SUBSIDIARY REGGE TRAJECTORIES WITH SINGULAR RESIDUES. NUCLEON-NUCLEON SCATTERING*

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It is pointed out that the existence of secondary Regge trajectories with intercepts $\alpha(0)$ which differ by integers from that of the leading trajectory at $t=0$, and residues which are singular at that point, is a very general phenomenon which should occur both in the scattering of particles with spin, and in the scattering of particles of unequal mass. Rules are given for determining if such trajectories exist, and for determining their properties. The case of nucleon-nucleon scattering is discussed as an example.

It was recognized recently by Freedman and Wang[1] that the assumed analyticity of scattering amplitudes as functions of the energy and momentum-transfer variables s and t imposes severe restrictions on the Regge-pole description of scattering processes in which the masses of some of the external particles differ. In particular, if a t-channel Regge pole is coupled to particles of unequal mass, it must be accompanied by a set of secondary trajectories[1,2] ("daughters") with trajectory intercepts $\alpha_n(0)$ at $t=0$ spaced by integers, $\alpha_n(0)=\alpha_1(0)-n+1$, $n=1,2,\cdots$. Furthermore, the reduced residues of the secondary poles are singular at $t=0$, $\bar{\beta}_n\propto t^{1-n}$. Although the individual singularities cancel in the complete scattering amplitude, the existence of the secondary trajectories, and the cancellation mechanism, are essential in obtaining the normal $s^{\alpha(0)}$ variation of the s-channel amplitude for $s\to\infty$, $t=0$.

In the present note, we wish to point out that the existence of secondary trajectories with singular residues is a very general phenomenon which can occur, for example, in the scattering of equal-mass particles with nonzero spin.[3] Our basic results are summarized in the following rules:

(I) Sets of secondary poles with angular momenta $\alpha_n(0)$ spaced by integers at $t=0$, and singular residues, will in general appear whenever the primary Regge pole is coupled to nonconserved (tensor) currents.

(II) The amplitudes to which the secondary trajectories contribute, the spacing rules for the trajectory intercepts, and the degree of singularity of the residues may be determined by examining the behavior of the amplitude for the exchange of elementary particles of arbitrary spin as the mass of that particle approaches zero.

The existence of such secondary poles appears to have a number of important experimental consequences which we shall mention later.

For definiteness, we will restrict our discussion to nucleon-nucleon scattering. The generalizations necessary for other situations are straightforward. The nucleon-nucleon scattering matrix may be expanded in terms of Dirac covariants as

$$M=\sum_i F_i\bar{u}(p_4)O_i u(p_2)u(p_3)O_i u(p_1),$$
$$i=S,P,V,A,T,\ O_S=1,\ O_P=i\gamma_5,$$
$$O_V=i\gamma_\mu,\ O_A=i\gamma_\mu\gamma_5,\ O_T=\sigma_{\mu\nu}. \tag{1}$$

The coefficient functions F_i are known to be free of kinematic singularities,[4] and presumably satisfy the Mandelstam representation. With our conventions, the Mandelstam variables s, t, and u are defined as

$$s=-(p_1+p_2)^2,\quad t=-(p_1-p_3)^2,\quad u=4m^2-s-t. \tag{2}$$

The contributions to the F_i of the t-channel Regge poles are obtained by considering the crossed reaction $\bar{4}+2\to\bar{1}+3$, expanding M in terms of the t-channel helicity amplitudes following Jacob and Wick,[5] and performing the usual Sommerfeld-Watson transformation on the resulting partial-wave series. For simplicity, we will omit the final step, and consider only the structure of the partial-wave series; complete results will be published elsewhere. After a lengthy but straightforward reduction, it is found that the F_i can be expressed in terms of the partial-wave helicity amplitudes for states of definite parity as[4,6]

$$F_S=\sum_j(2j+1)p_t^{-2}\{a_jp_j(x)+c_jx\eta_j(x)-d_jx\pi_j'(x)-[(t+4m^2)/(2m\sqrt{t})]b_jx\tilde{\pi}_j(x)\}, \tag{3}$$

