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EXPERIMENTAL TEST OF A HIDDEN-VARIABLE QUANTUM THEORY

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The probabilistic interpretation of the quantum mechanical measurement process, although extremely successful, seems to many to be unsatisfactory as an ultimate description of the physical universe.[1] One would like to believe that if a system is prepared with sufficient care, then its behavior in any well-defined experiment is completely predictable. If the system is only partially prepared, leaving some of its significant variables unspecified, then the system's behavior will not be completely predictable. The uncertainty in the outcome of an experiment would merely reflect the uncertainty of the unspecified variables. This explanation of the probabilistic results of measurement was proved by von Neumann to be incompatible with quantum mechanics.[2] The validity of the von Neumann proof has recently been contested, and several new descriptions of the measurement process, compatible with the existence of hidden variables,[3,4] have been proposed. One of these new descriptions, the Bohm-Bub theory, is capable of being tested experimentally.

The essential features of this theory are easily understood by considering a two-state quantum system. The physical realization of a two-state system used in this experiment is the photon,[5] which can exist in one of only two independent states of polarization (or a linear combination of these two states). Any experiment that can distinguish between the two states of polarization constitutes a measurement and is performed by means of linear polarizers.[6] The parts of the Bohm-Bub theory that are needed to understand this experiment are the following:

(1) Each photon has associated with it the usual quantum mechanical wave function $\Psi = \psi_1|a_1\rangle + \psi_2|a_2\rangle$, where $|a_1\rangle$ and $|a_2\rangle$ form the basis set of states and ψ_1, ψ_2 are complex numbers. The wave function can be normalized so that $|\psi_1|^2 + |\psi_2|^2 = 1$.

(2) Each photon has associated with it a pair of complex numbers (called the hidden variables) ξ_1, ξ_2. These numbers can be made to satisfy $|\xi_1|^2 + |\xi_2|^2 = 1$.

(3) The transformation properties of ξ_1, ξ_2 are the same as those of ψ_1, ψ_2 under a unitary transformation. This ensures that the theory is independent of the choice of the basis set.

(4) The dynamical equation that connects the quantum variables ψ_1, ψ_2 with the hidden variables ξ_1, ξ_2 during the measurement process is such that the normalization conditions are maintained.

(5) The outcome of a measurement S is completely predictable if $|\psi_1|$ and $|\xi_1|$ are known. If $|a_1\rangle$, $|a_2\rangle$ are eigenstates of the operator that corresponds to the measurement S, then the measurement S will take Ψ into $|a_1\rangle$ with certainty, if just before the measurement $|\psi_1| > |\xi_1|$. If just before the measurement $|\psi_1| < |\xi_1|$, then the measurement S will take Ψ into $|a_2\rangle$. In the case of plane polarizers, the photon can emerge only in one state, say $|a_1\rangle$, whereas state $|a_2\rangle$ is completely absorbed. This means that for polarizers, $|\psi_1| > |\xi_1|$ implies transmission and $|\psi_1| < |\xi_1|$ implies no transmission. The usual quantum mechanical results are obtained if ξ_1 is assumed to be uniformly distributed in its complex space inside the unit circle.

(6) The process of measurement can and usually does change the wave function, but the hidden variables remain approximately constant during the measurement, provided that the time for measurement is sufficiently short. In an ensemble of systems the distribution of the hidden variables relaxes to the uniform distribution with a characteristic time τ. Bohm and Bub estimate that $\tau \approx h/kT \approx 10^{-13}$ sec at room tem-

perature.

The purpose of the experiment is to look for evidence of the hidden variables as suggested by Bohm and Bub. This is done by making appropriate successive measurements within times of the order of 10^{-13} sec. Failure to observe the existence of these hidden variables allows us to set an upper bound on their relaxation time.

The experiment is performed by sending photons through a stack of three linear polarizers and then measuring how the transmission varies as the final polarizer is rotated (see Fig. 1). If the transmission axes of the first two polarizers are nearly crossed, the photons that get through these two polarizers are in a well-defined quantum state and have well-defined values for their hidden variables. The final polarizer, the one that is rotated, is placed as close as possible to the second polarizer so that ξ_1 will not have had sufficient time to relax. The time can be as short as 7.5×10^{-14} sec in this experiment.

