NONCLASSICAL THEORY OF MAGNETIC MONOPOLES

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Interest has been recently aroused[1] in using magnetic monopoles to make quarks invisible. It has also been suggested[2] that the existence of magnetic monopoles may be the reason for CP nonconservation in K_{20} decay. In order that magnetic monopoles are to be so satisfactory in tidying up two of the more unpleasant loose ends in particle physics, it is necessary that they exist. They have been searched for vigorously,[3] but no trace of them has been seen. Indeed they seem to be as elusive as the quarks they are meant to suppress. It is the purpose of this paper to present a strictly nonclassical model of magnetic monopoles which makes them far more invisible than expected. At the same time this model may be used to violate CP invariance, and we discuss this aspect briefly.

The classical theory of point monopoles was given in its entirety by Dirac.[4] This theory required the use of string variables which makes space multiply connected and allows $\vec{H}=\nabla\times\vec{A}$, $\vec{H}\neq 0$ to have a solution, albeit with $\vec{A}$ singular on the string. An alternative approach,[5] which avoided the use of potentials with singular lines, has been to introduce a further photon, the "magnetic" photon, described by a four-potential B_μ; the total electromagnetic field is now $F_{\mu\nu}=\partial A_\mu/\partial x_\nu-\partial A_\nu/\partial x_\mu+\frac{1}{2}\epsilon_{\mu\nu\lambda\tau}\partial B_\lambda/\partial x_\tau$. This extra photon makes the magnetic and electric sources essentially independent, and so one loses the very powerful quantization condition of Dirac,[6]

$$eg/\hbar=\tfrac{1}{2}n, \tag{1}$$

where n is an integer. We prefer to keep this condition here, since it gives rise to quantization of charge; so we accept the existence of potentials with singular lines. The basic equations of the monopole theory are the extended equations of Maxwell,

$$\partial_\mu F_{\mu\nu}=j_\nu,\quad \partial_\mu \hat{F}_{\mu\nu}=\hat{j}_\nu, \tag{2}$$

where $\hat{F}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\lambda\tau}F_{\lambda\tau}$. Evidently $\partial_\nu j_\nu=\partial_\nu\hat{j}_\nu=0$. We have to make a choice for the currents $j_\nu, \hat{j}_\nu$. Since the strength of the magnetic current may be large, following (1), it will be necessary to choose a parity-conserving current. Recent discussions of the evidence for C and T conservation[7] show that it is not necessary that they be conserved by (2). Thus we may assume that j_ν arises from a spin one-half particle of charge e, and $\hat{j}_\nu$ from a spin one-half particle of magnetic charge g. Denoting the wave functions of these particles by ψ and χ, respectively, we postulate

$$j_\nu=e\bar{\psi}\gamma_\nu\psi,\quad \hat{j}_\nu=g\bar{\chi}\gamma_\nu\gamma_5\chi. \tag{3}$$

In order that $\hat{j}_\nu$ be conserved, it is necessary that χ be massless. If we add the equations of motion for the ψ and χ fields of standard form, we may then quantize, following Schwinger,[8] with energy density

$$T^{00}=\tfrac{1}{2}(\vec{E}^2+\vec{H}^2)+\bar{\psi}\vec{\gamma}\cdot(-i\nabla-e\vec{A}^T-e\vec{A}_g)\psi+m_e\bar{\psi}\psi$$
$$+\bar{\chi}\vec{\gamma}\cdot(-i\nabla-g\gamma_5\vec{B}^T-\gamma_5 g\vec{B}e)\chi, \tag{4}$$

where $\vec{E}=\vec{E}^T-\nabla\varphi$, $\vec{H}=\vec{H}^T-\nabla\hat{\varphi}$, $\varphi(\vec{x})=\int d\vec{y}\,\mathfrak{D}(\vec{x}-\vec{y})j_0(\vec{y})$, $\hat{\varphi}(\vec{x})=\int d\vec{y}\,\mathfrak{D}(\vec{x}-\vec{y})\hat{j}_0(\vec{y})$, and $\vec{H}^T=\nabla\times\vec{A}^T$, $\vec{E}^T=-\nabla\times\vec{B}^T$ are the transverse components of the magnetic and electric field strengths, with $\mathfrak{D}(\vec{x})=(4\pi|\vec{x}|)^{-1}$. The quantization of the system may be performed exactly as in Ref. 8, and gives a relativistically invariant, "string"-independent theory if the gauge quantization condition $eg/\hbar c=\frac{1}{2}n$ is satisfied, where n is an integer.[9] Thus a quantized massless magnetic monopole may be described by our Hamiltonian (4).

