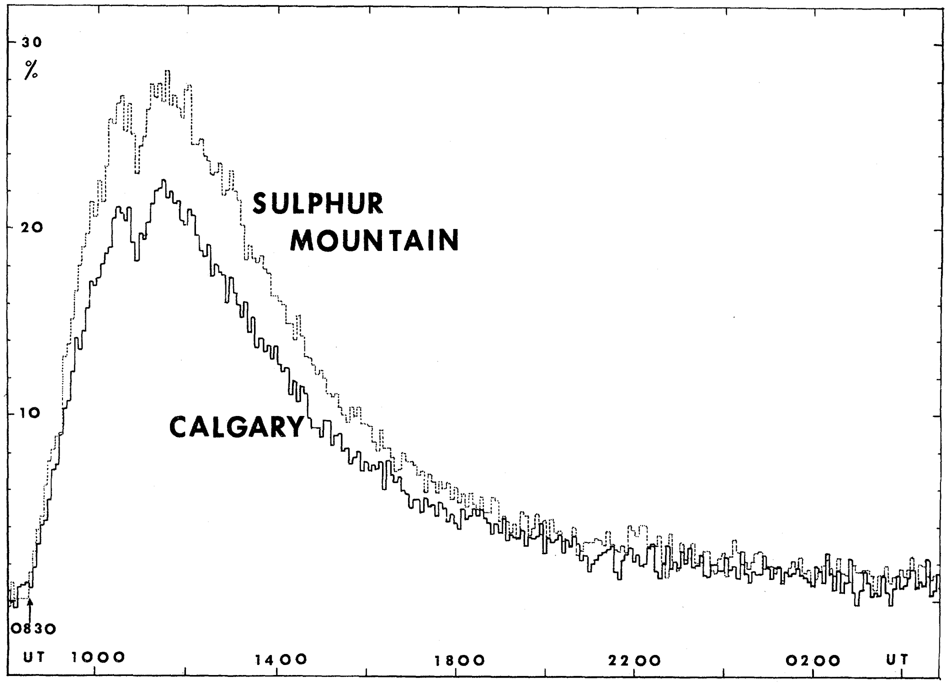

FIG. 1. Solar-flare increase as observed by superneutron monitors at Sulphur Mountain and Calgary (see text).

itors at this station and find the ratio of percentage increase of the supermonitor to that of the IGY monitor to be 1.03 ± 0.05, indicating that the effective responses are very similar. This suggests that the chain of supermonitors can be intercompared directly with the IGY network of stations.

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ELECTROMAGNETIC CORRECTIONS TO THE WEAK $\Delta S = 0$ VECTOR COUPLING CONSTANT*

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The electromagnetic renormalization of the $\Delta S = 0$ weak vector coupling constant G_V coming from the corrections to the vector part of the weak Hamiltonian is a universal divergent constant, independent of the details of the strong interactions.

An elegant feature of the $V-A$ theory of weak interactions is that the vector part of the $\Delta S = 0$ hadron current is proportional to the isospin current. This assumption, and the hypothesis that the isospin is conserved by the strong interactions, are known jointly as the conserved-vector-current (CVC) hypothesis. An important implication of this view is that the ratios of the renormalized to unrenormalized isovector coupling constants are equal for all processes. When supplemented with the notion of a universal coupling for the isovector current, these conclusions predict simple relations among the observed isovector coupling constants.[1-3]

In order to check the validity of this picture, it is important to calculate the corrections to the vector coupling constants arising from the electromagnetic interactions. The electromagnetic corrections to the decay $\mu \to e + \nu + \bar{\nu}$ have been calculated to order α.[4-7] Early attempts to calculate the corrections to G_V in neutron β decay essentially ignored the complications of the strong interactions and led to a logarithmically divergent result which could only be estimated in terms of a cutoff.[4-7] More recently it has been shown that this logarithmic divergence probably persists even if the effects of the strong interactions are included.[8] Equation (9.20) of Ref. 8 gives the "divergent part" of the isovector decay amplitude. This expression is universal in the sense that it is independent of the details of the strong interactions; except for simple factors dependent on the isospin of the decaying particle, it is the same for all amplitudes.

