NEW SET OF π-π PHASE SHIFTS*

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A set of s-wave phase shifts in the region of the ρ is deduced from a compilation of data on π-π scattering. Our results indicate slowly varying $I=2$ and $I=0$ s-wave phase shifts. The preferred set indicates that the $I=0$ phase shift might go through 90° in the neighborhood of 850-950 MeV.

Since very early in the study of the ρ meson, large differences have been noted in the angular distribution of the decay products of the ρ^0 and ρ^-.[1,2] The π^--π^0 from the ρ^- seem to have nearly a $\cos^2\theta$ angular distribution while the ρ^0 decays into a π^+ and π^- with an asymmetric distribution, albeit dominated by a $\cos^2\theta$ term. These features can be seen in Fig. 1. Also shown in Fig. 1 are the normalized square of the scattering amplitudes for π^+-π^- and π^--π^0. The amplitudes are obtained by first compiling the mass spectra from reactions $\pi^- + p \rightarrow \pi^- + \pi^+ + n$ and $\pi^- + p \rightarrow \pi^- + \pi^0 + p$ at energies around 2, 3, and 7 BeV from several sets of experiments.[3] The cross section is

$$\frac{d^2\sigma}{d\Delta dm^*} = Km^{*2}\sigma_{\pi-\pi} f(\Delta),$$

where $f(\Delta)$ is an empirically determined function where Δ = momentum transfer to the nucleon, K = pion momentum in the di-pion rest frame, m^* = mass of di-pion. Knowledge of $f(\Delta)$ enables us to correct for kinematic effects produced by differences in the bombarding energies and thus obtain $\sigma_{\pi-\pi}$. The quantities shown in Fig. 1 are proportional to $K^2\sigma_{\pi-\pi}$ and are normalized to be close to 3 at the peak of the ρ.

The p wave dominates both the (−0) and (+−) states in this energy region, and thus in the region of the ρ we can determine the p-wave phase shift directly from the amplitudes shown in Fig. 1. Previous investigators[4,5] have used the asymmetry ratio to estimate the mixture of s- and p-wave amplitudes in the π-π system. We have chosen instead to use the minimum point in the π-π angular distribution which has certain advantages in facilitating the analysis. We expect the interference with extraneous amplitudes to be greatest in the extreme forward and backward direction of the π-π system. We fit the angular distributions in the range $-0.8 \leq \cos\theta \leq 0.8$, for events with $\Delta^2 \leq 4m_\pi^2$. In writing the scattering amplitude we resolve the amplitudes with respect to the p-wave amplitude. By doing this we retain all of the p-wave amplitude in one term. Thus

$$|A|^2 = [\sin\delta_s \cos(\delta_p - \delta_s) + 3x \sin\delta_p]^2 + \sin^2\delta_s \sin^2(\delta_s - \delta_p)$$

where δ_s = s-wave π-π phase shift, δ_p = p-wave π-π phase shift, $x = \cos\theta_{\pi-\pi}$.

The minimum in the angular distribution occurs when $\sin\delta_s \cos(\delta_p - \delta_s) + 3x \sin\delta_p = 0$. By determining $x_{\min}$ and using our knowledge of δ_p we can determine δ_s. The above analysis can be used as is for the π^--π^0 system, where δ_s is then the $I=2$ s-wave phase shift.[4] The analysis is useful in the energy region in which the scattering is dominated by the p-wave amplitude. When one applies this method without further correction one discovers that the s-wave amplitude is greatly overestimated. The reason must have to do with the fact that the p-wave amplitude can be partially spin flipped. The spin-flipped amplitude will no longer interfere with the s wave since we average over the azimuth (Treiman-Yang) angle. We have estimated the corrections for the spin flip and absorption effects by comparing the reactions $\pi^+ + \pi^+ \rightarrow \pi^+ + \pi^+$ and $\pi^+ + \pi^0 \rightarrow \pi^+ + \pi^0$. We have compared the cross sections for the processes[5]

$$\pi^+ + p \rightarrow \pi^+ + \pi^+ + n,$$

$$\pi^+ + p \rightarrow \pi^+ + \pi^0 + p,$$

$$\pi^- + p \rightarrow \pi^- + \pi^+ + n.$$

We compute the cross sections for these processes requiring that m^* be between 650 and 850 MeV and that Δ^2 be less than $10\mu^2$. Thus we have a comparison of the $I=2$ amplitude in the region of the ρ.[6] The results of the comparison indicate an amplitude which corresponds to an $I=2$ s-wave phase shift of $-(18^{+6}_{-3})°$. We note also that the π^--π^0 angular distribution