Using the eigenstates $|b_1\rangle$, $|b_2\rangle$ of polarizer B, the wave function of a photon in region I (between polarizers A and B) is

$$\Psi=\psi_1|b_1\rangle+\psi_2|b_2\rangle=\sin\epsilon|b_1\rangle+\cos\epsilon|b_2\rangle,$$

where ϵ is the angle between the transmission axes of polarizers A and B. This angle should be small, and in this experiment $\epsilon=10°$. Now any photon in region I at polarizer B with $|\psi_1|<|\xi_1|$ will have to go into state $|b_2\rangle$ and hence be absorbed by polarizer B. This means that for a photon to get through polarizer B it must satisfy $|\psi_1|>|\xi_1|$. From the instant ξ_1 is defined by polarizer B, it starts relaxing with a characteristic relaxation time τ. If ξ_1 has not had sufficient time to relax, then in region II both the wave function and the hidden variable are known, $\Psi=|b_1\rangle$ and $|\xi_1|<\sin\epsilon$. The outcome of a subsequent measurement can thus be predicted with near certainty if ϵ is small, and is best described by using the eigenstates of polarizer C,

$$|b_1\rangle=\cos\theta|c_1\rangle+\sin\theta|c_2\rangle,$$

$$|b_2\rangle=-\sin\theta|c_1\rangle+\cos\theta|c_2\rangle.$$

In this new representation, indicated by the superscript (c),

$$\psi_1^{(c)}=\cos\theta\psi_1^{(b)}+\sin\theta\psi_2^{(b)}$$

and

$$\xi_1^{(c)}=\cos\theta\xi_1^{(b)}+\sin\theta\xi_2^{(b)}.$$

Now the wave function of any photon that gets through the polarizer C is $\Psi=|c_1\rangle$, in region III. This means that in region II $|\psi_1^{(c)}|>|\xi_1^{(c)}|$. Since $\Psi_1^{(b)}=1$ and $\psi_2^{(b)}=0$, in region II the above inequality becomes

$$|\cos\theta|>|\xi_1^{(b)}\cos\theta+\xi_2^{(b)}\sin\theta|.$$

Use of the normalization condition $|\xi_1^{(b)}|^2+|\xi_2^{(b)}|^2=1$ leads to

$$\frac{1-\tan^2\theta}{4\tan\theta}>\left|\frac{\xi_1^{(b)}}{\xi_2^{(b)}}\right|\cos\alpha,$$

where α is the phase angle between $\xi_1^{(b)}$ and $\xi_2^{(b)}$. Any photon whose variables satisfy this inequality will be transmitted through polarizer C with certainty. Now,

$$|\xi_1^{(b)}/\xi_2^{(b)}|_{\max}=\tan\epsilon,\quad[\cos\alpha]_{\max}=1.$$

This means that the inequality will definitely

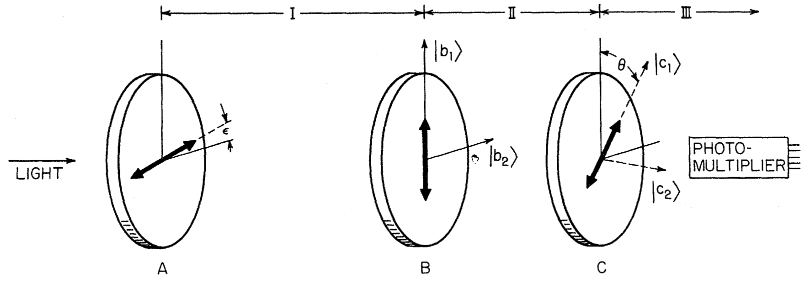

FIG. 1. Experimental arrangement of linear polarizers A, B, and C. The heavy arrows indicate the direction of polarization transmitted by each polarizer. The arrows labeled $|b_1\rangle$ and $|b_2\rangle$ indicate the direction of polarization for the eigenstates of polarizer B, and $|c_1\rangle$, $|c_2\rangle$ for the eigenstates of polarizer C. Angle $\epsilon=10°$.

be satisfied if

$$(1-\tan^2\theta)/4\tan\theta > \tan\epsilon.$$

A more direct relationship between θ and ϵ is obtained by using the identity

$$\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1-\tan\theta}{1+\tan\theta},$$

which leads to the inequality

$$\tan(\tfrac{1}{4}\pi-\theta)/[1-\tan^2(\tfrac{1}{4}\pi-\theta)] > \tan\epsilon.$$

It is now obvious that $\frac{1}{4}\pi-\theta > \epsilon$ will satisfy the inequality above.