We shall show here that to second order in e^2, to first order in G_V, and to zero order in the momentum carried off by the leptons, the matrix element of the isovector part of the weak Hamiltonian density $\mathcal{H}_W^V$ is given exactly by Eq. (9.20) of Ref. 8. That is, not only is the divergent part of the amplitude independent of the details of the strong interactions, but all the finite contributions cancel in the limit of zero momenta for the leptons. For leptonic decays (e.g., $\pi^- \to \pi^0 + e + \bar\nu$, etc.) involving hadrons belonging to the same isomultiplet, the lepton momenta are of order α, and hence our result implies that the electromagnetic corrections to the isovector part of all such decay amplitudes are given to order α by one universal (divergent) factor. That is, with the definition

$$\sqrt{Z_4} \equiv 1 - \frac{3}{2}\frac{ie^2}{(2\pi)^4}\int \frac{d^4k}{(k^2 - i\epsilon)^2}, \tag{1}$$

the contribution of the vector hadron current to amplitude for π_{e3} decay, for example, is

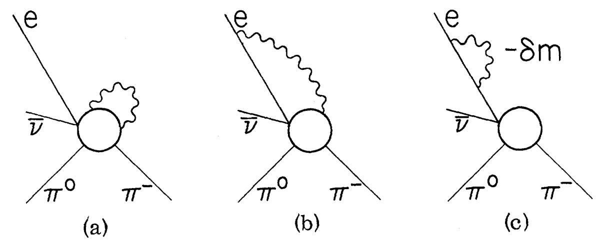

FIG. 1. The electromagnetic corrections to π_{e3} decay.

given to order α by[9]

$$\langle \pi^0 e\bar\nu | \mathcal{H}_W^{\ V} | \pi^- \rangle = (\sqrt{Z_4}) G_W \bar u(e)(\not\pi^0 + \not\pi^-)(1+\gamma_5) v(\nu), \tag{2}$$

where

$$\mathcal{H}_W^{\ V} = (G_W/\sqrt{2}) t_{+\lambda}[\bar\psi_e \gamma_\lambda (1+\gamma_5)\psi_{\nu_e} + \bar\psi_\mu \gamma_\lambda (1+\gamma_5)\psi_{\nu_\mu}] + \text{H.c.}, \tag{3}$$

and $t_{+\lambda}$ is the charge raising component of the isospin current. To order e^2, the total decay amplitude also includes a part $\langle \pi^0 e\bar\nu | \mathcal{H}_W^A | \bar\pi\rangle$ from the axial-vector hadron current.

In order to demonstrate our result, we consider for definiteness the decay $\pi^- \to \pi^0 + e + \bar\nu$. To order α there are three kinds of electromagnetic corrections to the decay amplitudes of Eqs. (2) and (3). These are indicated in Fig. 1. Figure 1(a) is the order-α part of the isospin matrix element $\langle \pi^0 | t_{+\mu} | \pi^-\rangle$. To order α ($\pi^0 - \pi^- \sim \alpha$),

$$\langle \pi^0 | t_{+\mu} | \pi^- \rangle = A(\pi^0 + \pi^-)_\mu.$$

The calculation of A can best be done using the methods of Fubini and Furlan.[10] Consider

$$0 = \int d^4x\, \partial_\lambda \langle \pi^- | T[t_{+\lambda}(x) t_{-4}(0)] | \pi^- \rangle = 2\langle \pi^- | t_{34}(0) | \pi^- \rangle + \int d^4x \langle \pi^- | T[\partial_\lambda t_{+\lambda}(x) t_{-4}(0)] | \pi^- \rangle. \tag{4}$$

Setting[11] $\langle \pi^- | t_{34}(0) | \pi^- \rangle = 2\pi_4^-$ and noting that the π^0 intermediate state yields a contribution (to order α) to the second term of Eq. (4) equal to $2|A|^2 \pi_4^-$, we can divide by $\pi_4^- = iE^-$ and obtain

$$|A|^2 - 2 = \frac{(2\pi)^3}{2E^-} \sum_n{}' \delta^3(n - \pi^-) \left[\frac{\langle \pi^- | \partial_\lambda t_{+\lambda} | n \rangle \langle n | t_{-4} | \pi^- \rangle}{E_n - E^- - i\epsilon} + \frac{\langle \pi^- | t_{-4} | n \rangle \langle n | \partial_\lambda t_{+\lambda} | \pi^- \rangle}{E_n - E^- - i\epsilon} \right], \tag{5}$$

where the prime on the sum indicates that the π^0 intermediate state is absent. The delta function in (5) allows one to replace t_{-4} by $\partial_\mu t_{-\mu}$ and divide by another power of $E_n - E^-$. Keeping only intermediate states $|n\rangle$ containing a photon (no others contribute to order α) and employing the relations[8,10]

