πN CHARGE-EXCHANGE POLARIZATION AND THE BARYON TRAJECTORIES IN THE DIRECT CHANNEL*

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It is predicted that baryon trajectories continue to rise for quite large energies. The ρ Regge amplitude, along with the direct-channel contribution written in the Khuri representation, gives excellent fits to the polarization and the differential cross-section data, even though the elasticity factors involved are quite small. At a given energy W, the direct-channel contribution is predominantly imaginary and comes mainly from partial waves in the neighborhood of $J \simeq \mathrm{Re}\alpha(W)$.

Polarization in πN charge-exchange scattering at 5.9 and 11.2 BeV/c has been found to be non-negligible (~15%) with a very slow variation as a function of energy.[1] Since only the ρ meson can be exchanged in this reaction, a simple Regge-pole model would predict an identically zero polarization. This is so because the spin-flip and the spin-nonflip amplitudes have the same signature factors and, therefore, the same phase. Recent attempts to explain the polarization in terms of the interference between a finite number of known low-energy direct-channel resonances and the ρ-exchange amplitude, even though successful at lower energies, predict much smaller polarization at the energies in question.[2] Regge cuts[3] and an additional hypothetical trajectory, ρ', with ρ quantum numbers[4] have been recently proposed as possible mechanisms to explain the nonvanishing polarization.

We would like to point out that if resonances in the direct channel continue to occur at high energies, then it is possible to explain the magnitude and the energy dependence of the polarization (as well as the differential cross section[5]) even though in the direct channel the elasticity factors involved are quite small.

On the basis of dispersion relations and the knowledge of the low-energy resonance parameters, we find that the baryon trajectories continue to rise for quite large energies, including those energies which are of interest.[6] The following relations are used (where W is the total center-of-mass energy and i stands for the N_α, N_γ, and Δ_δ trajectories):

$$\alpha_i(W) = a_i + b_i W + \frac{W^2}{\pi}\int_{(m+m_\pi)}^{\infty} dW' \frac{\mathrm{Im}\alpha_i(W')}{W'^2(W'-W)}, \qquad (1)$$

$$\mathrm{Im}\alpha_i(W) = (W-W_0)^{\epsilon_i}\frac{P_i(W)}{Q_i(W)}, \qquad (2)$$

where ϵ_i is the threshold power and W_0 the threshold energy. The quantities P_i and Q_i are polynomials in W. The parameters are fixed on the basis of the known low-energy resonances and it is found that no more than quadratic polynomials are needed.[7,8] It is also found that at least two subtractions are needed, because $\mathrm{Im}\alpha_i$ along with only one subtraction is unable, even at low energies to sustain the rising nature of $\mathrm{Re}\alpha_i$.[9] Note that the subtraction terms partly approximate the contribution of opposite-parity amplitudes which we have ignored because, except for N_α, no resonances in these amplitudes are found. Furthermore, since we have already made two subtractions, the behavior of $\mathrm{Im}\alpha_i$ at infinity plays an insignificant role at the energies in question. The $\mathrm{Re}\alpha_i$'s are plotted in Fig. 1(a). We finally note that no arbitrary parameters are needed in the above expressions.

The elasticity parameters $X_{\alpha_i(W)}$ $[=X_i(W)]$ play an important role in determining the direct-channel contribution.[10] The values of X_i are not known at high energies, and even at low energies they are generally poorly determined. Therefore, at 5.9 and 11.2 BeV/c we shall take X_i's as parameters to be determined from both the polarization and the charge-exchange differential cross-section data. If our basic idea makes any sense, then the X_i's at high energies must be quite small and smoothly connected to the low-energy values. We find, in fact, that this is so.

The direct-channel contribution is written in terms of the Khuri representation. In this representation a partial wave is given by

$$\frac{\beta_i(W)}{J-\alpha_i(W)}\exp[-(J-\alpha_i)\xi], \qquad (3)$$

where $\beta_i(W)$ is the residue. The factor ξ is taken to be the same for all these trajectories.

