is taken from V. Barger and D. Cline, Phys. Rev. Letters 16, 913 (1966); and to be published. The first four resonances are chosen for N_γ and Δ_δ, and for N_α the $N(938)$, $N^*(1688)$, and a possible $\frac{5}{2}^-$ resonance mentioned in Ref. 8 are taken.

[8]C. B. Chiu and J. D. Stack, Phys. Rev. 153, 1575 (1967). Our results reproduce their $W=0$ intercept of the N_α trajectory.

[9]The N_α and N_γ trajectories do not turn around, while Δ_δ turns around at $W\approx 35$ BeV and $\alpha\approx 125$. Whether or not the trajectories turn around depends on the detailed asymptotic behavior of Imα_i. Our results for the polarization in question (where $W \lesssim 5$ BeV) are insensitive to such details.

[10]Since Imα_i is small compared with Reα_i, the imaginary part of X_i is ignored.

[11]C. B. Chiu, R. J. N. Phillips, and W. Rarita, Phys. Rev. 153, 1485 (1967).

[12]One can use more complicated expressions for $X_i(W)$ which would, among other things, take account of its threshold properties. However, experimental information on these quantities at low energies is not very good and, consequently, no useful purpose is served in increasing the number of parameters. We have implicitly assumed that, asymptotically, X_i falls off as an exponential. This need not be true in practice.

[13]This is partly due to the fact that higher partial-wave amplitudes carry larger statistical weight.

[14]The direct-channel contribution is comparable with ρ only around the dip region, where the ρ amplitude is $\sim\frac{1}{5}$ its forward value. However, ρ is a rather weak secondary trajectory because at $t=0$ it is already $\sim\frac{1}{10}$ the Pomeranchuk (P) amplitude. Therefore, compared with P the direct channel is quite small and the basic Regge hypothesis of the asymptotic dominance of the leading poles in unhampered.

EVIDENCE FOR ΛNN REPULSIVE FORCES FROM p-SHELL HYPERNUCLEI

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A strongly repulsive three-body ΛNN force is proposed for the explanation of some exceptionally large binding-energy differences in the p-shell hypernuclei. It is argued that heavy hypernuclei maintain their stability.

Λ binding energies of hypernuclei in the p shell follow a linear trend when plotted against baryon number A (Fig. 1). The over-all linear increase with A is attributed, in various approaches,[1,2] to the strong spin-independent central part of the ΛN interaction. For each p-shell nucleon, an average of about 1- to 1.5-MeV binding has been obtained.[2,3] The spin-dependent components of the ΛN interaction are expected to produce deviations of about 0.5 MeV[3,4] from the straight line shown in Fig. 1.

There are some exceptions to this general description of p-shell hypernuclei, notably at A numbers 9 and 13. Both $_\Lambda\mathrm{Be}^9$ and $_\Lambda\mathrm{C}^{13}$ lie considerably below the straight line of Fig. 1. For these two A values surprisingly large binding-energy differences are found[5-7]:

$$\Delta B_\Lambda({}_\Lambda\mathrm{Li}^9-{}_\Lambda\mathrm{Be}^9)=1.62\pm0.19\ \mathrm{MeV},$$
$$\Delta B_\Lambda({}_\Lambda\mathrm{B}^{13}-{}_\Lambda\mathrm{C}^{13})=1.6\pm0.8\ \mathrm{MeV}. \qquad (1)$$

It is to be noticed that $B_\Lambda({}_\Lambda\mathrm{C}^{13})=10.9\pm0.3$ MeV[7] is even lower than $B_\Lambda({}_\Lambda\mathrm{B}^{12})=11.06\pm0.14$ MeV,[5] in spite of the smaller A value of $_\Lambda\mathrm{B}^{12}$. The third example we wish to consider is that of the $A=8$ hypernuclei. The "exotic" $_\Lambda\mathrm{He}^8$[8] lies considerably above the straight line of Fig. 1. Here we have[5,8]

$$\Delta B_\Lambda({}_\Lambda\mathrm{He}^8-{}_\Lambda\mathrm{Li}^8)=1.0\pm0.8\ \mathrm{MeV}, \qquad (2)$$

which might appear less problematic than in the previous examples (1). However, by coupling the particularly low first excited state of Li^7 ($J_N=\frac{1}{2}^-$, 0.478 MeV) to the Li^7 ground state ($J_N=\frac{3}{2}^-$), the spin-dependent effects in $_\Lambda\mathrm{Li}^8$ may reduce the "bare" ΔB_Λ.

