decay rate $A_2^+ \to \rho^+ + \pi$ and ρ dominance[4] is 1 ± 0.4 MeV. Thus Eq. (23) seems to be satisfied very well. The advantage of our approach is that relation (23) is independent of the parameters of the A_1 meson. From Eq. (20), we get the decay rate $\Gamma(A_1^+ \to \pi^+ + \gamma) \approx 2$ MeV which seems to be quite high. But our Eq. (23) should hold even if Eq. (20) gives a quite high value for the decay rate $\Gamma(A_1^+ \to \pi^+ + \gamma)$.

It may be noted that if we neglect the contribution of A_1 and A_2 mesons, we get from Eq. (19)

$$f_{\omega\pi\gamma}^{\ 2} = 4/(m_\omega^{\ 2} - m_\pi^{\ 2}) \approx 4/m_\omega^{\ 2}, \tag{24}$$

which gives $\Gamma(\omega \to \pi + \gamma) \approx 0.96$ in good agreement with the experimental value. This may be regarded as accidental or else it may be that the contribution of 0^- and 1^- mesons [which belong to an SU(6) multiplet] and 1^+ and 2^+ mesons separately saturate the sum rule (13) or (16). If this is true then we get, in addition to Eq. (24), the relation

$$f_{A_2\pi\gamma}^{\ 2} \approx 2/m_{A_2}^{\ 4}. \tag{25}$$

The above statement is certainly not true when we consider the commutator of axial-vector current with axial-vector current as we shall show.

If we assume that the decay $\omega \to \pi + \gamma$ or $A_2 \to \pi + \gamma$ proceeds through the ρ meson, we get $f_{\omega\pi\gamma} = \gamma_\rho g_{\omega\pi\rho}/m_\rho^{\ 2}$ and $f_{A_2\pi\gamma} = \gamma_\rho g_{A_2\pi\rho}/m_\rho^{\ 2}$, where $\gamma_\rho \gamma_{\rho\pi\pi}/m_\rho^{\ 2} = 1$. Hence from Eq. (22), we get

$$m_\omega^{\ 2} g_{\omega\rho^0\pi^0}^{\ 2} = 2 m_{A_2}^{\ 4} g_{A_2^+\rho^0\pi^+}^{\ 2}, \tag{26}$$

whereas Eqs. (24) and (25) give

$$g_{\omega\rho^0\pi^0}^{\ 2} = \frac{4\gamma_{\rho\pi\pi}^{\ 2}}{m_\omega^{\ 2}}, \quad g_{A_2^+\rho^0\pi^+}^{\ 2} = \frac{2}{m_{A_2}^{\ 4}} \gamma_{\rho\pi\pi}^{\ 2}. \tag{27}$$

(iii) Finally, we consider the matrix elements

$$M_{\mu\nu} = i\int d^4x e^{-iq\cdot x}\theta(x_0) \times \langle \pi^+, p' | [j_{\mu 5}^{\ +}(x), j_{\nu 5}^{\ -}(0)] | \pi^+, p\rangle.$$

Now $M_{\mu\nu}$ can be written

$$M_{\mu\nu} = A_1 P_\mu P_\nu + \cdots.$$

Using Fubini's technique, we get the sum rule

$$\frac{1}{2\pi i}\int a_1(\nu', q^2, q'^2, t) d\nu' = 4F(t), \tag{28}$$

where a_1 is the coefficient of $P_\mu P_\nu$ to be picked out from

$$m_{\nu\mu} = i(2\pi)^4 \sum_s [\delta(p+q-s)\langle \pi^+ | j_{\mu 5}^{\ +} | s\rangle \times \langle s | j_{\nu 5}^{\ -} | \pi^+\rangle - \delta(p-q'-s) \times \langle \pi^+ | j_{\nu 5}^{\ -} | s\rangle \langle s | j_{\mu 5}^{\ +} | \pi^+\rangle]. \tag{29}$$

As can be easily seen from Eq. (29), the only possible single-particle states are ρ^0, σ, and f. Hence we get from Eq. (28), at $q^2 = q'^2 = 0$ and $t = 0$, the sum rule

$$\frac{1}{2m_\rho^{\ 2}}[h_1 + (m_\rho^{\ 2} - m_\pi^{\ 2}) h_2]^2 + 4G_+^{\ 2} + \frac{1}{2} \times \frac{2}{3} \frac{(m_f^{\ 2} - m_\pi^{\ 2})^4}{m_f^{\ 4}} 2g_{A_2}^{\ 2} = 4, \tag{30}$$

where the weak-coupling constants h_1, h_2, etc., are related to the corresponding strong-coupling constants $\gamma_{\rho\pi\pi}$, etc., by Goldberger-Treiman-type relations:

$$[h_1 + (m_\rho^{\ 2} - m_\pi^{\ 2}) h_2]^2 = 4 f_\pi^{\ 2} \gamma_{\rho\pi\pi}^{\ 2},$$

$$G_+^{\ 2} = f_\pi^{\ 2} g_{\sigma\pi\pi}^{\ 2}/2(m_\sigma^{\ 2} - m_\pi^{\ 2})^2,$$

$$g_{A_2}^{\ 2} = 2 f_\pi^{\ 2} g_{f\pi\pi}^{\ 2}/(m_f^{\ 2} - m_\pi^{\ 2})^2.$$

The coupling constants $g_{\sigma\pi\pi}$ and $g_{f\pi\pi}$ are defined by

$$\langle \pi^+, q | j_\pi^{\ +} | p, \sigma\rangle = g_{\sigma\pi\pi},$$

$$\langle \pi^+, q | j_\pi^{\ +} | p, f\rangle = g_{f\pi\pi}(q-k)_\alpha (q-k)_\beta A_{\alpha\beta}.$$