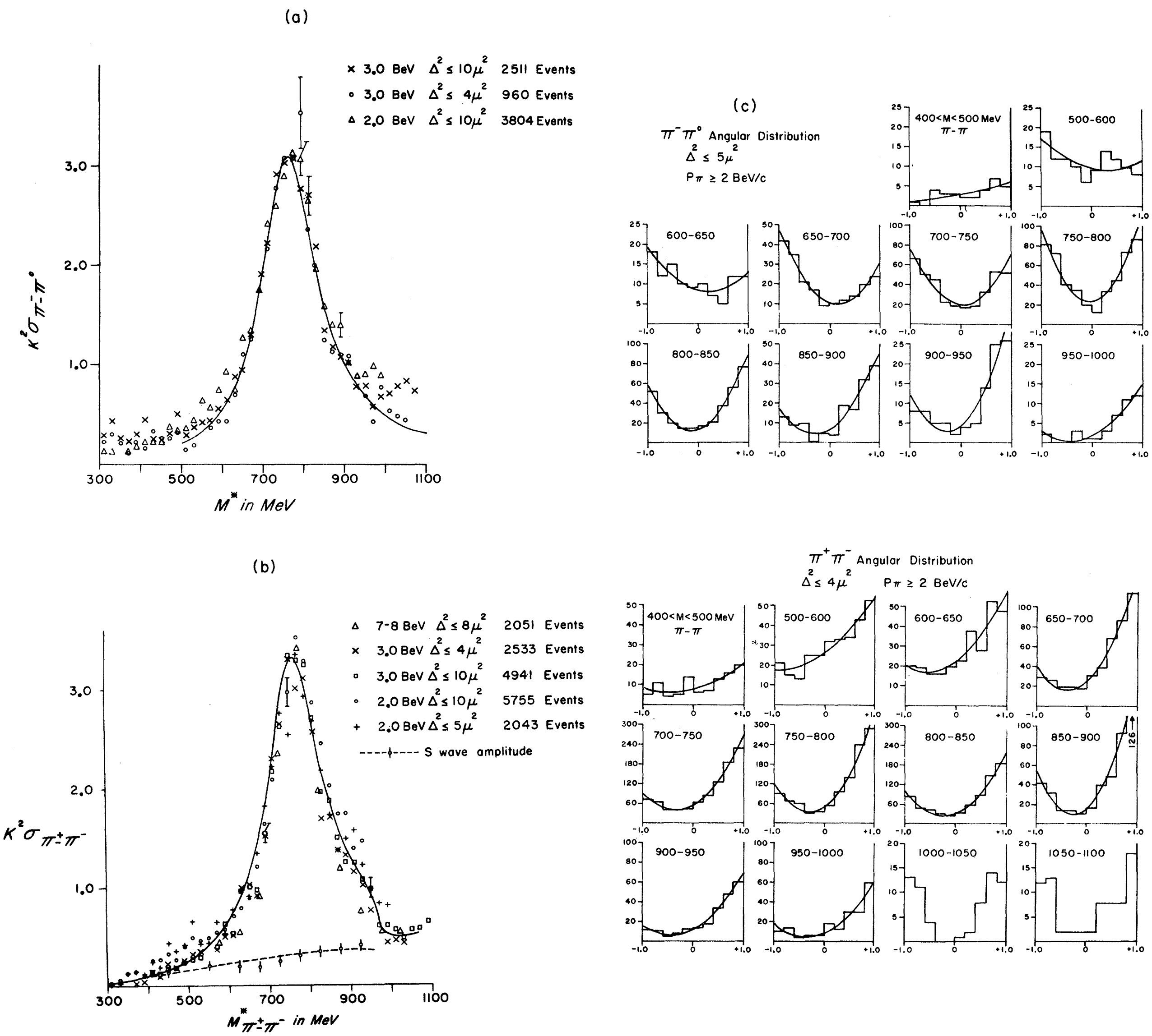

FIG. 1. (a) $K^2\sigma_{\pi^-\pi^0}$ as deduced from experiments around 2.0 and at 3.0 BeV/c. The reaction used was $\pi^-+p \rightarrow \pi^-+\pi^0+p$. The curve is the result of fitting a p-wave resonance to the amplitude. $\Gamma=160\pm10$ MeV, $m_R=760$ MeV. (b) $K^2\sigma_{\pi^+\pi^-}$ as deduced from experiments around 2.0, 3.0, and 7 BeV/c. The reaction used was $\pi^-+p\rightarrow\pi^-+\pi^++n$. The curve is the same as fitted to the π^--π^0 and added to the s-wave amplitude. The π^+-π^- mass spectrum seems to be shifted to higher mass values than π^--π^0 by 5-10 MeV. (c) The angular distributions of scattered π's in the di-pion rest frame for case of small momentum transfer to the nucleon.

