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POLARON EFFECTS IN THE CYCLOTRON-RESONANCE ABSORPTION OF InSb

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Comparison of experimental results for cyclotron-resonance measurements above and below the Reststrahl frequency reveals anomalies attributable to polarons.

We have observed a discontinuity in the magnetic-field dependence of the cyclotron-resonance energy in InSb which can be attributed to polaron effects. Because the transitions studied in the present experiment are between conduction-band Landau levels, the polaron theory developed earlier[1] can be expected to apply directly and quantitatively to the data. In contrast, quantitative interpretation of the interband magnetoabsorption data obtained previously[2] is complicated by valence-band and exciton effects. Our present data, however, confirm the essential correctness of attributing the interband anomalies to polaron effects and constitute the first direct observation of polaron effects on Landau levels.

It has been previously suggested[3] that the experimental investigation of cyclotron resonance in polar materials for frequencies above and below the Reststrahl frequency could be expected to show characteristic self-energy effects associated with the electron–LO-phonon interaction. Our recent theoretical examination of the polaron in a magnetic field has shown that (1) the experimental cyclotron-resonance energy, $h\nu_{\mathrm{CR1}}$, which is a linear function of magnetic field at low fields, should saturate at the value of the LO phonon energy, $\hbar\omega_0$, as the magnetic field increases; (2) a second resonance, whose energy, $h\nu_{\mathrm{CR2}}$, is never less than $\hbar\omega_0$, should appear at higher magnetic fields; (3) $h\nu_{\mathrm{CR2}}$ should lie above the position expected from the extrapolation of $h\nu_{\mathrm{CR1}}$ from the low-field region.

Experimentally, we have examined the variation with magnetic field of the cyclotron-resonance energy of electrons in InSb for photon energies above and below $\hbar\omega_0$. The separation of $h\nu_{\mathrm{CR2}}$ from the extrapolation of $h\nu_{\mathrm{CR1}}$ is clearly demonstrated. In addition, our measurements are consistent with predictions (1) and (2) above. The observation of cyclotron resonance for photon energies arbitrarily close to $\hbar\omega_0$ is hindered greatly by lattice absorption. The results of the present experiment are consistent with our previous study of the corresponding energy levels using interband magnetoabsorption,[1,2] where the lattice absorption problem does not arise. The latter results show more clearly effects corresponding to (1) and (2) but contain complications due to excitons and to valence band behavior.

Consider the predictions of Fröhlich's polaron theory. The energy of a conduction electron in the presence of weak coupling to the longitudinal optical phonons and in the limit of small k can be calculated using simple perturbation theory and is given by[4]

$$E \approx \frac{\hbar^2 k^2}{2m_e}\left(1-\frac{\alpha}{6}\right)-\alpha\hbar\omega_0, \tag{1}$$

where m_e is the electron effective mass, and α is the coupling constant (defined in Ref. 4) with characterizes the strength of the interaction. Polaron effects show up as an increase in effective mass by the factor $(1+\alpha/6)$ and a downward shift of the energy levels by an amount $\alpha\hbar\omega_0$. For InSb the former effect would be quite small (increasing the band mass by $<0.5\,\%$) and $\alpha\hbar\omega_0 \approx 0.02\times 24 \approx 0.5$ meV. In the absence of much more accurate knowledge of the rigid-lattice band structure than is presently available, these effects could not be verified experimentally by accurate measurement, say, of the band gap and the effective mass a at the bottom of the conduction band.

In the presence of a magnetic field, the polaron ground-state energy $E(0)$ and the $n=1$ Landau level energy $E(1)$, for weak coupling and in the limit of low magnetic field H, are given by[5]

$$E(n) \approx (n+\tfrac{1}{2})\hbar\omega_c(1-\alpha/6)-\alpha\hbar\omega_0, \tag{2}$$

where $\hbar\omega_c$ is the cyclotron energy in the absence of electron-phonon interaction.

