MIGDAL'S THEORY OF NUCLEAR STRUCTURE AND PARTIAL MUON CAPTURE RATES IN O^{16}

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The partial muon-capture rates in O^{16} leading to low-lying states in N^{16} are calculated in Migdal's theory of nuclear structure, with results that agree well with the available experiments for suitable values of $C_P = m_\mu F_P/F_A$. It is thus argued that the neglect of G-parity irregular terms was not the cause of previous apparent disagreement. The capture rates are in turn employed to examine the effect of the velocity-dependent terms in Migdal's effective amplitude.

There have been several measurements[1] on the muon-capture transitions from the ground state of O^{16} (0^+, $T=0$) to $J^\pi = 0^-$, 1^-, and 2^- ($T=1$) states in N^{16}. Though the different measurements are not in satisfactory accord with each other, they provide valuable information on the weak interaction process as well as on the nuclear structure. Whereas the processes $0^+ \to 0^-$ and $\to 2^-$ depend on the induced pseudoscalar coupling constant F_P, the transitions $0^+ \to 1^-$ and 3^- do not. Thus, the latter could be used to "determine" the nuclear wave function, and the former to deduce F_P. Several theoretical calculations[2] have been attempted in that spirit, but none of them has been able to give any reasonable estimate of F_P. The theoretical rates for the transitions $0^+ \to 1^-$ and 2^- are invariably too high, the latter by a factor of 2 or 3.

The purpose of this Letter is to show that the theory of a finite Fermi system developed by Migdal,[3] which takes into account phenomenologically the residual interactions between quasiparticles, can eliminate the persistent discrepancy between the theory and the experiments. This is hardly surprising in view of the considerable success of the theory in describing many other aspects of nuclear structure, notably the magnetic dipole moments[4] and the total muon capture rates.[5]

For the $T=1$ final states (which are reached from the $T=0$ ground state by the μ capture process), the residual interaction between quasiparticles can be given in momentum space by the amplitude[6]

$$\Gamma(1,2) = V_0(\vec{\tau}_1\cdot\vec{\tau}_2) \times \sum_K (f_K' + g_K'\vec{\sigma}_1\cdot\vec{\sigma}_2) P_K(\vec{p}_1\cdot\vec{p}_2/p_F^2), \quad (1)$$

where $V_0 = d\epsilon_F/d\rho$, ϵ_F is the Fermi energy, p_F is the Fermi momentum, ρ is the nuclear matter density, and f' and g' are the dimensionless coupling constants of order unity to be extracted from various experiments. Assuming that higher harmonics decrease sufficiently fast, one may restrict to $K=0$ and 1. Equation (1) constitutes the most fundamental Ansatz in Migdal's theory, and is used below to write the rigorous equation for the transition rates.

Consider the transition from $|0^+\rangle$, the ground state of O^{16}, to a state $|S,f\rangle$ in N^{16}, where S represents the total angular momentum J and isospin T, and f represents any other necessary quantum number. We introduce the notation for a pure particle-hole configuration as $|\lambda_1\lambda_2^{-1}S\rangle$, where $|\lambda_1\rangle \equiv |n_{\lambda_1} l_{\lambda_1} j_{\lambda_1} m_{\lambda_1}\rangle$ for a particle, and $|\lambda_2^{-1}\rangle = \langle\bar{\lambda}_2|$ (the bar represents time inversion) for the hole. For an arbitrary single-particle operator t, the corresponding transition rate $\Lambda(0^+ \to f)$ can be given in terms of the residue of the "polarization" operator[4] $\mathcal{P}(\omega)$ at the pole $\omega = \omega_f$ [see Eq. (5) below], where

$$\mathcal{P}(\omega) = \sum_{\lambda_1\lambda_2} \langle 0|\tilde{t}|\lambda_1\lambda_2^{-1}S\rangle \times A_{\lambda_1\bar{\lambda}_2}(\omega)\langle\lambda_1\lambda_2^{-1}S|\mathcal{T}(\omega)|0\rangle. \quad (2)$$

Here $\tilde{t} = e(t)t$, $e(t)$ is an "effective charge" for the operator t, $A(\omega)$ is the particle-hole propagator corresponding to the pole parts of the one-particle Green's functions, and $\mathcal{T}$ is the exact vertex

$$\langle\lambda_1\lambda_2^{-1}S|\mathcal{T}(\omega)|0\rangle = \langle\lambda_1\lambda_2^{-1}S|\tilde{t}|0\rangle + \sum_{\mu_1\mu_2}\langle\lambda_1\lambda_2^{-1}S|\Gamma|\mu_1\mu_2^{-1}S\rangle \times A_{\mu_1\bar{\mu}_2}(\omega)\langle\mu_1\mu_2^{-1}S|\mathrm{T}(\omega)|0\rangle. \quad (3)$$

