

AMPLITUDE DEPENDENCE OF ULTRASONIC ATTENUATION IN SINGLE-CRYSTAL COPPER

O. M. Mracek Mitchell and G. H. Yates

Bell Telephone Laboratories, Murray Hill, New Jersey

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Amplitude-dependent ultrasonic attenuation has been observed for shear waves (10–60 MHz) propagating in the [100] direction of a 99.999%-pure copper single crystal in the temperature range 4.2 to 300°K. The amplitude dependence was found to be consistent with breakaway of dislocations from pinning centers, but no experimental evidence of an electron viscosity damping of dislocations was seen at 4.2°K for frequencies up to 60 MHz.

Amplitude dependence of the attenuation of ultrasonic waves in superconductors has been reported by several investigators^{1–5} who found negligible or reduced amplitude dependence in the normal state. A dislocation mechanism has been proposed in which the difference between the normal and superconducting states is explained in terms of electron damping of dislocations.^{2,5,6} A recent investigation,⁷ however, has shown nearly the same amplitude dependence in Pb in the normal and superconducting states.⁸ This Letter presents observations of amplitude dependence at MHz frequencies in a nonsuperconducting metal—a single crystal of Cu,⁹ both in zero magnetic field and fields up to 9 kG applied to reduce the electron viscosity. Evidence is presented for the dislocation mechanism of the amplitude dependence and for the phonon damping of the dislocation motion, but no experimental evidence of electron damping of dislocations has been found.

The dislocation mechanism which has been used to account for the amplitude dependence of the attenuation is the breakaway of dislocations from pinning points.¹⁰ This model has been used to account for the amplitude-dependent damping studied extensively at lower frequencies (below 100 kHz), but until recently not observed at MHz frequencies. The shear stress in the glide plane required to cause breakaway is

$$T_{13} \propto [1 + (\omega B L^2 / \pi^2 G b^2)^2]^{1/2}, \quad (1)$$

where G is the shear modulus in the glide plane, b is the Burgers vector, L is the dislocation loop length, ω is the frequency, and B , the damping constant, is the sum of components proportional to the phonon¹¹ and electron⁶ viscosities. It is well known that the attenuation of an ultrasonic wave by electrons can be reduced considerably by application of a magnetic field of about 10 kG.¹² Since the electronic attenuation is proportional to the electron vis-

cosity, the electron viscosity is reduced by the field and hence the electron contribution to the damping constant should also be reduced by the field. It is this magnetic field dependence of the breakaway stress which was investigated in the present experiment.

Measurements of amplitude-dependent attenuation were made by means of a double-ended pulse-echo technique in which great care was taken to eliminate nonlinear effects in the electronic circuitry. The difference in attenuation between the first and second pulses transmitted through the specimen was taken as a measure of the attenuation, while the amplitude of the first pulse was taken as a measure of the corresponding strain amplitude of the wave. This method allows a more accurate observation of the amplitude dependence than previous methods.^{1–5,7} Measurements were made for shear waves propagating in the [100] direction¹³ in a 99.999%-pure Cu single crystal with a residual resistance ratio¹⁴ of 530 (eddy-current decay method) and 380 (four-point probe method). The magnetic field dependence of the attenuation at 20 MHz at low amplitudes yielded an estimate of the electron relaxation time of 1.4×10^{-11} sec which corresponds to a residual resistance ratio of 700.¹²

Figure 1 shows the strain-amplitude dependence of the attenuation of 20-MHz shear waves in a 1.24-cm-long specimen at 4.2°K both in zero applied magnetic field ($H=0$) and in a magnetic field of 9 kG applied perpendicular to the directions of propagation and particle motion ($H=9$ kG). In the curves labeled "before irradiation" the attenuation has a strong amplitude-dependent component at high amplitudes but is essentially independent of the amplitude at low amplitudes. Note that although these measurements were made at two different output levels of the pulsed rf source, the two sets of results form smooth curves. Application of the magnetic field reduces the amplitude-

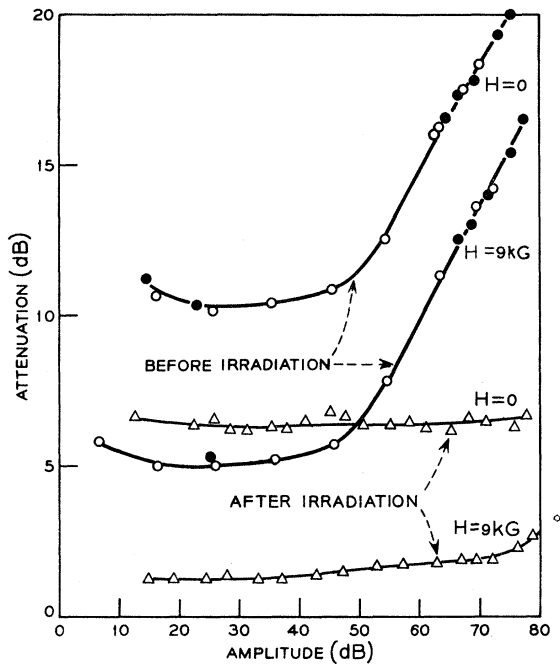


FIG. 1. The attenuation (difference in dB between the first and second transmitted pulses) of a 20-MHz shear wave in the [100] direction in Cu at 4.2°K versus the amplitude of the first pulse. Filled circles are data taken with maximum oscillator amplitude; open circles are with the rf amplitude reduced approximately 10 dB. The maximum peak to peak voltage on the 20-MHz Y-cut quartz transducer was 650 V.