At low temperatures, where only the ground state is occupied with electrons, the cyclotron-resonance energy ($h\nu_{\rm CR}$) is given by the difference $E(1)-E(0)$, i.e.,

$$h\nu_{\rm CR} \approx \hbar\omega_c\left(1-\frac{\alpha}{6}\right). \tag{3}$$

To calculate $h\nu_{\rm CR}$ in the neighborhood where $\hbar\omega_c \gtrsim \hbar\omega_0$ a more sophisticated theory is necessary. Such a theory, correct to order α for weak coupled polarons in a magnetic field, has been given recently.[1,2,5] The energy for the $n=1$ Landau level is given by the implicit relation

$$E(1)=(\tfrac{3}{2})\hbar\omega_c-\frac{\alpha\hbar\omega_0}{2\pi^2}\mathrm{P}\int d^3k \times\sum_n\frac{|H_{n,1}'(\vec{k})|^2}{E(0)+n\hbar\omega_c+\hbar\omega_0+\hbar^2k_z^2/2m_e-E(1)}, \tag{4}$$

where $H_{n,1}'(\vec{k})$ is a matrix element given by

$$H_{n,1}'(\vec{k})=\frac{r_0}{(n!)^{1/2}}\frac{n-a^2k_\rho^2}{(ak_\rho)^{1-n}}\frac{\exp(-\frac{1}{2}a^2k_\rho^2)}{k} \tag{5}$$

and

$$a=\left(\frac{\hbar^2/2m_e}{\hbar\omega_c}\right)^{1/2};\; r_0=\left(\frac{\hbar^2/2m_e}{\hbar\omega_0}\right)^{1/2}. \tag{6}$$

The quantity a is the cyclotron radius for an electron in the $n=0$ Landau level, and r_0 corresponds to the polaron radius. The magnetic field, H, is in the z direction.

The numerical results of this theory for InSb are shown in Fig. 1, along with the perturbation theory results[5] for $E(0)$. For $\hbar\omega_c \lesssim 0.8\,\hbar\omega_0$, Eq. (2) is seen to be a good approximation, both $E(0)$ and $E(1)$ lying approximately

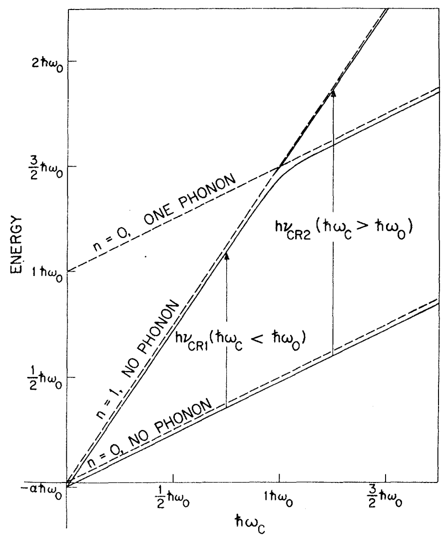

FIG. 1. Theoretical behavior of Landau levels in the presence of electron–LO-phonon interaction in InSb assuming $\alpha = 0.02$. The dashed lines are the unperturbed energy levels.

$\alpha\hbar\omega_0$ below the unpertrubed values. Note that $E(1)$ becomes double-valued for $\hbar\omega_c \approx \hbar\omega_0$ with the lower branch bending off and becoming parallel to $E(0)$ for $\hbar\omega_c > \hbar\omega_0$. For photon energies $h\nu_{\rm CR} > \hbar\omega_0$, cyclotron-resonance absorption corresponds to transitions to the upper branch of $E(1)$. This branch lies close to the unperturbed energy values, but, as seen in the figure, $h\nu_{\rm CR2}$ is increased by $\sim\alpha\hbar\omega_0$ above the value that one would obtain by extrapolating the lower branch of $E(1)$ from below $\hbar\omega_c \approx 0.8\,\hbar\omega_0$. We use the term "energy offset" or just "offset" to refer to this energy difference. We find that the theoretical offset is weakly field dependent and has a value close to, but slightly less than, $\alpha\hbar\omega_0$ over the range $\omega_0 < \omega_c < 1.5\omega_0$.