Some α-α-Λ models[9] for $_\Lambda\mathrm{Be}^9$ give a rearrangement energy of $\lesssim 1$ MeV due to the fact that Be^8 is unstable against α-α formation. This provides a possible explanation of the large $\Delta B_\Lambda({}_\Lambda\mathrm{Li}^9-{}_\Lambda\mathrm{Be}^9)$. However, the large rearrangement energy is not reproduced by a shell model calculation.[10] Clearly, the rearrangement arguments, when applied to the $A=8$ hypernuclei, affect $\Delta B_\Lambda({}_\Lambda\mathrm{He}^8-{}_\Lambda\mathrm{Li}^8)$ in the opposite direction, since He^7 is particle unstable. The $A=12$-13 hypernuclei are not affected by these considerations.

Here we propose that three-body ΛNN forc-

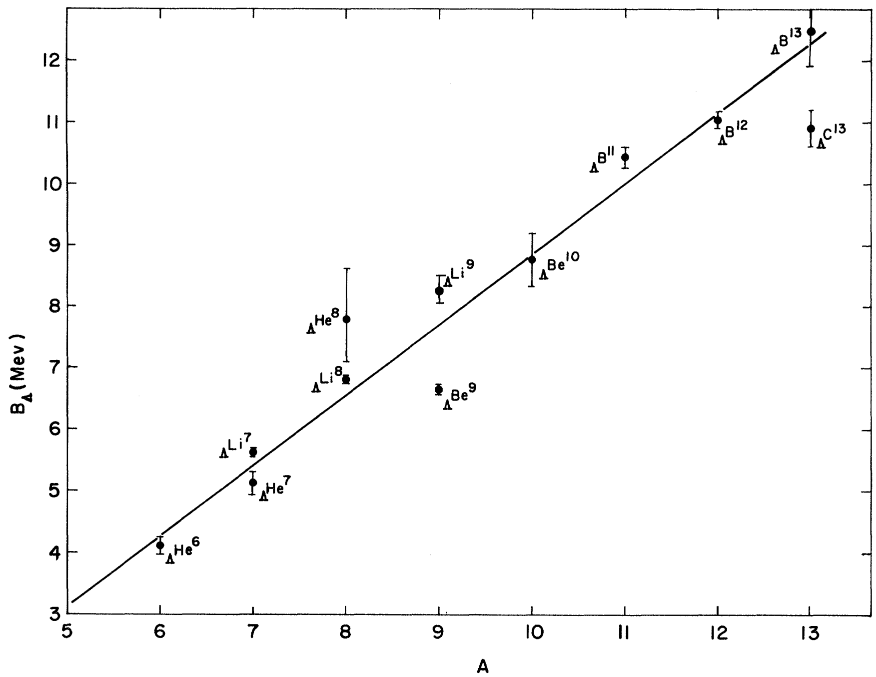

FIG. 1. Binding energies of hypernuclei in the p shell, extracted from Refs. 5, 6, 7, and 8; C. Mayeur et al., Nuovo Cimento 43A, 180 (1966); and W. Gajewski et al., Phys. Letters 23, 152 (1966).

