of considerably lower energy; for still higher Z, the spherical shape is not even a minimum. It may be seen from Fig. 2 that the minimum is always very shallow, and that the energy maximum encountered in a transition to the very elongated prolate shape is less than 1 MeV. Such nuclei must be extremely susceptible to spontaneous fission.[5]

In spite of the crudeness of the semiempirical formula, we believe that our predictions are fairly reliable. Deviations from that formula occur chiefly at closed shells, and it is generally agreed that around $Z = 100$ we are about in the middle of filling a shell. In this case, the Nilsson levels of individual nucleons are quite close together, forming almost a continuum, so that the liquid-drop model should be a good approximation. Furthermore, collective interactions have the tendency to make the shape of nuclei prolate spheroids even in regions of Z and A where the liquid-drop model predicts spheres. We therefore believe that a liquid-drop prediction of spheroidal shape should be taken seriously.

These results will be modified by shell structure. This has an important influence on the energy only for the spherical shape. The next magic nucleus after Pb^{208} is expected to have 184 neutrons and, to keep the ratio of neutrons to protons close to equilibrium, probably 114 protons (which is not really a good closed shell).

The energy reduction for this semimagic nucleus is likely to be considerably less than for Pb^{208} for which it is about 15 MeV. On the other hand, without shell structure, the elongated shape has an energy about 18 MeV less than the sphere for $Z = 114$. We therefore believe that shell effects are unlikely to make the nucleus $Z = 114$ stable.

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FINE-STRUCTURE ANALYSIS OF ANALOG RESONANCES IN K^{41} †

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This Letter reports an analysis of the fine structure observed in a high-resolution study of analog resonances in K^{41} using $Ar^{40}(p,p)Ar^{40}$.[1] The analysis uses a K-matrix theory of fine structure in nuclear reactions.[2] The S matrix is related to the K matrix by

$$S_{cc'} = \exp(i\delta_c)[(1-i\pi K)(1+i\pi K)^{-1}]_{cc'}\exp(i\delta_{c'}). \quad (1)$$

The K matrix is derived from a shell-model approach to reaction theory and has a resonant part

$$K_{cc'}^{R} = (2\pi)^{-1}\sum_{\lambda}\Gamma_{\lambda c}^{\frac{1}{2}}\Gamma_{\lambda c'}^{\frac{1}{2}}/(E-E_{\lambda}), \quad (2)$$

where $\Gamma_{\lambda c} = 2\pi|\langle X_{\lambda}|V_e|c\rangle|^2$ is the partial width for decay of the state X_{λ} through channel c. The V_e is the shell-model effective interaction and X_{λ} is obtained by diagonalizing $H_0 + V_e$ on the set of discrete eigenstates of H_0. This H_0 is an independent-particle Hamiltonian with a Saxon-Woods potential, which is used to generate the nonresonant phase shifts appearing in Eq. (1).

The resonance energies E_{λ} and the widths $\Gamma_{\lambda c}$ can be treated as parameters which are determined by fitting the experimentally observed resonances. This analysis has been carried out on the Duke data. The spin, parity, resonance energy, proton width, and alpha widths of every fine-structure resonance in the range of $E_p = 1.6432$ MeV to $E_p = 2.6020$ MeV have been tabulated.[3] From this tabulation we have calculated the reduced widths $\gamma_{\lambda}^2 = \Gamma_{\lambda}/2kRP$ which are shown in Fig. 1.

The model chosen to describe this resonance structure is the following. Strongly coupled to the incident proton channel is what one calls the analog state, which acts as a doorway state.