Note that if one were able to observe cyclotron resonance for energies arbitrarily close to $\hbar\omega_0$, one would observe an absorption peak associated with the lower branch of $E(1)$ whose photon energy would saturate at the value $\hbar\omega_0$ with increasing magnetic field. Electron transitions to the $n=1$ Landau level can also be observed in the interband magnetoabsorption, and such a saturation effect has previously been reported.[1,2]

The experimental results for an InSb sample, 30 μ thick, with carrier concentration of approximately 10^{14} cm^{-3} are shown in Fig. 2.

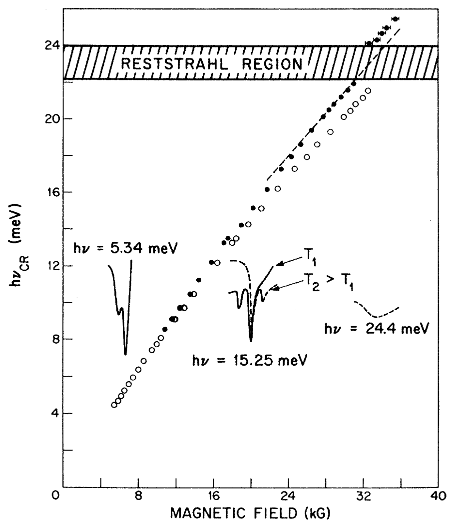

FIG. 2. Observed cyclotron-resonance energy versus magnetic field for InSb. The solid points correspond to the central component (spin-up transition) and the open points, to the high-H satellite (spin-down transition); the points are a composite of data taken at various temperatures. The insets show the sample transmission versus magnetic field observed at fixed photon energies. We estimate the temperatures T_1 and T_2 to be 15 and 60°K, respectively.

The transmission of unpolarized light was measured with the magnetic field parallel to the ⟨110⟩ crystal direction and perpendicular to the wave vector of the radiation. The sample was supported by a slotted piece of InSb in thermal contact with a copper block at liquid-helium temperature. In some cases, the sample was thermally insulated from the block with Mylar sheet. This allowed the incident room radiation to raise the sample temperature above that of the block, the actual temperature being determined by the thickness of the Mylar. We estimate that the range in temperature obtained was about 15-60°K.

The data were taken by setting at a fixed photon energy and varying the magnetic field. Son typical traces are shown as insets to Fig. 2. Over most of the region covered, three absorption lines were observed at each photon energy as shown for 15.25 meV. The relative amplitudes of these absorption lines depend strongly on temperature. Two lines were resolved at photon energies as low as 5 meV. The strong central component is the CR (cyclotron resonance) line of principal interest. The satellite at low H is due to impurities and has no connection with structure observed in the interband magnetoabsorption. The impurity transitions will be discussed in a future publication. The satellite at high H is due to cyclotron resonance involving the higher energy spin states. This line grows as the temperature increases and the $n=0$, spin-down level becomes populated.

The sample is opaque in the neighborhood of 23 meV due to Reststrahl reflection. For $h\nu > \hbar\omega_0$, the absorption lines become broadened and only one broad band is observed as shown for 24.4 meV. The broadening is apparently due to a decrease in lifetime since now the excited state can decay via optical phonon emission. The cyclotron-resonance energy is shown plotted versus magnetic field in Fig. 2. By taking measurements at various temperatures we have been able to determine that the maximum of the broad absorption plotted in Fig. 2 corresponds to the strong central component.