es, effectively of the form[11]

$$\sum_{i<j}^{A-1} -\frac{W}{3}(\vec{\sigma}_i\cdot\vec{\sigma}_j)(\vec{\tau}_i\cdot\vec{\tau}_j)\varphi(r_{i\Lambda}, r_{j\Lambda}), \qquad (3)$$

are the source of the above reviewed irregularities of binding energies. Investigations of the role of such forces in the s-shell hypernuclei suggest that they are strongly repulsive ($W>0$).[12] Due to its exchange nature, the interaction (3), with $W>0$, is repulsive in the symmetric S and D states of the p^2 configuration and attractive in the antisymmetric P states. Hence, the more spatial symmetry is inherent in the nuclear core state, the more pronouncedly one expects repulsive ΛNN contributions to lower the hypernuclear binding energy.[13]

Let us consider in some detail the case $A=9$. Be^8 (the core of ${}_\Lambda Be^9$) has predominantly the maximum orbital symmetry, [4], in its ground state,[14] while Li^8 (the core of ${}_\Lambda Li^9$), because of the Pauli principle, cannot have more spatial symmetry than the [3, 1] type. The interaction (3) is capable therefore of lowering the binding energy of ${}_\Lambda Be^9$ relative to that of ${}_\Lambda Li^9$. Also for the other pairs considered by us we find a similar situation. For C^{12} the predominant symmetry is of the [4, 4] type, the maximum orbital symmetry compatible with the Pauli principle, while that of B^{12} cannot exceed [4, 3, 1]. In the $A=8$ hypernuclear case we know that Li^7 is predominantly of the [3] orbital symmetry type,[14] while the maximum orbital symmetry of He^7 is [2, 1].

These qualitative remarks may be made quantitative by computing the coefficients a_0 and a_2 of the two Slater integrals F^0 and F^2. The ΛNN contribution to the hypernuclear binding energy is then given by $a_0F^0+a_2F^2$. The contribution of the F^0 term is expected to be the more important one.[2] Some of these coefficients have been given in the literature[2] and the rest of them relevant to our purpose were evaluated by us for intermediate-coupling wave functions.[14,15] They are shown in Table I. The most marked variations of a_0 and a_2 occur at the $A=8$ and $A=9$ hypernuclei. For the $A=12$-13 case the variation is large between ${}_\Lambda B^{12}$ and ${}_\Lambda C^{13}$. A variation in a_0 of about 7 units is common to these three cases. No analogous variations are found in the energy expression because of noncentral ΛN forces. We would like, therefore, to attribute the large binding-energy differences (1) and (2) to a three-body repulsive ΛNN interaction of the type (3). After allowance has been made for spin-dependence

Table I. The coefficients with which F^0 and F^2 appear in the energy expression due to ΛNN forces of the type (3). a_0 is the coefficient of F^0 and a_2 is that of F^2.

Hypernucleus	a_0	a_2
$_\Lambda He^7$ [a]	−2.49	−0.21
$_\Lambda Li^7$ [a]	−2.73	−0.24
$_\Lambda He^8$ [a]	−2.83	−0.20
$_\Lambda Li^8$ [b]	−8.72	−0.42
$_\Lambda Li^9$ [b]	−9.84	−0.53
$_\Lambda Be^9$ [b]	−17.55	−0.87
$_\Lambda Be^{10}$ [a]	−17.55	−0.98
$_\Lambda B^{11}$ [a]	−18.31	−1.08
$_\Lambda B^{12}$ [b]	−21.8	−1.42
$_\Lambda B^{13}$ [a]	−24.5	−1.97
$_\Lambda C^{13}$ [b]	−28.9	−2.15

[a]Calculated by us.
[b]Calculated by Bodmer and Murphy, Ref. 2.

effects, we identify a mean binding-energy difference of about 1 MeV with approximately (1/7) $\times a_0F^0$.