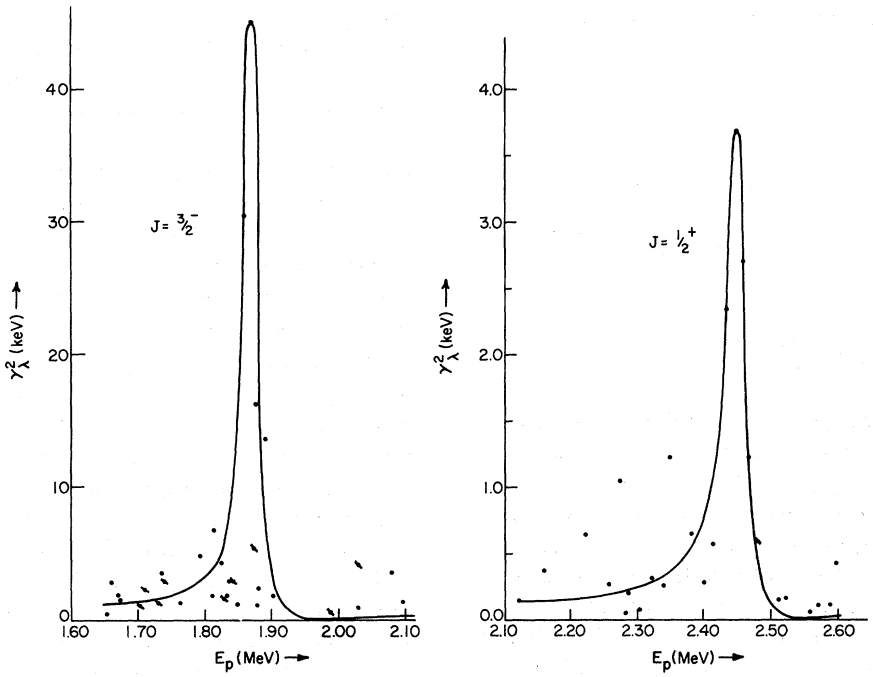

FIG. 1. The fine-structure widths as a function of energy. Dots represent experimental values of the widths of hallway states, while dots with lines through them are widths of hallway states whose spin is uncertain. The solid curve is the theoretical fit of Eq. (4) using the values of the parameters as stated.

Weakly coupled to the continuum are hallway states,[4] which are seen far away from the analog as very narrow resonances with widths of the order of tens of electron volts. These hallway states are also coupled to the analog state, and they acquire an additional width through their coupling to the continuum via the analog. The width of the analog is in this way distributed to the nearest hallway states, and the widths about the analog resonance energy show an enhancement. This enhancement is at the expense of the analog, of course, which actually "dissolves" among the hallways. Otherwise stated, the coupling between the hallway states and the analog state results in a new set of eigenmodes, each of which contains some portion of the original analog state.

The hallway states are approximately isospin $T=\frac{3}{2}$ states, while the analog corresponds to $T=\frac{5}{2}$. Since the average Coulomb interaction requires spatially different radial wave functions for neutrons and protons, the appropriate wave functions for describing both the analog state and the hallway states are actually eigenfunctions of analog spin and not of isospin.[5] The coupling matrix elements between the analog state and the hallway states will therefore contain both the "residual" Coulomb interaction and the charge-independent effective nuclear interaction. In addition to the direct coupling between the analog state and the hallway states it is important to consider the coupling through the continuum as intermediate states. Only a calculation can tell which coupling is more important.

Under certain conditions the width distribution will show an asymmetry as first observed by Robson[6] who gave a formula for the distribution of widths about the analog. The qualitative explanation of this asymmetry is that the "intrinsic" width of the hallway state and the width gained by coupling to the analog state contribute coherently when a certain phase relation is satisfied and, therefore, interfere constructively or destructively over the resonance region. The $Ar^{40}(p,p)Ar^{40}$ fine-structure widths show this asymmetric pattern in Fig. 1.