$$F_P=\sum_j(2j+1)p_t^{-2}\{(4p_t^2/t)e_jp_j(x)+d_j[(4m^2/t)\pi_j(x)+x\pi_j'(x)]$$
$$-c_j[(4m^2/t)\pi_j'(x)+(4p_t^2/t)x\eta_j(x)]+(2m/\sqrt{t})b_jx\tilde{\pi}_j(x)\}, \quad (4)$$

$$F_V=\sum_j(2j+1)p_t^{-2}\{c_j\eta_j(x)-d_j\pi_j'(x)-(2m/\sqrt{t})b_j\tilde{\pi}_j(x)\}, \quad (5)$$

$$F_A=\sum_j(2j+1)p_t^{-2}\{-c_j\pi_j'(x)+d_j\eta_j(x)\}, \quad (6)$$

$$F_T=\sum_j(2j+1)p_t^{-2}\{c_j\eta_j(x)-d_j\pi_j'(x)-(t^{1/2}/2m)b_j\tilde{\pi}_j(x)\}. \quad (7)$$

In these equations, x is the cosine of the t-channel scattering angle, $x=(u-s)/(t-4m^2)$, and p_t is the three-momentum of the particles in that system, $p_t=\frac{1}{2}(t-4m^2)^{1/2}$. The Legendre functions are defined as

$$\pi_j(x)=\tilde{\pi}_j(x)/[j(j+1)]^{1/2}=P_j'(x)/j(j+1),$$
$$\eta_j(x)=(d/dx)[x\pi_j(x)]. \quad (8)$$

The partial-wave amplitudes a_j, b_j, and c_j describe the scattering in the states with parity $(-1)^j$ [the coupled triplet states of the $\bar{N}N$ system]; the amplitudes d_j and e_j, the scattering in the states with parity $-(-1)^j$ [the uncoupled triplet and the singlet states, respectively].[7]

The analyticity of the F_i imposes some restrictions on the partial-wave amplitudes. Near $p_t=0$, a_j, b_j, and c_j have the normal threshold behavior p_t^{2j-2}, subject to the restrictions $b_j\to[j/(j+1)]^{1/2}c_j$, $a_j\to[j/(j+1)]c_j$, $p_t\to0$, while d_j and e_j vary as p_t^{2j}.[8] It is readily checked that the Reggeized amplitudes are nonsingular at $p_t=0$ if these conditions are satisfied.

The restrictions of primary interest occur at $t=0$. It is evident from the appearance of factors $t^{-1/2}$ and t^{-1} in Eqs. (3)-(5) that the partial-wave amplitudes cannot be completely arbitrary at this point. In fact, it is easily seen that b_j must be proportional to $t^{1/2}$ for $t\to0$, hence, from the factorization theorem for the Regge residues, $a_jc_j=b_j^2\propto t$ at a $t=0$ Regge pole. The remaining restriction arises from the analyticity of M_P for $t\to0$. There are two possibilities. Either (A) c_j, d_j, and e_j are all proportional to t for $t\to0$, or (B) a cancellation occurs such that the complete amplitude is regular despite singularities in the individual terms. Only the first possibility was considered by Sharp and Wagner[6] in their parametrization of the nucleon-nucleon scattering amplitude. The second possibility, considered some time ago by Volkov and Gribov,[9] was rediscovered recently by Gell-Mann and Leader,[10] and by the author. Unfortunately, the specific cancellation mechanism discussed in Ref. 9 does not appear to be correct.