$$\partial_\mu t_{\pm\mu} = \mp ieA_\mu t_{\pm\mu} \tag{6}$$

to contract out the photons, we obtain after some algebra[12]

$$A=\sqrt{2}\left\{1+\frac{e^2}{8E(2\pi)^4}\int d^4kD_{\lambda\mu}(k)\frac{\partial}{\partial k_0}\int d^4xe^{-ik\cdot x}\langle\pi^-|T[t_{+\lambda}(x)t_{-\mu}(0)]|\pi^-\rangle\right\},\tag{7}$$

where $D_{\lambda\mu}(k)$ is the Feynman propagator of the photon. This result also appears in Ref. 8.

Since e^2 multiplies the integral in Eq. (7), we can use isospin invariance to manipulate the integrand into a form more convenient for comparing with the corrections of Figs. 1(a) and 1(c). Isospin symmetry gives

$$\int d^4xe^{-ik\cdot x}\langle\pi^-|T[t_{+\lambda}(x)t_{-\mu}(0)]|\pi^-\rangle$$
$$=\sqrt{2}\int d^4xe^{ik\cdot x}\langle\pi^0|T[j_\mu(x)t_{+\lambda}(0)]|\pi^-\rangle+2\int d^4xe^{ik\cdot x}\langle\pi^-|T[j_\mu(x)t_{3\lambda}(0)]|\pi^-\rangle,\tag{8}$$

where j_μ is the electromagnetic current. The second term on the right-hand side of Eq. (8) does not contribute to the integral in Eq. (7). This follows from TP invariance[12] and does not depend upon the external particle being a pion. Therefore, if we define

$$T_{\lambda\mu}(k,\pi)\equiv i\int d^4xe^{ik\cdot x}\langle\pi^0|T[j_\mu(x)t_{+\lambda}(0)]|\pi^-\rangle,\tag{9}$$

the expression in Eq. (7) for the correction of Fig. 1(a) can be written as

$$A=\sqrt{2}-\frac{ie^2}{4E(2\pi)^4}\int d^4kD_{\lambda\mu}(k)\frac{\partial}{\partial k_0}T_{\lambda\mu}(k,\pi).\tag{10}$$

The corrections shown in Figs. 1(b) and 1(c) can also be obtained by using current commutators and an expression for $\partial_\mu t_{3\mu}$ analogous to Eq. (6). However, for present purposes let us simply write down the formal expression represented by Figs. 1(b) and 1(c).[8] Combining these contributions with that of Eq. (10), the matrix element in Eq. (2) becomes

$$\langle\pi^0e\bar{\nu}|\mathcal{H}_W{}^V|\pi^-\rangle=\frac{G_W}{\sqrt{2}}\bar{u}(e)\left\{\left[\sqrt{2}-\frac{ie^2}{4E(2\pi)^4}\int d^4kD_{\lambda\mu}(k)\frac{\partial}{\partial k_0}(T_{\lambda\mu}-S_{\lambda\mu})\right](\not{\pi}^0+\not{\pi}^-)\right.$$
$$+\frac{ie^2}{(2\pi)^4}\int d^4kD_{\lambda\nu}(k)\gamma_\nu\frac{m-\not{e}-\not{k}}{k^2+2e\cdot k-i\epsilon}\gamma_\mu(T_{\lambda\mu}-S_{\lambda\mu})$$
$$\left.-\frac{\sqrt{2}}{2}\left[\frac{ie^2}{(2\pi)^4}\int d^4kD_{\lambda\nu}(k)\gamma_\nu\frac{m-\not{e}-\not{k}}{k^2+2e\cdot k-i\epsilon}\gamma_\lambda-\delta m\right]\frac{1}{\not{e}+m-i\epsilon}(\not{\pi}^-+\not{\pi}^0)\right\}(1+\gamma_5)v(\nu),\tag{11}$$

where $S_{\lambda\mu}$ is a constant related to the Schwinger term. It occurs only in theories with a local A^2 coupling and gives the part of the Compton amplitude not contained in the time-ordered product. Since it is independent of k it contributes nothing to the first term of Eq. (11) and is included only to simplify our remarks.