The experimental evidence[3] has been considered on the basis of a classical theory of monopoles: A static monopole is supposed to produce a magnetic field of strength g/r^2 and be acted on by a force $g\vec{H}$ in a magnetic field $\vec{H}$. Since g is large ($g^2/\hbar c\geq 137/16$), the effect of even a weak magnetic field will be large, and galactic magnetic fields will accelerate monopoles strongly; so they may be expected to arrive at the surface of the earth with high energy. Also the associated electric field of a moving monopole should be large, so that a monopole will have a high rate of energy loss on passing through matter.

A monopole theory with magnetic current

$g\bar{\chi}\gamma_\mu\chi$ will have the above classical properties, and so will a massless monopole with current $g\bar{\chi}\gamma_\mu\gamma_5\chi$ (though this cannot be made static). In both cases the effective monopole charge is g, giving large effects which are ruled out by experiment. In order to keep a monopole theory we must attempt to make the effective monopole charge 0. There are apparently two possibilities of doing this, based on the axial-vector magnetic current $\hat{j}_\mu = g\bar{\chi}\gamma_\mu\gamma_5\chi$. The first of these is to introduce mass m for the monopole by adding a nonlocal term to $\hat{j}_\mu$: $\hat{J}_\mu = g\bar{\chi}\gamma_\mu\gamma_5\chi + 2mg(\Box^2)^{-1}\partial_\mu\bar{\chi}\gamma_5\chi$. Then $\hat{J}_\mu$ is conserved for massive monopoles and a static monopole has a zero effective charge, since the matrix elements of γ_5 and $\gamma_\mu\gamma_5$ are then 0. This method has the disadvantage of introducing a massless pseudoscalar meson, which should be strongly produced in electromagnetic processes. Thus we will not consider it further here.

The alternative possibility is to introduce mass for the monopole by spontaneously breaking the γ_5 symmetry, keeping the monopole current $\hat{j}_\mu = g\bar{\chi}\gamma_\mu\gamma_5\chi$. The matrix element of $\hat{j}_\mu$ taken between single monopole states will have the form[10]

$$\langle p'|\hat{j}_\mu|p\rangle = \bar{u}(p')X_\mu(p',p)u(p), \tag{5}$$

where[11]

$$X_\mu(p',p) = F_1(q^2)[\gamma_\mu\gamma_5 + (2m/q^2)\gamma_5 q_\mu] + i(p_\mu' + p_\mu)\gamma_5 F_2(q^2).$$

We have used current and parity conservation in (5), where the current conservation no longer requires $F_2(q^2) \equiv 0$ on the monopole mass shells, as it does in the vector-current case.[7] Hermiticity of $\hat{j}_\mu$ requires F_1 and F_2 to be real, while invariance under time reversal T requires F_1 to be real, F_2 to be imaginary. Since T and C are both not conserved by (3), (4), and (5) (which we denote by an axial monopole theory), there is no need to take F_2 zero. We still have a strongly coupled massless pseudoscalar meson, with effective coupling $F_1(0)$. We may remove this unwanted particle by the condition

$$F_1(0) = 0, \tag{6}$$

and we are left with an effective coupling constant $2mF_2(0)$. We see that (6) and the Dirac quantization condition (1) determine both e and g. Assuming that we may renormalize the axial monopole theory along similar lines to conventional spin-$\frac{1}{2}$ electrodynamics,[12] then (1) and (6) determine both the renormalized charge and pole strength. We note that (6) is just the case excluded by Nambu,[13] because he considered T-conserving theories only.

In our case the effective pole strength of a static monopole is still 0, though between monopole states for which the matrix elements of γ_5 is appreciable the effective pole strength will be of order $2mF_2(0)$. It is no longer necessary that this be as strong as the value of g given by the Dirac quantization condition. It seems very difficult to obtain a value for $2mF_2(0)$ which does not depend very strongly on the approximation made. This is true, in particular, for the value of m.[13] It is evident that conditions (1) and (6) may play an important role in determining this value. It is necessary to set up a nonperturbative approximation scheme which includes (1) and (6) at each step. Till this is done we can only say that there is no evidence either way for presence or absence of axial magnetic monopoles. In particular, the contribution of our theory to CP nonconservation may be small enough to be evident only in K_2^0 decay; for example, $F_2(0)$ may be of order e. Thus we have a way around the difficulty raised by Salam[2] that the axial monopole would give large C-nonconserving effects in atomic physics.

It is still to be expected that the pole coupling constant g, though not the effective coupling constant, may give large values for F_1 and F_2 for large q^2. We may conjecture that monopoles are strongly interacting particles, in the absence of any other strongly coupled systems in nature; this is partly suggested by the fact that $\chi_\mu(p,p)$ is purely pseudoscalar, and so corresponds to pseudoscalar (massive) meson absorption. Such a theory corresponds to a model for the "mismatch" theory of Lee,[14] since the strongly interacting part, coupled to the magnetic current, is invariant under magnetic-charge conjugation C_m, so that $C_{\text{strong}} = C_m$ in this model. It may then be possible to interpret the monopole charge as hypercharge; this will not be conserved in a spontaneously broken-symmetry theory.