To clarify the situation, it is useful to consider the structure of the partial-wave series which result from the exchange in the t channel of elementary particles of arbitrary spin j, as we expect this structure to be preserved in the Sommerfeld-Watson interpolation. This construction requires the general form of the propagator for a spin-j boson,[11]

$$P^j_{\mu\cdots\nu;\,\mu'\cdots\nu'}$$
$$=\sum_r(-1)^r2^r[(2j-2r)!/(2j)!(j-r)!]$$
$$\times\{\tilde{\delta}_{\mu\mu'}\cdots\tilde{\delta}_{\nu\nu'}\}_r(\mu^2-t)^{-1}. \quad (9)$$

In this expression,

$$\tilde{\delta}_{\lambda\lambda'}=\delta_{\lambda\lambda'}+q_\lambda q_{\lambda'}/\mu^2, \quad (10)$$

where q_λ is the four-momentum of the boson, $t=-q^2$, and μ is its mass. The bracket $\{\cdots\}_r$ is a product of j $\tilde{\delta}$ symbols, completely symmetrized with respect to either the right or left indices, after which, in r distinct pairs of $\tilde{\delta}$ symbols, the left index of one symbol is interchanged with the right index of the other, $\tilde{\delta}_{\lambda\lambda'}\tilde{\delta}_{\sigma\sigma'}\to\tilde{\delta}_{\lambda\sigma}\tilde{\delta}_{\lambda'\sigma'}$. The vertex factors for the coupling of the nucleon-antinucleon system to the particle exchanged are linear combinations of $P_\mu\cdots P_\sigma i\gamma_\lambda$ and $P_\mu\cdots P_\sigma P_\lambda$ (j factors in each term) for the coupled triplet states with parity $(-1)^j$, $P_\mu\cdots P_\sigma i\gamma_\lambda\gamma_5$ for the uncoupled triplet states with parity $-(-1)^j$, and $P_\mu\cdots P_\sigma\times P_\lambda i\gamma_5$ for the singlet states with parity $-(-1)^j$. In these expressions, $P_\mu=(p_3-\bar{p}_1)_\mu$ at the 1, 3 vertex, and $P_\mu=(p_2-\bar{p}_4)$ at the 2, 4 vertex.

The construction of the amplitudes for elementary particle exchange is now straightforward. The tensor currents for the coupled triplet states are conserved for equal-mass external particles, and the $\tilde{\delta}_{\mu\mu'}\cdots$ can therefore be replaced by $\delta_{\mu\mu'}\cdots$ in Eq. (9) in dealing with the corresponding amplitudes. When the coupled triplet amplitudes are expressed in terms of the covariants in Eq. (1), the results for the a-, b-, and c-dependent terms in Eqs. (3)-(7) are reproduced as expected. Since the reduction of the amplitudes to standard form does not introduce any factors of t^{-1} or $t^{-1/2}$, it is clear from the foregoing construction that the coupled triplet contributions to the F_i are nonsingular at $t=0$, and in particular, that $b_j \propto \sqrt{t}$ as noted previously, and $c_j \propto t$. The amplitude a_j remains finite at $t=0$, as implied by the factorization theorem.[12] A similar argument for the normal singlet amplitudes indicates that $e_j \propto t$ for $t\to 0$.

The situation is quite different for the uncoupled triplet amplitudes. The tensor currents in this case are not conserved because of the factors $i\gamma_\lambda\gamma_5$. As a result, the momentum dependent terms in the $\tilde{\delta}$ symbols cannot be neglected, and lead, in fact, to a contribution to F_P proportional to $4m^2/\mu^2$. Furthermore, the d-dependent term with the coefficient $4m^2/t$ in F_P is missing.[13] The results of Eqs. (3)-(7) are otherwise reproduced correctly, with d_j finite for $t\to 0$. The elementary exchange results have the correct analyticity properties at $t=0$. On the other hand, if d_j remains finite for $t\to 0$, F_P as given by Eq. (4) is singular at that point. The difference between the two results for a fixed j,

$$[F_P(\text{elementary})-F_P(\text{Eq. 4})]_j$$
$$=(2j+1)(4m^2/p_t^{\,2})[(t-\mu^2)/\mu^2 t]d_j\pi_j(x)$$
$$=\textstyle\sum_r (2j-4r-1)e'_{j-2r-1}P_{j-2r-1}(x), \tag{11}$$

with

$$e_{j-2r-1}'=[(2j+1)/j(j+1)][(t-\mu^2)/\mu^2 t]$$
$$\times(4m^2/p_t^{\,2})d_j, \tag{12}$$

represents the contribution to F_P of a set of singlet terms with t^{-1} singularities necessary to cancel the t^{-1} singularity in the uncoupled triplet term for $t\to 0$.