It is well known that the conduction band of InSb is nonparabolic. Therefore, to obtain the energy offset properly one must first fit the low-magnetic-field data to a suitable nonparabolic theory. A simple argument, however, shows that the central component, indicated by the dark circles in Fig. 2, does not obey the rigid-lattice band theory. Band theories for InSb predict that $d^2(h\nu_{CR})/dH^2 < 0$. Thus band theory requires that the five points in Fig. 2 with energy above the Reststrahl energy should lie below the linear extrapolation indicated by the dashed line in Fig. 2. Inspection of Fig. 2 shows that the high-energy points lie well above the dashed line. Such behavior is expected for polarons, as can be deduced from Fig. 1.

The foregoing analysis of the experimental results can be considerably refined and the nature of the satellite lines demonstrated. In the absence of electron-phonon interaction, the conduction-band Landau levels in InSb have been given theoretically by[6]

$$E(n) = \tfrac{1}{2}E_g\left\{-1+\left[1+\frac{4}{E_g}\left((2n+1)\frac{m}{m_e}\beta H \pm \tfrac{1}{2}g_0\beta H\right)\right]^{1/2}\right\}. \quad (7)$$

Using expressions of the form of (7), $h\nu_{CR}$ was calculated using $E_g = 236$ meV. The effective mass and g factor were varied independently to obtain a semiempirical fit to the data. A good least-squares fit can be obtained only if the data for $h\nu > 18$ meV are excluded. This indicates that the data for $h\nu > 18$ meV contain additional effects. We attribute these effects to the electron–LO-phonon interaction and use only the low-energy points to obtain a fit to the band theory. The best fit is obtained using $m_e = 0.0138\ m$. This is in good agreement with the value of $0.0139\ m$ obtained independently by Bell and Rogers[7] using the interband data of Pidgeon and Brown.[8] Our fit is not sensitive to the precise choice of g factor.

The deviations of the experimental points from the semiempirical fit to band theory[9] are shown in Fig. 3. The deviations are random and relatively small at low energies. Systematic deviations occur above and just below the Reststrahl frequency. The offset which occurs at high energies is approximately +0.4 meV, in satisfactory agreement with a value of 0.02 for α and 24 meV for $\hbar\omega_0$. Furthermore, the depression of the observed absorption energy just below the Reststrahl is consistent with the polaron bend-off predicted by theory using these values of α and $\hbar\omega_0$.

Kaplan et al.[10] have studied the coupling of plasmons and optical phonons in cyclotron-resonance transitions. Their theory, which neglects polaron effects, predicts a small offset in the observed resonance energy which very rapidly vanishes with increasing field, and a bend-off of the lower absorption branch which would not be observable at our low carrier concentration (10^{14} cm^{-3}). Therefore, we conclude that our results are not significantly influenced by plasmon effects. Polaron effects were also observed in the impurity absorption. These will be discussed in a future publication.

Figure 1 and the interband results[1,2] clearly indicate the desirability of measuring cyclotron resonance for photon energies closer to $\hbar\omega_0$. It is possible that Landau-level light scattering[11] may accomplish this. Such are experiment avoids Reststrahl effects and effects due to excitons. At present, however, improvements in sensitivity and linewidth appear to be necessary.

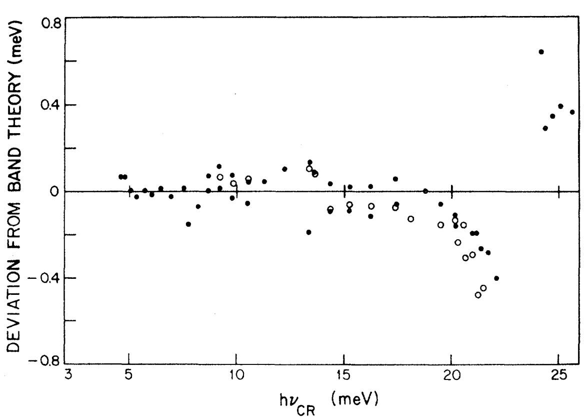

FIG. 3. Deviations of observed cyclotron-resonance energy from a least-squares fit of Eq. (7) to the low-energy data. Solid points are spin-up transitions and open points are spin-down transitions.

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