In terms of the pure one-particle, one-hole configuration $|S_0\rangle \equiv |\alpha_0\beta_0^{-1}S\rangle$, which would be an exact eigenstate of the Hamiltonian if there were no quasiparticle interactions,[7] the residue of $\mathcal{P}(\omega)$ is given by

$$[\mathrm{Res}\mathcal{P}(\omega)]_{\omega=\omega_f} = [1-\langle S_0|(\partial\tilde{\Gamma}/\partial\omega)_{\omega=\omega_f}|S_0\rangle]^{-1} \times |\langle S_0|\tilde{\tau}(\omega_f)|0\rangle|^2, \qquad (4)$$

where ω_f for a given S is determined from

$$\omega_f = \epsilon_{\alpha_0} - \epsilon_{\overline{\beta}_0} + \langle S_0|\tilde{\Gamma}(\omega_f)|S_0\rangle. \qquad (5)$$

$\tilde{\Gamma}$ and $\tilde{\tau}$ contain the contributions from all the configurations other than $|S_0\rangle$ and have the role of "effective" operators.[8] In matrix form, $\tilde{\Gamma} = \tilde{\Gamma} + \Gamma\tilde{A}\tilde{\Gamma}$ and $\tilde{\tau} = (1+\tilde{\Gamma}\tilde{A})t$, where $\tilde{A}$ is the same as the diagonal matrix A except that $\langle S_0|\tilde{A}|S_0\rangle = 0$. Note that $\tilde{\Gamma}$, $\tilde{A}$, and $\tilde{\tau}$ are functions of ω.

For the weak interaction process, we assume (a) the hypothesis of conserved vector current (CVC),[9] (b) the time-reversal invariance, and (c) the absence of G-parity–nonconserving terms.[10] Using the effective Hamiltonian of Fujii and Primakoff[11] with the lowest order recoil correction and defining

$$\langle S_0|\mathfrak{M}(Q)|0\rangle \equiv \langle S_0|[1+\tilde{\Gamma}(\omega_f)\tilde{A}(\omega_f)] \times [\textstyle\sum_i \tau_i^{(-)} \exp(-i\vec{\nu}_f\cdot\vec{r}_i)Q_i]|0\rangle,$$

we obtain, for the transition $|0^+\rangle \to |f\rangle$,[12]

$$\Lambda(0^+\to f) = \frac{\nu_f^2}{2\pi}|\varphi_\mu|_{\mathrm{Av}}^2\left(1+\frac{\nu_f}{AM}\right)^{-1}\left[1-\left\langle S_0\left|\left(\frac{\partial\tilde{\Gamma}}{\partial\omega}\right)_{\omega_f}\right|S_0\right\rangle\right]^{-1}$$
$$\times\sum_{M_f}\int\frac{d\hat{\nu}}{4\pi}\{G_V^2|\langle S_0|\mathfrak{M}(1)|0\rangle|^2 + G_A^2|\langle S_0|\mathfrak{M}(\vec{\sigma})|0\rangle|^2 + (G_P^2-2G_PG_A)|\langle S_0|\mathfrak{M}(\hat{\nu}\cdot\vec{\sigma})|0\rangle|^2$$
$$-2G_Vg_V\langle S_0|\mathfrak{M}(1)|0\rangle^*\langle S_0|\mathfrak{M}(\hat{\nu}\cdot\vec{p}/M)|0\rangle - 2g_A(G_A-G_P)\langle S_0|\mathfrak{M}(\hat{\nu}\cdot\vec{\sigma})|0\rangle^*\langle S_0|\mathfrak{M}(\vec{\sigma}\cdot\vec{p}/M)|0\rangle$$
$$-2G_Ag_Vi\hat{\nu}\cdot[\langle S_0|\vec{\mathfrak{M}}(\vec{\sigma})|0\rangle\times\langle S_0|\vec{\mathfrak{M}}(\vec{p}/M)|0\rangle^*]\}, \qquad (6)$$

where M is the nucleon mass, $\vec{p}$ is the nucleon momentum, φ_μ is the muon wave function,[13] and ν_f is the neutrino energy which may be written in the case of O^{16} as[14] $\nu_f = 108.03 - \omega_f$. With this definition of ν_f, ω_f corresponds to the excitation energy of the analog states $|T=1, T_0 = 0, J^-\rangle$ in O^{16} obtainable from Eq. (5) by taking ϵ_{α_0} and $\epsilon_{\overline{\beta}_0}$ to be the neutron particle and hole energies, respectively. The true transition energy ($\equiv E_f$) is obtained from ω_f by subtracting Coulomb energy and the neutron-proton mass difference.