If the denominator (μ^2-t) in Eq. (9) is interpreted as the leading term in the expansion of $j-\alpha(t)$ for $\alpha(t)\sim j$, the elementary-particle-exchange amplitude for the uncoupled triplet channel may be shown to Reggeize in the usual fashion. From the foregoing discussion, we would expect each Regge pole in that channel to be accompanied by a set of secondary poles of opposite signature in the singlet channel with the following properties.[14]

(i) The angular momenta $\alpha_r(0)$ associated with the secondary poles at $t=0$ differ from the triplet intercept $\alpha(0)$ by integers,

$$\alpha_r(0)=\alpha(0)-2r-1,\quad r=0,1,\ldots. \tag{13}$$

(ii) The residues of the secondary poles diverge as t^{-1} for $t\to 0$,

$$e'(\alpha_r)\to -t^{-1}\{[2\alpha(0)+1]/\alpha(0)[\alpha(0)+1]\}$$
$$\times(4m^2/p_t^{\,2})d(\alpha(0)). \tag{14}$$

Despite the singularities in the individual terms, the complete amplitude F_P is finite at $t=0$. It appears unlikely that possibility A for insuring that F_P is nonsingular at $t=0$ is realized,[6] there being no apparent reason why d_j should vanish for $t\to 0$.

It is not clear that the secondary poles move with t: The analyticity of the scattering amplitude at $t=0$ requires only that the secondary terms appear as poles in the j plane, hence contribute along with the leading Regge poles to the Sommerfeld-Watson transformation. If the factor (μ^2-t) in the numerators of Eqs. (11) and (12) is associated as above with $j-\alpha$, the secondary terms are nonsingular at $j=\alpha$, and do not contribute as poles in the transformation. This suggests strongly that the secondary poles are fixed at the positions $j=\alpha_r(0)$ given in Eq. (13). If this interpretation is correct, the secondary poles may contribute to scattering amplitudes, but would not appear as resonances. Such a result would appear more plausible physically than the speculations in Refs. 1 and 2, as the theory would than not contain particles which could not couple to equal-mass particles or conserved currents. It is important that this question be investigated using more powerful techniques.

The generalization of the present argument to other situations is straightforward; the results for general external spins and masses are summarized in rules I and II noted above. In particular, for spinless particles of unequal mass, the present argument leads at once to the results of Freedman and Wang.[1,2]

The existence of secondary Regge trajectories with singular residues appears to have a number of important experimental consequences. An example within the context of nucleon-nucleon scattering is provided by the neutron-proton charge-exchange reaction $n+p\to p+n$. This reaction is characterized by an extremely sharp forward peak, with a width $\sim m_\pi^2$ in t. Although it is natural to ascribe this peak to pion exchange, such an explanation is not possible using the standard parametrization of the Regge-pole model[6]: The pion contribution to the scattering amplitude, and other singlet channel contributions with which it could interfere, all vanish at $t=0$. Alternative explanations in terms of ρ or A_2 exchange do not appear to be tenable. This difficulty has been a matter of some embarrassment, as simple dynamical calculations using the single-pion exchange model modified by rescattering corrections in the initial and final state give remarkably good results for the cross section in the peak region.[15] It appears, in fact, that the peak can be ascribed in the Regge-pole model to interference between the rapidly varying contribution of the normal pion Regge trajectory and less rapidly varying contributions from the singlet trajectories secondary to the A_1 trajectory. Because of the factor t^{-1} in the residues of the latter, their contribution to the scattering amplitude is nonzero at $t=0$. If $\alpha(0)<0$ for the A_1 trajectory, a result consistent with the mass of the A_1 meson and the general slope of the known Regge trajectories, the pion and the secondary A_1 terms interfere destructively for $t<0$, producing a sharp forward peak in the scattering amplitude, hence, in the charge exchange cross section. Details of this analysis will be discussed elsewhere. Important effects may also appear in such reactions as $\pi+N\to\rho+N$, $\pi+N\to\rho+N^*$, and $\bar{N}+N\to\bar{N}^*+N^*$.