We now show that the effects of the strong interactions, which are hidden in the $T_{\lambda\mu}$, cancel completely to zero order in the electron momentum. First let us note the relation

$$k_\lambda(T_{\lambda\mu}-S_{\lambda\mu})=k_\lambda(T_{\mu\lambda}-S_{\mu\lambda})=-2\sqrt{2}\pi_\mu,\tag{12}$$

which is readily derived from Eq. (9) and the usual equal-time commutator of $j_0(x)$ and $t_{+\lambda}(0)$. Using Eq. (12) it is fairly easy to check that the matrix element in Eq. (11) is gauge invariant.[13] We are thus free to use the Feynman gauge, $D_{\lambda\mu}(k)=\delta_{\lambda\mu}/(k^2-i\epsilon)$. If we integrate by parts the first term of Eq. (11), write the result covariantly, and keep only the part of the second term which contributes to zero order in the electron momentum, we obtain

$$\langle\pi^0 e\bar{\nu}|\mathcal{H}_W{}^V|\pi^-\rangle=\frac{G_W}{\sqrt{2}}\bar{u}(e)\biggl[\sqrt{2}\gamma(\pi^0+\pi^-)-\frac{ie^2}{m_\pi{}^2(2\pi)^4}\int\frac{d^4k}{(k^2-i\epsilon)^2}(k\cdot\pi T_{\lambda\lambda}\not{\pi}+m_\pi{}^2\gamma_\lambda\not{k}\gamma_\mu T_{\lambda\mu})$$

$$-\sqrt{2}\left(\frac{ie^2}{(2\pi)^4}\int\frac{d^4k}{k^2-i\epsilon}\gamma_\lambda\frac{m-\not{e}-\not{k}}{k^2+2e\cdot k-i\epsilon}\gamma_\lambda-\delta m\right)\frac{1}{\not{e}+m-i\epsilon}\not{\pi}\biggr](1+\gamma_5)v(\nu). \qquad (13)$$

But

$$\int\frac{d^4k}{(k^2-i\epsilon)^2}\gamma_\lambda\not{k}\gamma_\mu T_{\lambda\mu}=2\gamma_\mu\int\frac{d^4k}{(k^2-i\epsilon)^2}k_\lambda T_{\lambda\mu}-\frac{1}{m_\pi{}^2}\int\frac{d^4k}{(k^2-i\epsilon)^2}k\cdot\pi T_{\lambda\lambda}\not{\pi}. \qquad (14)$$

Using (12) to simplify the first term on the right-hand side of Eq. (14), substituting (14) into (13), and taking the limit of zero electron momentum in the third term of Eq. (13) leads to the result stated in Eqs. (1) and (2). The two terms containing $T_{\lambda\lambda}$ exactly cancel.

A similar calculation can be performed for any β decay in which the mass difference of the hadrons is of electromagnetic origin; for example, $K^0\to K^++e+\bar{\nu}$, $n\to p+e+\bar{\nu}$, $\Sigma^-\to\Sigma^0+e+\bar{\nu}$, $O^{14}\to N^{14}+e^++\nu$, etc. For all the spin-zero hadrons the argument is analogous to the one presented for the pion. When the hadrons have spin, our remarks apply to the "charge" form factor (e.g., the coefficient of γ_μ for spin $\frac{1}{2}$). This can be seen by carrying out the same calculation for the spin average of the amplitude.

In all these decays the ratio of the renormalized to unrenormalized isovector coupling constants is given by the one divergent factor defined in Eq. (1).

Concerning the input leading to our result, essentially everything follows from Eq. (6) and the equal-time commutation relations among the isospin and hypercharge currents. Only the time-time and time-space components of the commutation rules have been used, and presumably these are the most reliable. We believe that Eq. (6) is equivalent to minimal coupling of the electromagnetic field,[12] and hence that we have assumed the charge-conjugation invariance of electromagnetic interactions. Equation (6) is reminiscent of Yang-Mills conditions; whereas our result resembles the Ward-Takahashi identity. This suggests that it might be possible to prove it from some general principle.

Early calculations of electromagnetic mass splittings are also logarithmically divergent.[14] It is now believed that this divergence is due to the use of perturbation theory, and that inclusion of the strong interactions to all orders will give finite results, which depend, through tensors like our $T_{\mu\nu}$, on the details of the strong interactions.[15] We have shown here that the situation for the electromagnetic corrections to that part of G_V arising from the vector hadron current, at least up to order α, is just the reverse; namely, that they are described by one infinite constant, which is completely independent of the strong interactions.