This identification leads then to very important consequences. In $_\Lambda C^{13}$, the heaviest among the well-known p-shell hypernuclei, the ΛNN repulsion due to p-shell nucleons alone is expected to be about 4 MeV, which is not negligible at all compared to the total binding. Such a large three-body repulsion was discarded, although its connection with the binding energy differences in the p-shell hypernuclei was considered, in the very detailed analysis made by Bodmer and Murphy.[2] However, these authors put limits on the ΛNN contribution by treating the $J_N = 0$ hypernuclei, not taking into account two-body contributions arising from noncentral ΛN forces. The order of magnitude we have established here for the ΛNN interaction seems to be the same as that proposed for the s-shell hypernuclei,[12] where it leads to about 3 MeV ΛNN repulsion in $_\Lambda He^5$. These considerations will necessitate the strengthening of the triplet central ΛN interaction used in hypernuclear calculations, as indicated by Λ-p scattering.[16]

In view of the strong ΛNN interaction which we propose, it might seem that hypernuclear stability would be destroyed at an early stage, since the number of ΛNN bonds increases quadratically with A. Rather, upon closer inspection the opposite appears more probable. The expectation values of both ΛN and ΛNN interactions are expected to reach an asymptotic value for high A values. In terms of a naive shell-model picture, the Λ wave function in heavy hypernuclei extends over the nuclear core, and hence its frequency ν_Λ (for harmonic-oscillator wave function) behaves like $A^{-2/3}$, while the nuclear frequency ν decreases only as $A^{-1/3}$. The overlap between the Λ and the inner nuclear shells becomes poorer with higher A values, thus leading to an asymptotic behavior. The same conclusions may be reached, of course, also by nuclear-matter calculations. However, the effect of the exchange factor $(\vec{\sigma}_i\cdot\vec{\sigma}_j)\times(\vec{\tau}_i\cdot\vec{\tau}_j)$ in further reductions is much more pronounced in a shell-model context than in nuclear-matter calculations. In the former all three-body combinations of the form $\sum_i \Lambda N_i N_j$, where j runs over a filled LS shell and i belongs to a different shell, are strictly 0, due to the exchange factor. A rough calculation[17] indicates that, with ΛNN forces of the type (3) which are strong enough to fit the binding-energy differences discussed above, we get a total of about 10 MeV repulsion for $_\Lambda Ag^{109}$. By the same method ΛN forces, of a strength indicated by Λ-p scattering,[16] yield about 30 MeV attraction, the net being $B_\Lambda \sim 20$ MeV, which is not too far from what is observed in Ag and Br emulsions.[18] It is generally believed[18] that for such heavy systems we are quite close to asymptotic behavior. Therefore we expect hypernuclear stability not to be destroyed even for heavier systems.

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[1]R. Lawson and M. Rotenberg, Nuovo Cimento 17, 449 (1960).
[2]A. R. Bodmer and J. W. Murphy, Nucl. Phys. 64, 593 (1965).
[3]R. H. Dalitz, in Proceedings of the International School of Physics "Enrico Fermi," Varenna Lectures, 1966 (to be published).
[4]R. H. Dalitz and A. Gal, Nucl. Phys. 1B, 1 (1967).
[5]J. Sacton, in Proceedings of the International School of Physics "Enrico Fermi," Varenna Lectures, 1966 (to be published).
[6]S. Mora, Phys. Letters 20, 89 (1966).
[7]A. Z. M. Ismail et al., Nuovo Cimento 28, 219 (1963).
[8]J. Lemonne et al., Phys. Letters 11, 342 (1964); C. Détraz, J. Cerny, and R. H. Pehl, Phys. Rev. Letters 14, 708 (1965). Our considerations bear the same results even if the identified event of Lemonne et al.

turns out to be $_\Lambda He^9$ instead of $_\Lambda He^8$.

[9]A. R. Bodmer and S. Ali, Nucl. Phys. 56, 657 (1964); Y. C. Tang and R. C. Herndon, Phys. Rev. 138, B637 (1965).

[10]R. Lawson and J. M. Soper, private communication.

[11]For a general discussion of this particular form see R. H. Dalitz, Nuclear Interactions of the Hyperons (Oxford University Press, New York, 1965), pp. 77-80, as well as the review of Ref. 3.