By a similar argument it can be shown that a photon in region II will definitely not be transmitted to region III if $\theta-\frac{1}{4}\pi > \epsilon$. Summarizing, $0<\theta<\frac{1}{4}\pi-\epsilon$ implies certain transmission, while $\frac{1}{4}\pi+\epsilon<\theta<\frac{1}{2}\pi$ implies certain absorption. The transmission in the range $\frac{1}{4}\pi-\epsilon<\theta<\frac{1}{4}\pi+\epsilon$ is of no concern in the subsequent discussion but is represented by a straight line from 100 to 0% (see Fig. 2).

The data were taken by measuring the output of the photomultiplier as θ was varied in 10° steps from 0° to 90°. This was done under two conditions: one in which polarizers B and C were touching, and one in which B and C were separated by 76×10^{-4} cm. If the transmission scale in Fig. 2 is normalized to 100 units at $\theta=0°$, the measurements for each θ under the two conditions mentioned are within 1 unit of each other and within 1.5 units of the quantum-theory curve. The additional 0.5-unit discrepancy could be due to inhomogeneities in the polarizers or to a slight systematic error (less than $\frac{1}{2}°$) in the determination of the angle θ.

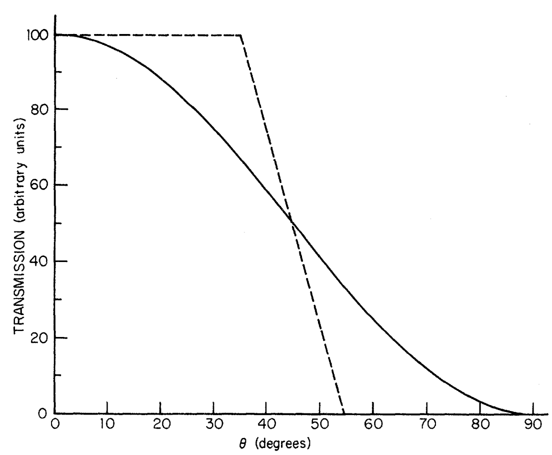

FIG. 2. The solid curve indicates transmission versus θ according to quantum mechanics and is proportional to $\cos^2\theta$. The dotted curve is that predicted by the Bohm-Bub theory for $\epsilon=10°$, assuming no relaxation of the hidden variables. The data, taken at a relaxation time of 7.5×10^{-14} sec, agree with the quantum-theory curve to within 1%.

Since there was considerable polarization dependence in the monochromator originally placed between polarizer C and the photomultiplier, the monochromator was eliminated and the data here reported were taken with white light (from a tungsten-ribbon filament lamp). The power flux density of the light used in this experiment was 10^{-10} W/cm^2, which is low enough to consider this experiment as being done with single photons.

The data taken support the usual quantum mechanical theory, but the results can be interpreted in terms of the Bohm-Bub theory by assuming a suitably short relaxation time for the hidden variables. If each point in the Bohm-Bub curve is assumed to relax exponentially to the quantum-theory curve, the measurements indicate

$$0.01 > e^{-t/\tau}(1-\cos^2\theta), \quad \theta<\tfrac{1}{4}\pi-\epsilon;$$

$$> e^{-t/\tau}\cos^2\theta, \quad \theta>\tfrac{1}{4}\pi+\epsilon;$$

where t is the transit time[7] from polarizer B to polarizer C. This inequality is strongest for the data taken at $\theta=30°$, 60° and result in

$$\tau < 2.4\times10^{-14} \text{ sec}.$$

An experiment is now in progress to set an even lower, upper bound on τ by using a thinner polarizer B. This technique can be carried to the point where there is insufficient polarization by polarizer B. It is also possible to perform a more definitive test of Bohm and Bub's choice of h/kT as the relaxation time, by repeating the experiment at lower temperatures. The lack of a theoretical understanding of this choice of τ, however, does not at this time justify cooling the apparatus to liquid air (or lower) temperatures.

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[2]J. von Neumann, The Mathematical Foundations of Quantum Mechanics (Princeton University Press, Princeton, New Jersey, 1955), pp. 295-328.

[3]Bell, Ref. 1.

[4]Bohm and Bub, Ref. 1.

[5]A good discussion of the photon as a two-state quantum system may be found in P. A. M. Dirac, The Principles of Quantum Mechanics (Oxford University Press, Oxford, 1958), 4th ed., Chap. I.