This work seems to eliminate the hope of checking the Cabibbo form of universality, unless, as in μ decay,[4-7] the contribution to the renormalized G_V coming from the axial-vector current is also divergent in such a way as to cancel the ultraviolet divergence in Eq. (1).

We remark that the apparent infrared divergence of Eq. (1) is spurious and can be eliminated by more careful treatment of the electron momentum dependence near $k=0$. These and other details will be published elsewhere.

After completion of this work, Professor J. Bjorken informed us that Mrs. Helen Quinn has obtained a similar result. We also thank Professor Bjorken for a discussion of our conclusions, and, in particular, for emphasizing to us the importance of the axial-vector current. We want to express our gratitude to Professor J. M. Cornwall for a number of helpful discussions.

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[9]The symbols $\not{\pi}^0$, $\not{\pi}$, etc., stand for $\gamma\cdot p$, where p is the momentum of the indicated particle. Our pion states are normalized so that $\langle\pi|\pi'\rangle=(2\pi)^3 2E\,\delta_3(\vec{p}-\vec{p}')$.

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[11]The equal-time commutator $[t_{+0}(\vec{x},t),t_{-0}(\vec{y},t)]=2t_{30}(\vec{x},t)\delta_3(\vec{x}-\vec{y})$ has been used to obtain the first term on the right-hand side of Eq. (4). The correct expression includes a factor of Z_3 in this term, which we have omitted. The diagrams to which it corresponds have also been omitted, so that our calculation is in fact correct. We shall explain this fine point elsewhere.

[12]We are assuming here that A is real, which follows from TP conservation. Since the proof of Eq. (6) [see S. L. Adler, Phys. Rev. 139, B1638 (1965)] depends upon minimal electromagnetic coupling, which implies C invariance, we have assumed TP symmetry from the beginning.

[13]Simply set $D_{\mu\nu}(k)=a(k^2)k_\mu k_\nu$ in (11). The first term can be evaluated using $k_\mu k_\nu(\partial/\partial k^0)T_{\mu\nu}=2\sqrt{2}E$, which follows from (9) and (12). Then the three terms proportional to e^2 vanish.

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SPIN AND PARITY OF THE $K^*(1420)$ MESON†

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The existence of the $I=\frac{1}{2}$ $K^*(1420)$ meson has been established in many experiments.[1-8] Studies of the $K^*(1420)$ decay angular correlations[1,2,4] have ruled out $J^P=0^+$ and suggested 2^+ as the likely J^P value. However, uncertainties due to large background and/or the absence of independent information of the $K^*(1420)$ alignment do not permit one to rule out 1^- and 3^-. We report here an analysis of a relatively clean $K^*(1420)$ sample, in which we make use of the production dynamics to ascertain the resonance alignment. Although model-dependent assumptions are used, the inherent discrimination among spin-parity values is so marked that we consider the analysis sufficient to rule out 1^- conclusively, and to make 3^- unlikely.

The data discussed below come from 4.6- and 5-BeV/c K^-p interactions, obtained in an exposure of ~300 000 pictures in the Brookhaven National Laboratory 80-inch hydrogen chamber.[9] The production of the $K^*(1420)$ is observed in the two readily identifiable reactions

$$K^-+p\to\bar{K}^0+\pi^-+p \qquad (1)$$

and

$$K^-+p\to\bar{K}^0+\pi^++\pi^-+n, \qquad (2)$$

containing 1180 and 1500 events, respectively. Investigation of all two- and three-particle mass combinations reveals that there is little N^*, ρ, or $Y_0{}^*$ formation, both final states being dominated by $K^*(890)$ production. Moreover, there appear to be no kinematically induced complications[10] (e.g., Deck mechanisms) in Reaction (2). These circumstances simplify the interpretation of the mass spectra relevant to the identification of the $K^*(1420)$, i.e., the $M(K\pi)$ spectrum from Reaction (1) and the $M(K\pi\pi)$ spectrum from Reaction (2). These spectra are shown in Figs. 1(a) and 1(d), respectively. From these figures one sees that Reactions (1) and (2) contain the quasi-two-body reactions

$$K^-+p\to K^{*-}(1420)+p,\quad K^{*-}(1420)\to\bar{K}^0+\pi^- \qquad (3)$$

and

$$K^-+p\to K^{*0}(1420)+n,\quad K^{*0}(1420)\to\bar{K}^0+\pi^++\pi^- \qquad (4)$$

with about 70 $K^*(1420)$ events in each channel. The combined enhancements yield a mass and