From Eq. (30), we get the sum rule[5,7]

$$\frac{1}{2m_\rho^{\ 2}} f_\pi^{\ 2} \gamma_{\rho\pi\pi}^{\ 2} + \frac{f_\pi^{\ 2} g_{\sigma\pi\pi}^{\ 2}}{2(m_\sigma^{\ 2} - m_\pi^{\ 2})^2} + \frac{1}{3} \frac{(m_f^{\ 2} - m_\pi^{\ 2})^2 f_\pi^{\ 2} g_{f\pi\pi}^{\ 2}}{m_f^{\ 4}} = 1. \tag{31}$$

First we note that, if we neglect the contribution of σ and f mesons and use the relation[6] $\gamma_{\rho\pi\pi} = m_\rho/m_\pi$ which is well satisfied experimentally, ρ gives just half the contribution to the sum rule (28). Neglecting the term m_π^2/m_f^2, we get from Eq. (31)

$$\frac{g_{\sigma\pi\pi}{}^2 m_\rho{}^2}{(m_\sigma{}^2-m_\pi{}^2)^2}+\tfrac{2}{3}m_\rho{}^2 g_{f\pi\pi}{}^2-\gamma_{\rho\pi\pi}{}^2=0. \qquad (32)$$

Now $\gamma_{\rho\pi\pi}/4\pi=2.5$ and $g_{f\pi\pi}{}^2/4\pi\approx 0.02/m_\pi{}^2$ (calculated from $\Gamma_f=0.72m_\pi$ and $m_f{}^2=80.0m_\pi{}^2$) and we see that the contribution of σ is essential to satisfy the sum rule (32), although the experimental evidence for σ is doubtful.

It may be seen that we have derived the sum rule (16) under the assumptions of conservation of electromagnetic current, CVC, unsubtracted dispersion relations, and the commutation relation (9). In our approach and that of Harari and Pagels, all other assumptions are common, except CVC and the last assumption. CVC is well established experimentally, therefore as far as the derivation of sum rule (16) is concerned, the current algebra, the quark model, and Regge-pole theory lead to the same result.

*This work supported in part by the U. S. Atomic Energy Commission.

†On leave of absence from Atomic Energy Centre, Lahore, Pakistan.

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SUPERCONVERGENT SUM RULES FOR PHOTOPRODUCTION*

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Assuming that the Regge trajectories $\alpha_{10}(0)$, $\alpha_{10^*}(0)$, and $\alpha_{27}(0)$ are less than 0, we derive sum rules for photoproduction. Comparison with the experiment is given.

De Alfaro et al.[1] have recently derived a class of "superconvergent" sum rules for strong-interaction scattering amplitudes on the basis of analyticity and reasonable arguments about the high-energy behavior. The subject has attracted considerable attention ever since.[2] For the purpose of verifying the relations and the assumptions involved, a number of authors[3] have considered the case of meson-baryon scattering and analyzed the one superconvergent sum rule assuming the high-energy behavior given by the Regge-pole model. It seems to us of importance to verify these assumptions in other processes as well. We have analyzed the superconvergence relations for the process of photoproduction of mesons from baryons. In this case, we obtain more than one nontrivial sum rule so that the question of mutual consistency can also be examined. In deriving these sum rules the basic assumption made is that the Regge trajectories $\alpha(t)$ have $\alpha_{27}(0)$, $\alpha_{10}(0)$, and $\alpha_{10^*}(0)$ less than 0. If the sum rules are valid, we may regard this as strong evidence for the correctness of our assumption.

Let k, q, p_1, and p_2 be the four-momenta of the photon, the meson, the initial baryon, and the final baryon, respectively. We decompose the T matrix in terms of the four invariant amplitudes, A, B, C, and D.[4] They are functions of the invariants

$$\nu=-\frac{(p_1+p_2)\cdot k}{2M}=-\frac{(p_1+p_2)\cdot q}{2M}$$

and $t=-(p-p')^2$, where M is the baryon mass. In the Regge-pole model, the invariant amplitudes $A,\cdots,D$ all behave[5] like $\nu^{\alpha(t)-1}$ as $\nu\to\infty$, where $\alpha(t)$ refers to the dominant Regge trajectory in the t channel, $\gamma+\bar\pi\to N+\bar N$. Since, there is no experimental evidence for the existence of any low-lying mesons with $I=\frac{3}{2}$ or 2, we may assume that[3]

$$\alpha_{10}(0),\ \alpha_{10^*}(0),\ \alpha_{27}(0)<0.$$

We are thus led to consider the following five nontrivial (i.e., those which are not trivially satisfied due to the crossing properties of A,