[6]The linear polarizers are "HN-32 stripable polarizers," supplied by Polaroid Land Corporation, Cambridge, Massachusetts. The 15×10^{-4}-cm-thick polarizing material was epoxied onto optical flats. The index of refraction of the polarizing sheet is 1.5, resulting in a transit time of $\sim7.5\times10^{-14}$ sec.

[7]Since the extinction coefficient of the HN-32 polarizer is about 3×10^{-5}, it can be shown that ~90% of the photons entering the sheet interact in the first 3×10^{-4} cm of the polarizing sheet. This means that the distance between polarizers B and C should be taken as the distance between their front surfaces (surfaces facing the light source). When polarizers B and C are touching, this distance is just the thickness of polarizer B; i.e., 15×10^{-4} cm.

NEW SUM RULES AND SINGULARITIES IN THE COMPLEX J PLANE

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Exact sum rules to investigate singularities in the complex angular momentum plane are obtained.

Remarkable diffraction shrinkage at high energy for the reaction $\pi^-+p\to\pi^0+n$ has been successfully explained by the Regge-pole model based on a single ρ-meson exchange.[1-3] In addition, the dip phenomena observed in the above and other reactions have been clearly explained in the same model with a vanishing helicity-flip amplitude at $\alpha=0$.[4,5] On the other hand, the single-ρ-exchange model for the above reaction predicts no polarization, which is not consistent with the observed nonzero polarization.[6] Recently, some models, including a ρ' pole[7] or a cut[8] in addition to the ρ pole, were proposed to explain the above polarization. Theoretically, it has not been definitely proved yet whether there are other singularities like a ρ' pole or a cut with the same quantum numbers as the ρ meson in addition to the ρ pole in the complex-J plane.

The purpose of this Letter is to propose a new sum rule to obtain a definite answer for the above question. We consider the πp helicity-nonflip forward scattering amplitude with charge exchange,[9]

$$f^{(-)}(\nu)\equiv(4\pi)^{-1}[A^{(-)}+\nu B^{(-)}], \qquad (1)$$

whose asymptotic behavior will be controlled by the ρ pole. Let us assume, at first, that there are no other singularities except the ρ pole in the complex-J plane for $\alpha_\rho\geq\alpha\geq-1$.[10] (No definite candidate is known among boson resonances with the same quantum numbers as those of the ρ, except on the ρ trajectory.)

Using the same technique[11] introduced by one of the authors, we separate $f^{(-)}(\nu)$ into the ρ-pole term $f_\rho(\nu)$ which behaves as ν^{α_ρ} at infinity, and the remaining term $f^{(-)\prime}(\nu)$ which vanishes faster than ν^{-1} at infinity due to our above mentioned assumption:

$$f^{(-)}(\nu)\equiv f_\rho(\nu)+f^{(-)\prime}(\nu). \qquad (2)$$

Here we define

$$f_\rho(\nu)\equiv-\beta_\rho\frac{P_{\alpha_\rho}(-\nu/\mu)-P_{\alpha_\rho}(\nu/\mu)}{2\sin\pi\alpha_\rho} \qquad (3)$$

with pion mass μ. Then, the dispersion relation for the $f^{(-)\prime}(\nu)$ is obtained as

$$f^{(-)\prime}(\nu)=\frac{g_r^2}{4\pi}\frac{\nu_B}{2m}\left(\frac{1}{\nu_B-\nu}-\frac{1}{\nu_B+\nu}\right)+\frac{1}{\pi}\int_\mu^\infty d\nu'\left(\frac{1}{\nu'-\nu}-\frac{1}{\nu'+\nu}\right)\mathrm{Im}f^{(-)\prime}(\nu'), \qquad (4)$$

$$\mathrm{Im}f^{(-)\prime}(\nu)=\frac{(\nu^2-\mu^2)^{1/2}}{4\pi}\tfrac{1}{2}[\sigma_{\pi^-p}(\nu)-\sigma_{\pi^+p}(\nu)]-\tfrac{1}{2}\beta_\rho P_{\alpha_\rho}(\nu/\mu), \qquad (5)$$

and

$$\nu_B=\mu^2/2m. \qquad (6)$$

In deriving Eq. (4), the crossing symmetry

$$\mathrm{Im}f^{(-)\prime}(\nu)=\mathrm{Im}f^{(-)\prime}(-\nu) \qquad (7)$$