The effective weak coupling constants G_V, G_A, G_P, g_V, and g_A are defined in the same way as done by Foldy and Walecka.[14] The relevant form factors are evaluated in accordance with the CVC.[15] For the induced pseudoscalar (PS) coupling constant, we define the ratio $C_P = m_\mu F_P/F_A$ (where F_A is the axial-vector form factor) which lies at $6 \lesssim C_P \lesssim 8$ if one takes the hypothesis of the one-pion-pole dominance.[16]

Let us now turn to the essential points of our calculation.

(1) The unperturbed one-particle, one-hole configurations and their energies needed to evaluate $\tilde{\Gamma}$ and $\tilde{\tau}$ are obtained from neighboring odd-A nuclei. This procedure in part justifies the procedure of separating the pole parts[4] of the Green's functions to obtain a simple form for $A(\omega)$. The configurations taken into account are $1d_{5/2}1p_{1/2}^{-1}$ (11.52), $2s_{1/2}1p_{1/2}^{-1}$ (12.39), $1d_{3/2}1p_{1/2}^{-1}$ (16.60), $2s_{1/2}1p_{3/2}^{-1}$ (18.55), $1d_{5/2}$-$1p_{3/2}^{-1}$ (17.68), and $1d_{3/2}1p_{3/2}^{-1}$ (22.76), where the numbers in parenthesis represent the particle-hole energies in MeV. The effect of multi-quasiparticle configurations is in principle already included in Γ and in the effective charge $e(Q)$. With this choice of configuration space, $|S_0\rangle$ introduced above corresponds to $2s_{1/2}1p_{1/2}^{-1}$ for the 0^- and 1^- states and to $1d_{5/2}1p_{1/2}^{-1}$ for the 2^- and 3^- states. The corresponding radial wave function is taken for convenience to be of harmonic oscillator type with the oscillator-length parameter $b = (M\omega)^{-1/2} = 1.75$ F consistent with electron-scattering data.[2]

Table I. Partial capture rates in 10^3 sec^{-1} for $\mu + O^{16}(0^+) \rightarrow \nu + N^{16}(J^\pi)$.

	0^-			1^-			2^-			3^-		
C_P	a	b	c	a	b	c	a	b	c	a	b	c
−12	⋯	4.00	4.08	2.36	1.89	1.84	⋯	12.1	12.2	0.182	0.109	0.115
−8	4.81	3.26	3.32				22.7	10.6	10.6			
−4	3.91	2.59	2.63				20.0	9.22	9.27			
0	3.10	1.99	2.02				17.6	8.07	8.12			
4	2.38	1.46	1.48				15.6	7.12	7.16			
6	⋯	1.23	1.24				⋯	6.72	6.75			
8	1.76	1.01	1.02				14.1	6.36	6.40			
12	1.23	0.618	0.623				12.9	5.80	5.83			
16	0.795	0.302	0.301				12.0	5.43	5.46			
d		1.1 ± 0.2			1.88 ± 0.10			6.3 ± 0.7			⋯	
e		1.6 ± 0.2			1.4 ± 0.2			⋯			⋯	

[a]Results of Gillet and Jenkins (Ref. 2) for the nuclear wave functions calculated in random phase approximation.

[b]Migdal theory with $f_0' = 0.35$, $g_0' = 0.50$, and $f_1' = g_1' = 0$.

[c]Same as (b) with $f_1' = -0.40$ and $g_1' = -0.10$.

[d]Columbia measurements (Ref. 1).

[e]Berkeley measurements (Ref. 1).

(2) The nuclear coupling constants f_0' and g_0' have been determined reliably from the magnetic moments,[4] the β-decay ft values,[17] and the total muon capture rates.[5] With the normalization $V_0 = 4\pi \times 35$ MeV F^3, one has $0.35 \lesssim f_0' \lesssim 0.40$ and $g_0' = 0.50$.[18] At this moment, there is no information available on the f_1' and g_1' terms other than the vacuum and nuclear matter estimates.