The evident problems raised by the above remarks are being considered further.

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[1]L. I. Schiff, Phys. Rev. Letters 17, 714 (1966); A. Peres, Phys. Rev. Letters 18, 50 (1967); J. G. Taylor, "Quarks and Magnetic Monopoles," to be published.

[2]A. Salam, Phys. Letters 22, 683 (1966), and independently by the author (unpublished).

[3]E. Goto, H. Kolm, and K. Ford, Phys. Rev. 132, 387 (1963); E. Purcell et al., Phys. Rev. 129, 2326 (1963); E. Amaldi et al., Nuovo Cimento 28, 773 (1963).

[4]P. A. M. Dirac, Phys. Rev. 74, 817 (1948).

[5]N. Cabibbo and E. Ferarri, Nuovo Cimento 23, 1147 (1962).

[6]P. A. M. Dirac, Proc. Roy. Soc. (London) A133, 60 (1931).

[7]J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. 139, B1650 (1965).

[8]J. Schwinger, Phys. Rev. 144, 1087 (1966).

[9]If ψ and χ are identical it is necessary to require $eg/\hbar c=\frac{1}{4}n$. The restriction of n to even integers only is required if the strings are allowed to go to infinity at both ends, and if all possible orientations of the string are considered.

[10]Y. Nambu, Phys. Rev. Letters 4, 380 (1960).

[11]We use Hermitian γ_0, γ_5, and anti-Hermitian γ_1, γ_2, γ_3, with $\gamma_0^2=-\gamma_1^2=-\gamma_2^2=-\gamma_3^2=\gamma_5^2=1$; $q=p'-p$.

[12]J. Schwinger, to be published.

[13]See Ref. 10, and Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).

[14]T. D. Lee, Phys. Rev. 139, 1415 (1965).

CURRENT ALGEBRA AND RADIATIVE DECAYS OF MESONS*

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Some time back Fubini[1] derived a new method of obtaining sum rules of interest in strong-interaction physics. In this note we apply Fubini's method to electromagnetic and weak processes. In particular we obtain sum rules for radiative decays of mesons. Making contact with strong interaction via Goldberger-Treiman-type relations and ρ dominance, we also get sum rules for strong decays of mesons.

(i) We begin by considering the matrix elements

$$M_{\nu\mu}=i\int d^4xe^{-ik\cdot x}\theta(x_0)\langle 0|[j_\nu^{\,\mathrm{el}}(x),j_{\mu5}^{\,-}(0)]|\pi^+,p\rangle$$

which are related to the process $\pi^+\to l^++\nu+\gamma$. Here we have two independent four-momenta p and k and one invariant $\nu=-p\cdot k$. Using the commutation relation

$$\delta(x_0)[j_0^{\,\mathrm{el}}(x),j_{\mu5}^{\,-}(0)]=-\delta^4(x)j_{\mu5}^{\,-}(x)$$

and the current conservation, we get

$$ik_\nu M_{\nu\mu}=-i\langle 0|j_{\mu5}^{\,-}|\pi^+\rangle=-f_\pi p_\mu. \tag{1}$$

The most general form of the matrix elements $M_{\nu\mu}$ is given by

$$M_{\nu\mu}=i[H_1(\nu)p_\nu p_\mu+H_2(\nu)p_\nu k_\mu +H_3(\nu)k_\nu P_\mu+H_4(\nu)k_\nu k_\mu+H_5(\nu)\delta_{\nu\mu}].$$

We separate the Born term and write Eq. (1) as

$$ik_\nu M_{\nu\mu}(\mathrm{Born})+ik_\nu\tilde{M}_{\nu\mu}=-f_\pi p_\mu, \tag{2}$$

where

$$M_{\nu\mu}(\mathrm{Born})=-if_\pi(2p-k)_\nu(p-k)_\mu/2\nu.$$

Hence from Eq. (2), we obtain

$$-f_\pi+\nu\tilde{H}_1(\nu)=-f_\pi,$$
$$f_\pi-\tilde{H}_5(\nu)+\nu\tilde{H}_2(\nu)=0, \tag{3}$$

First we note that at $\nu=0$, we get $\tilde{H}_5(0)=f_\pi$. From Eq. (3), we get $\tilde{H}_1(\nu)=0$ at $\nu\neq 0$ and the sum rule

$$\frac{1}{2\pi i}\int\tilde{h}_2(\nu')d\nu'=f_\pi, \tag{4}$$

where $\tilde{h}_2(\nu)$ is the coefficient of $p_\nu k_\mu$ to be picked