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[1]D. Z. Freedman and J. M. Wang, Phys. Rev. Letters 17, 569 (1966), and to be published; D. Z. Freedman, C. E. Jones, and J. M. Wang, to be published.

[2]L. Durand, Phys. Rev. (to be published).

[3]The basic ideas of the present argument were presented for spinless particles in Ref. 2. From the present point of view, the arguments in Refs. 1 and 2 based, respectively, on the Bethe-Salpeter equation and on potential scattering are rather special, applying only to the case of unequal masses. These specific models nevertheless support the analyticity arguments of Freedman and Wang and the present paper, and contain additional information not readily obtained by the present method.

[4]M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. 120, 2250 (1960).

[5]M. Jacob and G. C. Wick, Ann. Phys. (N.Y.) 7, 404 (1959).

[6]D. H. Sharp and W. G. Wagner, Phys. Rev. 131, 2226 (1963).

[7]Our phase conventions and notation are those of L. Durand, III, and J. Sandweiss, Phys. Rev. 135, B540 (1964), Appendix B. The eigenstates of parity are defined in terms of the Jacob-Wick helicity states by

$$|jm\pm;\bar{\lambda}\lambda\rangle=(1/\sqrt{2})[|jm\bar{\lambda}\lambda\rangle\pm|jm,-\bar{\lambda},-\lambda\rangle],\quad \lambda>0,$$

for parity $\pm(-1)^j$. The partial-wave amplitudes are related to the T-matrix elements $T_j=(S_j-1)/i$ in this representation by

$$a_j=\pi(t^{1/2}/2p_t)\langle +;++|T_j|+;++\rangle,$$
$$b_j=\pi(t^{1/2}/2p_t)\langle +;++|T_j|+;+-\rangle,$$
$$c_j=\pi(t^{1/2}/2p_t)\langle +;+-|T_j|+;+-\rangle,$$
$$d_j=\pi(t^{1/2}/2p_t)\langle -;+-|T_j|-;+-\rangle,$$
$$e_j=\pi(t^{1/2}/2p_t)\langle -;++|T_j|-;++\rangle.$$

[8]The particular ratios of a_j, b_j, and c_j noted above result from the fact that the coupling through orbital angular momentum $l=j+1$ vanishes more rapidly near threshold than that which involves $l=j-1$. Thus, the three helicity amplitudes are given at threshold by the $l=j-1$ amplitude multiplied by appropriate Clebsch-Gordan coefficients. The threshold behavior of the partial-wave amplitudes assumed in Ref. 6 is incorrect.

[9]D. V. Volkov and V. N. Gribov, Zh. Eksperim. i Teor. Fiz. 44, 1068 (1961) [translation: Soviet Phys.–JETP 17, 720 (1963)].

[10]M. Gell-Mann, in Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, California, 1966 (unpublished).

[11]This result, and the details of the present calculations, will be discussed elsewhere.