independent component of the attenuation considerably but does not appear to affect the breakaway stress or the amplitude dependence. After an irradiation at room temperature of 3.6×10^7 rad in a Co^{60} source, an irradiation known to pin dislocations nearly completely,¹⁵ the amplitude dependence is eliminated, thus providing the most convincing evidence available for the dislocation mechanism proposed for this phenomenon. These results are typical of measurements made also at 10, 30, and 60 MHz at 4.2°K. The amplitude-dependent component of attenuation was found to be proportional to frequency in this range in agreement with the theory of Granato and Lücke.¹⁰

The effect of electron viscosity on the damping constant, however, appears to be negligible at the frequencies studied as seen from the lack of effect of the magnetic field both on the breakaway stress and on the component of attenuation due to low-amplitude dislocation motion (difference between the attenuation at low amplitudes before and after irradiation).

The magnetic field was observed to decrease the attenuation by approximately the amount of the electronic contribution calculated from Pippard's expression,¹⁶ i.e., the electron viscosity is eliminated by the magnetic field; hence the damping constant should also be eliminated by the field. Since no effect of magnetic field on the breakaway stress was observed for frequencies up to 60 MHz, the second term in (1) must be smaller than unity at 4.2°K. The magnitude of the dislocation loop length contained in this term can be estimated from the effect of phonon damping of the dislocation motion observed in this crystal at higher temperatures. It was found that the breakaway stress at 30 MHz was independent of temperature up to 77°K, where the phonon contribution to B is calculated¹¹ to be 1.2×10^{-4} , and increased with temperature at higher temperatures. If we neglect the small thermal activation effect,¹⁷ the second term becomes comparable with unity for this value of B , and L is then estimated to be 2.5×10^{-4} cm. An upper limit for the electron-viscosity contribution to B is then estimated to be 6×10^{-5} , which is considerably less than the value of 10^{-2} calculated⁶ for Cu with a residual resistance ratio of 500. The negligible effect of magnetic field on the low-amplitude dislocation component suggests that the electron viscosity damping at 4.2°K is considerably smaller than this estimated upper limit, probably smaller than the phonon contribution to B of 4×10^{-7} . A change in B of one order of magnitude would result in a comparable change in the low-amplitude loss.¹⁸

In summary, we have provided additional evidence that the breakaway of dislocations from pinning centers is responsible for the amplitude dependence of ultrasonic attenuation at MHz frequencies in Cu, but have found no experimental evidence of an electron viscosity damping of the vibration of dislocations for frequencies up to 60 MHz.

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¹³A pure shear wave can be propagated along the [100] direction with any particle motion in the (100) plane.

The orientation factor for dislocation motion produced by any shear wave propagating along [100] is 0.205 for copper as calculated from the equations of R. E. Green and T. Hinton, *Trans. AIME* **236**, 435 (1966).

¹⁴The residual resistance ratio is the resistivity at 300°K divided by the resistivity at 4.2°K.

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¹⁸Reference 10, Figs. 4 and 6.

EXACTLY SOLUBLE MODEL OF THE FERROELECTRIC PHASE TRANSITION IN TWO DIMENSIONS

F. Y. Wu

Department of Physics, Virginia Polytechnic Institute, Blacksburg, Virginia
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The Slater model of the two-dimensional potassium dihydrogen phosphate crystal is solved exactly under the additional assumption that the dipoles are excluded from pointing along one direction of the crystal axis. The Curie temperature T_c is not affected by this additional assumption but the phase change becomes a second-order transition. Complete polarization occurs below T_c with specific heat $\sim (T - T_c)^{-1/2}$ near and above the Curie point.

This Letter reports a model of the ferroelectric phase transition which is exactly soluble in the two-dimensional case. We fix our attention on the potassium dihydrogen phosphate (KDP) crystal, KH_2PO_4 , which undergoes a second-order phase transition at 123°K. Slater¹ was the first to point out the important role played by the hydrogen atoms in the mechanism of this phase transition. The detailed structure of the KDP crystal proposed by him allows six possible configurations for the four hydrogen atoms attached to each PO_4 group. This simplified picture permits one to construct a well-defined mathematical model for the KDP crystal by associating arrows to the lattice bonds and energies to the lattice sites.² However, in spite of the simplicity of the statement of this problem, rigorous approaches to the solution have been lacking. Most of the previous

treatments based on the Slater model and its modifications have been essentially mean-field methods yielding a first-order phase transition,²⁻⁵ while the experimentally observed transition is a second-order one. The best statistical mechanical treatment to date has been given by Nagle,⁵ who obtained both the high and low temperature expansions of the partition function and located the Curie point. These expansions, however, yield no information about the behavior of the specific heat, which is of considerable theoretical interest. It therefore seems desirable to have an exactly soluble model which can exhibit the character of the discontinuity, while serving as a model for testing the validity of other approximation procedures.

We first describe the Slater KDP model.^{1,2} Consider a diamond-type lattice (four nearest