(3) The effective charge $e(Q)$ which reflects the renormalization of a "bare" operator Q in the presence of multi-quasiparticle interactions can be deduced in most cases from conservation laws.[3,4] In Eq. (6), the operators $\vec{\sigma}$ and $\vec{p}$ require $e(Q) \neq 1$. The analysis of magnetic moments[4] yields $e(\vec{\sigma}) = 0.90$. In our calculation, we assume $e(\vec{p}) \approx 1$. Though the renormalization $[e(\vec{p})-1]$ can be written in terms of effective mass M^* and $f_1 - f_1'$, where f_1 is the constant for $T = 0, k = 1$ amplitude, the latter is not known, and a rough estimate suggests that it is negligible.

The calculated capture rates are given in Table I along with the "best" results of Gillet and Jenkins.[2] They can be summarized briefly as follows:

(A) The transition $0^+ \rightarrow 0^-$. Here the f_0' and f_1' amplitudes vanish identically. Thus the nuclear interaction can be described by g_0' and g_1'. Though the transition rate is not sensitive to g_1', the transition energy E_f is and seems to require $g_1' < 0$ as can be seen from Table II. The sensitive dependence on C_P could be used to determine the PS coupling constant. Unfortunately, the large discrepancy between the two available measurements yields nonoverlapping ranges of C_P, $6 \lesssim C_P \lesssim 10$ for the Columbia data and $2 \lesssim C_P \lesssim 5$ for the Berkeley result. The upper limits of these ranges are expected to go up slightly if terms quadratic in P/M are included in the capture rate.

(B) The transition $0^- \rightarrow 1^-$. This transition is independent of C_P and hence enables us to verify the correctness of the nuclear coupling constants used in the calculation. Even if one neglects the f_1' and g_1' terms, one still obtains the rate in reasonable agreement with the Columbia datum (but larger than the Berkeley result). The transition energy E_f is, however, found to be too large compared with experiment. The role of the f_1' and g_1' terms is then

Table II. Eigenenergy E_f (the energy difference between the final states and O^{16} ground state) in MeV; $f_0' = 0.35$, $g_0' = 0.50$; $E_f = \omega_f - 2.56$, where ω_f is obtained from Eq. (5).

J^π	a	b	c	d
0^-	12.15	12.19	11.74	10.53
1^-	11.88	11.32	10.93	10.80
2^-	10.86	10.86	10.84	10.41
3^-	11.21	11.11	11.07	10.70

[a]Calculated with $f_1' = g_1' = 0$.

[b]Calculated with $f_1' = -0.40$, $g_1' = 0.01$.

[c]Calculated with $f_1' = -0.40$, $g_1' = -0.1$.

[d]Experimental values.

to lower the transition energy without much affecting the rate. This feature can be seen in Table II. This seems to indicate that the f_1' and g_1' terms, though less important in the transition rate, should be included if one wants to calculate the transition energies in agreement with experiment. The reason why Migdal and his collaborators have neglected these terms is that they have not been concerned with the transition energies in detail.

(C) The transition $0^+ \to 2^-$. Depending mainly on g_0' and C_P (and negligibly on g_1'), it provides a crucial test of the Migdal's theory, as no other nuclear models have succeeded in predicting correctly both this and the $0^+ \to 0^-$ transitions. The large suppression of the transition from the prediction of the independent particle model comes in the present calculation from two sources: (a) about 19% reduction of the dominant axial-vector matrix element [$\mathfrak{M}(\vec{\sigma})$ term] through the renormalization of the $\vec{\sigma}$ operator and (b) the effect of $T=1$ quasiparticle interaction which reduces considerably the matrix element of $\tilde{\tau}$ from that of τ. The effect (a) is entirely absent in the conventional calculations of Ref. 2. Note that the difference between Eq. (1) and the forces used in Ref. 2 can be manifested through the effect (b).

The large error in the Columbia measurements makes it difficult to narrow the C_P range. From our calculation, we find that $5 \lesssim C_P \lesssim 15$ and $30 \lesssim C_P \lesssim 38$. The former is quite compatible with the range for the $0^+ \to 0^-$ transition, but the latter seems to be ruled out.

(D) The transition $0^+ \to 3^-$. Being free of C_P, it could be used to check the f_1' and g_1' constants in conjunction with the process $0^+ \to 1^-$. A precise measurement of this transition if feasible at all would be desirable for a further check of the calculations.