The qualitative statements made above can be derived rigorously.[2] For simplicity we consider only one open channel, a good approximation in the $Ar^{40}(p,p)Ar^{40}$ data. For the dis-

tribution of widths in the model considered above, we have

$$\Gamma_\lambda \equiv 2\pi |\langle X_\lambda | V_e | c\rangle|^2 = \left| \Gamma_h^{1/2} - \Gamma_a^{1/2} \frac{V_{ha}}{E_\lambda - E_a} \right|^2 \frac{(E_\lambda - E_a)^2}{(E_\lambda - E_a)^2 + \Gamma_s^2/4}, \quad (3)$$

where

$$\Gamma_s \equiv \Gamma_{in} [1 + D^2/\pi^2 |V_{ha}|^2]^{1/2}.$$

Here D is the average level spacing of the fine structure, E_λ the energies of the fine-structure resonances, $V_{ha} = \langle \varphi_h | V_e | \varphi_a \rangle$ the average coupling matrix element of the hallway state $|\varphi_h\rangle$ to the analog state $|\varphi_a\rangle$, $\Gamma_h^{1/2}$ the average amplitude for decay of a hallway state before coupling to the analog, $\Gamma_a^{1/2}$ the analog-state amplitude for decay, and $\Gamma_{in} = 2\pi |V_{ha}|^2/D$.[7]

Near the analog energy E_a the distribution of widths follows a Lorentzian with a "spreading width" Γ_s. This width is determined by the coupling matrix elements V_{ha} between the analog state and the hallway states. An asymmetry appears in the distribution of widths when the matrix elements V_{ha} and $\Gamma_h^{1/2}$ have the same relative phase across the whole region of enhanced fine structure.[8] When this situation obtains, it is useful to write Eq. (3) in a different form,

$$\Gamma_\lambda = \Gamma_h \frac{(E_\lambda - E_a - \Delta)^2}{(E_\lambda - E_a)^2 + \Gamma_s^2/4}, \quad (4)$$

$$\Delta \equiv [V_{ha}/(\Gamma_h \Gamma_a)^{1/2}] \Gamma_a.$$

The enhancement factor which multiplies the intrinsic width Γ_h has a width of Γ_s and predicts a suppression of fine structure at an energy displaced from E_a by a multiple of the analog width Γ_a. This function has the general form of that given by Robson,[6] but the parameters are quite different. The width of Robson's enhancement function is Γ_a, which is generally expected to be considerably smaller than Γ_s.

In applying Eq. (4) to the analysis of the widths of fine structure in analog resonances we first remark that Γ_h and Γ_a increase with energy due to penetrability and phase-space factors. These factors are the same in these two widths, however, and therefore Δ is a constant. Since Γ_s is also a constant, the enhancement factor multiplying Γ_h contains only energy-independent constants. Thus the fine-structure reduced widths γ_λ^2 are determined by four energy-independent constants: the reduced width γ_h^2, Δ, Γ_s, and E_a. The value of γ_h^2 can be read from the distribution as the asymptotic (average) value of γ_λ^2 far from E_a. The energy E_a is the position of the maximum γ_{max}^2 of the reduced width distribution, if $\gamma_h^2 \ll \gamma_{max}^2$. The quantity $\Delta = E_S - E_a$ is found by reading the suppression energy E_S at which the reduced widths go to zero. Γ_s is approximately the width of the γ_λ^2 distribution at half-maximum. A simple relation which must be satisfied by these parameters is that $(\Gamma_s/2\Delta)^2 = \gamma_h^2/\gamma_{max}^2$. This relation can also be used to determine γ_h^2.

Once the four parameters γ_h^2, Δ, Γ_s, and E_a have been determined by fitting the γ_λ^2 distribution, the analog width Γ_a and the coupling matrix element V_{ha} are easily found. A fourth parameter now enters, however, the level spacing D. This is determined by counting the number of resonances in a suitable energy interval. V_{ha} is then found from the definition of Γ_s. The analog width follows from the definitions of Δ and Γ_s, which give $\Gamma_a = (2\pi\Delta^2/D)(\Gamma_h/\Gamma_s)$.