[12]A. Gal, Phys. Rev. 152, 975 (1966), where a value of $W \sim 17$ MeV is found for $\varphi(r_{i\Lambda}, r_{j\Lambda}) = (e^{-\mu r_{i\Lambda}}/\mu r_{i\Lambda}) \times (e^{-\mu r_{j\Lambda}}/\mu r_{j\Lambda})$, μ^{-1} being the pion Compton wavelength, by requiring $a_s \sim a_t \sim -2$ F as indicated by Λ-p low-energy scattering. The theoretical considerations of Ref. 11 give rather weakly repulsive force of strength $W \sim 2$ MeV.

[13]This had already been observed in Ref. 2.

[14]F. C. Barker, Nucl. Phys. 83, 418 (1966).

[15]J. M. Soper, private communication.

[16]G. Alexander et al., Phys. Letters 19, 715 (1966).

[17]A. Gal, unpublished.

[18]D. P. Goyal, Nucl. Phys. 83, 639 (1966).

π^+ PHOTOPRODUCTION BETWEEN 1.2 AND 3 GeV AT VERY SMALL ANGLES

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The reaction $\gamma + p \rightarrow \pi^+ + n$ has been investigated for photon energies between 1.2 and 3 GeV and pion c.m. angles from 2.5 to 15°. The cross section is strongly peaked in the forward direction and shows resonance structure in the region of the $N_{3/2}^*(1920)$ and $N_{1/2}^*(2190)$.

We have measured the photoproduction of single positive pions from hydrogen at angles ranging from 1° to 6° in the lab. A modified version of the magnetic spectrometer and scintillation counter-hodoscope system described earlier[1] was used. In order to reject the large electromagnetic background produced at small angles, the following changes in the apparatus[1] had to be made:

(i) The counter hodoscope $H1$ which previously measured the production angle (θ) of particles passing through the spectrometer was replaced by a collimator of variable width and height at the first angular focus in the horizontal plane ($H1$). The collimator size was chosen to optimize solid-angle acceptance while maintaining an angular resolution $\Delta\theta = \pm 4.5$ mrad at all production angles except for the one-degree measurements, where $\Delta\theta = \pm 2.5$ mrad was chosen to allow a more detailed investigation of the expected rapid variation of the cross section with angle.

(ii) Five scintillation counters, S_1 to S_5, were used to define the geometry of the beam. They were all placed behind the magnetic system to limit the highest instantaneous singles counting rate to $\lesssim 1$ Mc/sec.

(iii) A threshold gas Cherenkov counter[2] (C_e) of 25 cm diam and 2.40-m radiator length filled with ethylene at 1.2 atm was placed between S_1 and S_2 to detect positrons passing through the spectrometer. Its efficiency was $(99.93 \pm 0.03)\%$.

(iv) A second threshold gas Cherenkov counter (C_π) of the same diameter, but 3.40 m long, located between S_2 and S_3 and filled with ethylene at 3.5 atm, detected pions with momentum $p_\pi > 2.1$ GeV/c with an efficiency $\epsilon_\pi > 99\%$.

(v) The time of flight of particles was measured between counters S_1 and S_4 (7.7 m distance) with a resolution of 1.3-nsec full width at half-maximum, permitting the separation of pions from protons below 2.1 GeV/c, where the C_π counter becomes inefficient.

An event was defined as the passage of a charged particle other than a positron through the spectrometer. Its occurrence was indicated by an anticoincidence ($G\overline{C}_e$) of C_e with $G = (S_1S_2S_3S_4S_5)$, the geometry-defining coincidence between all trigger counters. Among these events, pions were distinguished from protons either by a coincidence of $G\overline{C}_e$ with C_π or by using the time-of-flight information, depending on momentum.

The number of positrons not rejected by $G\overline{C}_e$ because of the inefficiency of C_e contributed less than 1% to the pion rate except at $\theta_\pi^{\text{lab}} = 1°$, $E_\gamma = 1.37$ GeV, where it contributed 2%. Muons from pair production contribute a neg-