becomes symmetric between 700 and 750 MeV. This means that the s- and p-wave amplitudes are 90° out of phase and from this we can deduce an $I=2$ s-wave phase shift of $-20\pm7°$ in this mass range. On the basis of the above results we deduce a correction factor of $0.66^{+0.12}_{-0.06}$ for $x_{\min}$ in the amplitude formulas. If one uses the values of the density matrix elements determined for the ρ^- production process, a correction factor of $[(\rho_{00}-\rho_{11})/(\rho_{00}+\rho_{11})]^{1/2}$ is indicated which gives a smaller correction factor (~0.85). The value of $x_{\min}$ used in our calculations is 0.8 times the value obtained in the fitting procedure.

In order to make a better determination of the π-π phase shifts it is necessary to measure the charge-exchange cross section $\pi^-+\pi^+\rightarrow\pi^0+\pi^0$. We have preliminary data on this process obtained from an experiment in which we look at the process $\pi^-+p\rightarrow\pi^0+\pi^0+\pi^-+p$. These results are consistent with the mass spectrum published by Corbett et al.[7] Our cross section relative to ρ^0 production is much larger, however, and indicates opposite signs of δ_0 and δ_2 in the mass region of 300-500 MeV.[8] Our results on the $\pi^0\pi^0$ spectrum are consistent with the s phase shifts given in Fig. 2.

The phase shifts agree quite well in the 300- to 500-MeV range with those of Jones et al.[9] They considered their results to be an upper limit on the value of the phase shift. Our analysis indicates a possible 90° phase shift for δ_0 in the energy range 850-950 MeV. There is another family of phase shifts indicated in Fig. 2 which pass rapidly through 90° at 750 MeV. We believe that our amplitudes determined from the mass spectra would rule out such a set. There are some anomalies in the mass spectra between 750 and 800 MeV but they are dependent on bombarding energy and are likely produced by the amplitude for $\omega^0 \to 2\pi$. Were the set of phases (in which the s wave leads the p wave) correct, the charge-exchange amplitude would show a sharp decrease above 750 MeV. There is no evidence for such a behavior. Our results are valid up to about 900 MeV, as our method of analysis does not include effects of d waves in the $I=0$ state (our method does account in first order for the effect of any $I=2$, d wave so far as the $I=0$, s-wave computation is concerned). Although our results might be consistent with the sort of sharply resonant δ_0 in the 700-750-MeV range that might be expected from the data of Feldman et al.[10] or proposed by Hagopian et al.[11] or Wolf,[12] we favor the slowly varying set. There is some hint of a shoulder on the high-energy side of the ρ^0 which might indicate some more rapid variation of a phase shift but this is in the 850-950-MeV mass range. If we use our fitted curve for the ρ^-, as shown in Fig. 1, we find a p-wave scattering length $a_1 = 0.14$ in units of pion Compton wavelengths cubed. This value is larger than that deduced recently by Olsson[13] from dispersion relations, or by Weinberg from current algebra.[14] Our results concerning the s-wave phase shifts are in qualitative agreement with Olsson's results.

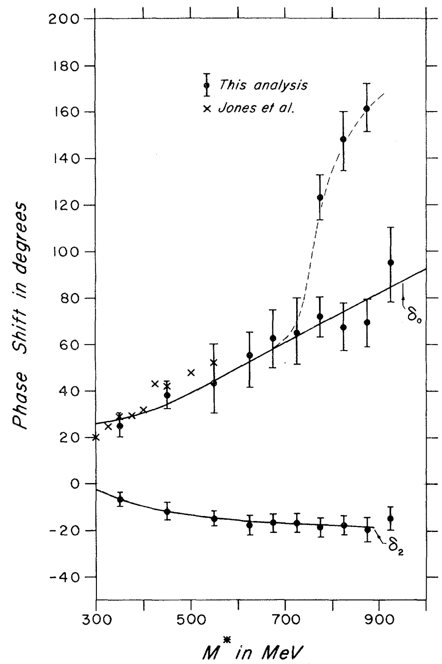

FIG. 2. The phase shift as deduced by methods described in the text. The branch of δ_0 which passes through 90° at 750 is thought to be unlikely. That set of phase shifts would show a resonance of width about 75 MeV.