[12]In order for c_j to be nonzero at $t=0$, the form factor B_j associated with $P_\mu\cdots P_\sigma i\gamma_\lambda$ must diverge as $t^{-1/2}$ for $t\to 0$. The coefficients a_j and b_j will then diverge at $t=0$ unless the form factor A_j associated with the factor $P_\mu\cdots P_\lambda$ also varies as $t^{-1/2}$ for $t\to 0$, subject to the condition $B_j+4A_j=0$, $t=0$; then $b_j\propto\sqrt{t}$, $a_j\propto t$. The requisite $t^{-1/2}$ variation of the form factors violates the normal analyticity properties of form factors in theories without zero mass particles. It may be noted also that any Regge pole which can contribute to to-

tal cross sections in the s channel must necessarily have a_j nonzero at $t=0$.

[13]This difference between the elementary-particle-exchange result for F_P and the formal result of the partial-wave decomposition, Eq. (4), arises from the fact that the latter requires that the tensor currents at the vertices be conserved (the tensor operator for angular momentum j satisfies a set of divergence conditions $\partial_\beta T_{\alpha \cdots \beta \cdots}=0$). These conditions lead to the appearance in the partial-wave series of singular terms which may be associated with unphysical zero-mass particles in the singlet channel.

[14]Although the present argument is purely heuristic, it was shown in Ref. 2 to reproduce the results of Freedman and Wang for the scattering of particles of unequal mass. The existence of secondary trajectories in that case is also associated with the nonconserved character of the tensor currents.

[15]G. A. Ringland and R. J. N. Phillips, Phys. Letters 12, 62 (1964); E. M. Henley and I. J. Muzinich, Phys. Rev. 136, B1783 (1964); L. Durand, III, and Y. T. Chiu, Phys. Rev. 137, B1530 (1965).

STRUCTURE IN THE K^--NUCLEON TOTAL CROSS SECTIONS BETWEEN 600 AND 1400 MeV/c

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The K^-p and K^-d total cross sections have been measured between 600 and 1400 MeV/c, at intervals of about 25 MeV/c, with a statistical accuracy typically of ±0.3 mb and with a momentum resolution of ±0.6%. In addition to the well-known resonances, a Y_0^* resonance with a mass of 1698 ± 5 MeV/c^2 is observed, and evidence is found for a $Y_1^*(1905)$, confirming an observation by Cool *et al.*[1]

The total cross sections have been measured by the conventional transmission technique using 55-cm-long targets of liquid hydrogen and deuterium. The targets, transmission counters, and electronics were similar to those used in a previous experiment on nucleon-nucleon total cross sections.[2]

Measurements were made in a 19-m unseparated beam, produced from an external target at Nimrod. The K^- meson flux ranged from 20 to 500 per 1.5×10^{11} extracted protons, while the total beam varied between 1.6×10^5 and 2.0×10^5. The K^- mesons were identified by a DISC[3] differential Cherenkov counter, using 5-cm thick liquid radiators and nine RCA 8575 photomultipliers in coincidence. The efficiency for K detection was over 90% and the rejection of unwanted particles was better than 2×10^{-6}. The beam contamination was thus insignificant at all momenta.

Accidental coincidences between particles in the beam were reduced to a negligible level by means of an electronic system[2] which vetoed, with efficiency better than 98%, any particle accompanied within the resolving time of the transmission counters by one or more others. Randoms arising from background particles not from the beam were also negligible.

Measurements at a given momentum consisted of several runs with each of the three targets (hydrogen, deuterium, and vacuum), and the runs were checked for consistency before averaging. Corrections made for additional decay, due to energy loss in the hydrogen and deuterium targets, varied between 4.0 mb at 600 MeV/c and 0.4 mb at 1400 MeV/c. Corrections were also made for Coulomb scattering, but no allowance has yet been made for Coulomb-nuclear interference. This correction will be small ($\lesssim$0.5 mb) and its principal effect is to decrease by 1 or 2 MeV/c^2 the apparent masses of all peaks in the total cross section.

Figures 1(a) and 1(b) show $\sigma(K^-p)$ and $\sigma(K^-d)$. For the sake of clarity, earlier results are omitted. Above 1 GeV/c, our results are in agreement with the recent accurate experiment of Cool *et al.*[1] At lower momenta the present