In conclusion, Migdal's theory is seen to eliminate the difficulty encountered with the nuclear structure, and the ratio $6 \lesssim C_P \lesssim 10$ seems to be quite compatible with all three observed partial rates in O^{16}. It is found that the velocity-dependent Migdal amplitudes are essential to guarantee that the transition frequencies also come out correctly. It is proposed that $f_1' < 0$ and $g_1' \lesssim 0$ with their phase defined by Eq. (1). Finally, there appears to be no cause to believe that the G-parity irregular terms in the weak Hamiltonian are necessary to reconcile with the O^{16} data. The nontrivial question as to what makes the Migdal theory work well while other phenomenological theories fail will be dealt with in a future paper.

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[6]In coordinate representation Γ is given by a contact (δ function) interaction and its derivative, respectively, for the velocity-independent and velocity-dependent terms.

[7]In the wave-function language, this would correspond to the dominant component.

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is defined by Eq. (5), E_f is the true transition energy, and E_B is the binding energy of the muon in the atom. By examining 2^- states ($T=1$) in O^{16} and N^{16}, one gets $\omega f - Ef = 2.56$ MeV.

[15]The form factors in the notation of Ref. 14 are $F_1 = 0.972$, $F_2 = 3.602/2M$, and $F_A = -1.179$.

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[18]For this normalization, the radius parameter r_0 (i.e., $R = r_0 A^{1/3}$) is $r_0 = 1.25$ F. This comes out from $(1/4\pi)V_0 = 4\epsilon_F r_0^3/9 = (160/9)r_0^3$ with $\epsilon_F = 40$ MeV.

INTERCOMPARISON OF NEUTRON MONITORS DURING SOLAR-FLARE INCREASES

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The energy spectrum of a solar-flare cosmic-ray increase is usually much steeper than that of galactic cosmic rays, and McCracken[1] has pointed out the advantage of using two different absorption lengths for the two components present in a flare increase. This Letter reports a direct measurement of the absorption length of solar energetic particles detected by conventional neutron monitors.

The recent flare of 28 January 1967 has been observed by a superneutron monitor and an IGY-type monitor at Sulphur Mountain (51.20°N, 115.61°W) and by a superneutron monitor at Calgary (51.08°N, 114.09°W). These two stations, at altitudes of 2283 and 1128 m, respectively, have threshold rigidities and asymptotic cones of acceptance which are very similar so that it is possible to determine the absorption length of the solar particles directly.

We have used the relative increases at Sulphur Mountain and Calgary to determine the absorption length of the particles producing the increase with the assumption that the differences intensity observed are a function only of the atmospheric absorption between the two observing altitudes. The normal counting rates of the supermonitors are 10^6/h. Since both galactic and solar components are exponentially absorbed, we may write

$$I_s(\text{Cal}) = I_s(\text{S.M.}) \exp(-\alpha\Delta p),$$

$$I_g(\text{Cal}) = I_g(\text{S.M.}) \exp(-\beta\Delta p),$$

where I_s and I_g are the intensities of the solar and galactic particles, respectively, and α and β are the respective barometric coefficients. Δp is the pressure differential between the two stations. It is easily shown that

$$\frac{\Delta I_s(\text{S.M.})}{\Delta I_s(\text{Cal})} = \exp[(\alpha-\beta)\Delta p], \quad (1)$$

where ΔI_s, the percentage increase observed, is equal to $(I_0 - I_g)/I_g \times 100$, I_0 being the measured total intensity.

The general cosmic-ray level was relatively undisturbed before and after the flare event, and we have taken I_g to be constant during the event and equal to the average corrected counting rate during the seven hours preceding the event. Using the pressure values during the event, we have subtracted the galactic component from the total intensity at each station to obtain the flare particle intensity as a function of time (Fig. 1). The ratio of percentage increases at the two stations (using the supermonitor data) has been calculated over 20-min intervals. The average ratio $I_s(\text{S.M.})/I_s(\text{Cal})$ between 0900 and 1700 U.T. was 1.26 ± 0.02, and this ratio appears to have remained constant throughout the event. Using Eq. (1) this gives $1/\alpha$ a value of 103 ± 3 g/cm^2 for the absorption length of the solar particles, close to the 100-g/cm^2 value used by McCracken.[1] The absorption length for galactic particles at these stations is 131 g/cm^2. While the value of $1/\alpha$ may vary between events, it is clear that this method enables intercomparison of all stations independent of altitude for each event.

The increase was also measured at Sulphur Mountain with an IGY-type monitor (Fenton, Fenton, and Rose[2]). We have compared the magnitudes of the increases between the two mon-