We wish to thank the many groups which have very graciously cooperated in sending their data for this tabulation. In particular we wish to thank Dr. W. Selove, Dr. V. Hagopian, Dr. D. H. Miller (Purdue), and Dr. Y. Y. Lee for their cooperation. We also wish to acknowledge the help and cooperation of Dr. J. Prentice and Dr. E. West in tabulating some of the data. We also thank R. Morse and R. Raghavan for their help. We have benefited from conversations with Professor C. Goebel and Professor L. Durand.

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[2]A preliminary version of these results was presented by W. D. Walker, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (to be published).

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[4]For the (+−) system the expression is $\{[\frac{1}{3}\sin\delta_2 \times\cos(\delta_p-\delta_2)+\frac{2}{3}\sin\delta_0\cos(\delta_p-\delta_0)]+3x\sin\delta_p\}^2+[\frac{1}{3}\sin\delta_2 \times\sin(\delta_p-\delta_2)+\frac{2}{3}\sin\delta_0\sin(\delta_p-\delta_0)]^2$.

[5]In making this comparison we used the data of D. D. Carmony, D. N. Hoa, R. L. Lander, P. M. Yager, and N. Xuong, unpublished; Y. Y. Lee, thesis, University of Michigan, 1964 (unpublished); N. Armenise et al., Nuovo Cimento 37, 361 (1965).

[6]L. J. Gutay, P. B. Johnson, F. S. Loeffler, R. L. McIlwain, D. H. Miller, R. B. Willmann, and P. L. Csonka, Phys. Rev. Letters 18, 142 (1967). More recently Gutay et al. have calculated phase shifts by making an expansion $A+B\cos\theta+C\cos^2\theta$ and correcting A, B, and C for absorption effects. We have tried to estimate the corrections in two ways. Our results are in qualitative but not quantitative agreement with theirs. Their neglect of the $I=2$ s wave produces a serious error below 700 MeV and could account for a large part of the difference.

[7]I. F. Corbett, C. J. S. Damerell, N. Middlemas, D. Newton, A. B. Clegg, W. S. C. Williams, and A. S. Carroll, Nuovo Cimento 39, 979 (1965). We detect γ's from the decay of the π^0's in Pb plates in the bubble chamber. This makes it possible to discriminate easily against single π^0 production which would contaminate our sample of low π^0-π^0 masses. Our results so far as the shape of the mass spectrum are concerned are in qualitative agreement with those of Corbett et al. Our relative cross section seems to be higher than theirs. We wish to thank Dr. Arthur Clegg for a more complete manuscript of this work.

[8]In the low-mass ($m^* < 600$ MeV) range we estimate the size of the p-wave amplitude from the π-π angular distribution and then subtract this from our (amplitude)^2 shown in Fig. 1. We then use the s-wave amplitude thus deduced to estimate the $I=0$, s-wave phase shift.

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[13]M. Olsson, to be published.

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DYNAMICS OF POSITIVE-PARITY BARYON EXCITED STATES*

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We study the dynamics of positive-parity excited baryon states with a bootstrap model involving high-spin baryons. A spectrum of broken SU(2)⊗O(3) multiplets is implied by the calculation.

Several recent experiments[1] have shown that the baryon resonance structure continues to exist even above 3 BeV center-of-mass energy, leading one to suspect that the spectrum of excited states may continue indefinitely. In this Letter we present a bootstrap model to explain the physics of the excitation process for positive-parity baryons. In particular, we assert that if, as is likely, an important constituent of the excited state is one in which a virtual high-spin baryon and low-spin meson[2] coexist in a state of low orbital angular momentum, then as a consequence of the forces, an approximate SU(2)⊗O(3) multiplet structure may emerge.[3] Hence, several previously unobserved particles are predicted to exist, as, for instance, in the πN system, where the $T=\frac{1}{2}$, $N(938)$ and $T=\frac{3}{2}$, $N^*(1238)$ particles would have as first excited states $T=\frac{1}{2}$, $J^P=\frac{3}{2}^+, \frac{5}{2}^+$ and $T=\frac{3}{2}$, $J^P=\frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+$ entities. Further, we assert that high orbital angular momentum states involving low-spin baryon-meson composites produce forces which break the multiplet structure in a way completely compatible with experiment.

For the remainder of this Letter, we restrict ourselves for simplicity to the πN system, our results being rigorously extendable to SU(3).