The value of Γ_a can also be obtained from a sum rule satisfied by the Γ_λ, or rather by the energy-independent γ_λ^2,[2]

$$\sum_\lambda \gamma_\lambda^2 - \sum_h \gamma_h^2 = \gamma_a^2. \quad (5)$$

The left-hand side approaches zero away from E_a and, therefore, the contributions to γ_a^2 come from resonances near the analog energy. The analog width can therefore also be found directly from the γ_λ^2 distribution. If we replace the sums by integrals and use the smooth distribution of Eq. (4), we find exactly the equation given for Γ_a in the preceding paragraph. Therefore, the distribution of Eq. (4) automatically satisfies the sum rule.

Note that although there appears a certain arbitrariness throughout this paper in the conversion factor relating widths and reduced widths, the absolute value of this factor does not affect the results of the analysis of the experimental widths Γ_λ in any way. Only the energy dependence enters, and this affects only the quality of the fits to the data far from E_a. Thus, penetration factors for a Saxon well should be used in defining the reduced widths, but we have used those for a square well as a convenient approximation.

The fits to the fine structure associated with analog resonances at $E_p = 1.87$ MeV and $E_p = 2.45$ MeV in $Ar^{40}(p,p)Ar^{40}$ are shown in Fig. 1. These are the analogs of the fourth and sixth excited states of Ar^{41} at excitation energies of 1.35 and 1.87 MeV. The data were fitted with $V_{ha} = 5.8$ keV, $D = 10$ keV, $\Gamma_h = 12$ eV, and $\Gamma_a = 5.5$ keV for the $J = \frac{3}{2}^-$ resonances near $E_p = 1.87$ MeV. The $J = \frac{1}{2}^+$ fine-structure width distribution was fitted with $V_{ha} = 7.6$ keV, $D = 12$ keV, $\Gamma_h = 23$ eV, and $\Gamma_a = 5$ keV.

The width Γ_a of the analog state can be related to the single-particle width Γ_p^{sp} of a proton scattering resonance at $E_p = E_a$. The relation is[9]

$$\Gamma_a = (2T+1)^{-1}\beta^2\Gamma_p^{sp}, \quad (6)$$

where β is the fraction of the Ar^{40} ground state which is the parent of the state in Ar^{41} corresponding to the analog resonance in K^{41}. Since Γ_a is determined from the sum rule of Eq. (5), a calculation of Γ_p^{sp} will lead to a value for the parentage coefficient β. This coefficient is also determined by stripping data[10] from $Ar^{40}(d,p)Ar^{41}$, although the ratios of such coefficients are more reliably obtained from stripping data than the absolute values.

Calculation of Γ_p^{sp} from an optical potential of the Saxon-Woods form adjusted in depth to give a $\frac{3}{2}^-$ resonance at $E_p = 1.87$ MeV and a $\frac{1}{2}^+$ resonance at $E_p = 2.45$ MeV gives 30 and 300 keV, respectively, for these two resonances.[11] Since these are $T = \frac{5}{2}$ states and the sum-rule value of Γ_a for each is about 5 keV, we find $\beta^2 = 1.0$ for the 1.87-MeV level and $\beta^2 = 0.1$ for the 2.45-MeV level. The ratio of the reduced stripping widths to the corresponding two levels in Ar^{41} is 10, in agreement with this result.[10]

Both values of β^2 are consistent with simple shell-model interpretations of these two levels in Ar^{41}. The $J = \frac{3}{2}^-$ state is suggested to be a $2p_{3/2}$ neutron coupled to the $J = 0^+$ ground state of Ar^{40}. The $J = \frac{1}{2}^+$ state, which is seen in $Ar^{40}(p,p)Ar^{40}$ as an s-wave proton resonance, is most simply interpreted as a $2s_{1/2}$ neutron coupled to a $J = 0^+$ excited configuration of Ar^{40} formed by exciting two neutrons from the $2s_{1/2}$ shell to the $1f_{7/2}$ shells. The analog of this state can be excited by a proton incident on Ar^{40} only if the ground state of Ar^{40} also contains $\beta^2 \simeq 10\%$ of this excited configuration. The presence of this configuration as core "excitation" of Ar^{40} in this amount appears to be reasonable.[12]

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