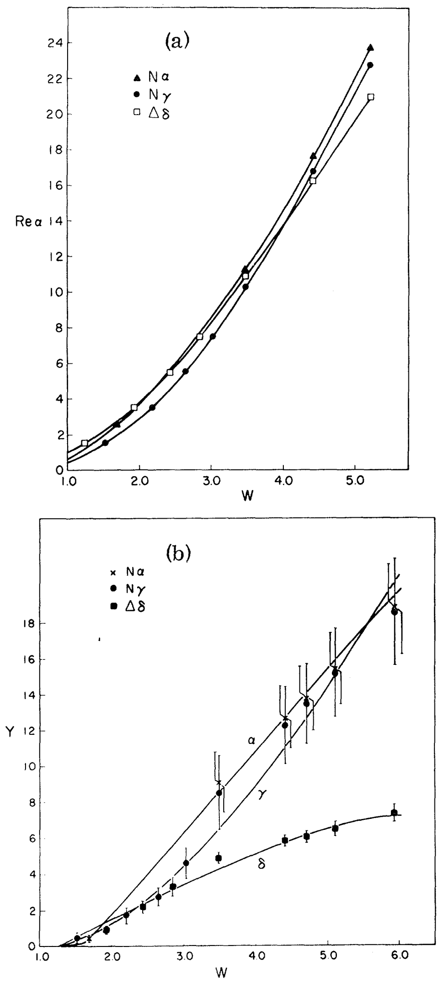

FIG. 1. (a) Reα versus total center-of-mass energy W (BeV). The last three points are the values obtained from Eqs. (1) and (2), while the others are experimental values at known low energy resonances. (b) Y versus total center-of-mass energy W (BeV). The last five points are obtained from polarization fits (first and third points, 5.9 and 11.2 BeV/c respectively) and differential cross-section fits (first, second, fourth, and fifth points, 5.9, 9.8, 13.3, and 18.2, respectively). The remaining points are from experimental data at known low-energy resonances. The over-all χ^2 is quite insensitive to substantial changes in Y for N_α and N_γ.

On the basis of the nonrelativistic theory, ζ has the form

$$\zeta = m_0/W \tag{4}$$

for large values of W. Here m_0 is the interaction radius in the πN system. It is known from crude considerations that $m_0 = (m_\pi m)^{1/2}$ (= 0.36 BeV). We shall fix m_0 at this value. In order that the above partial wave represent the appropriate Breit-Wigner form at a resonance, the residue is taken to be

$$|\beta_i| = K_i(W)\,\mathrm{Im}\,\alpha_i(W), \tag{5}$$

where $K_i(W)$ represents appropriate kinematic factors.[10] The phase of β_i is assumed to cancel the phase of the Khuri factor (which actually turns out to be very small anyway). The total direct-channel amplitude is given as an infinite sum over the partial waves with $\Delta J = 2$ together with a sum over the three possible values of the index i.

With the above prescription for calculating the direct channel amplitude $f^D(W,t)$ (where t is the momentum-transfer variable) and the conventional prescription for writing the ρ-exchange amplitude $f^\rho(W,t)$,[11] we assume the total amplitude to be given by the simple sum

$$f(W,t) = f^D(W,t) + f^\rho(W,t). \tag{6}$$

It is found that at a given W, the major contribution to the direct-channel amplitude $f^D(W, t)$ comes from partial waves in the neighborhood of $J \simeq \mathrm{Re}\,\alpha_i(W)$. This is a rather fortunate circumstance because it means that our results should not depend sensitively on the nature of ζ and the choice of m_0 as long as the latter is not very small ($\ll$ 0.36 BeV). It turns out that the real part is small because it gets contribution from above and below the resonance with opposite signs. The imaginary parts, on the other hand, are always positive. Therefore, the direct-channel contribution is predominantly imaginary. Since the contribution comes mainly from partial waves that are resonating or nearly resonating, the danger of double counting involved in (6) is minimized. Thus in our model those partial waves that are not resonating have in any case very little effect, and to those that are resonating, ρ serves as a background.

The fits to the polarization and scattering data are given in Figs. 2(a) and 2(b), and are excellent. In the forward direction, the direct-channel contribution is small compared with ρ but becomes comparable with ρ in the dip region. The X_i's are quite consistent with the extrapolated values from the low-energy data. For extrapolation purposes the following function consistent with an exponential asymptotic behavior is used[12]:

$$X_i(W) = \exp[-Y_i(W)], \tag{7}$$

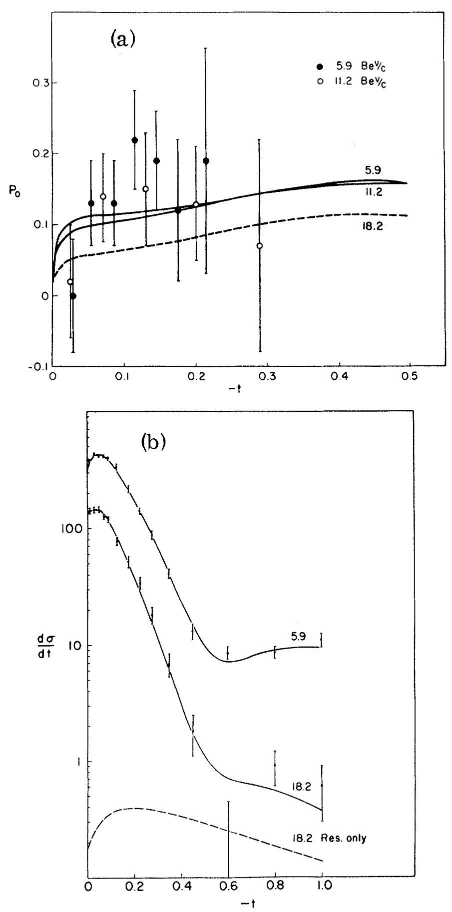

FIG. 2 (a) Polarization parameter P_0 versus momentum transfer variable $-t$ $(\mathrm{BeV}/c)^2$. Experimental points are from Ref. 1. The open circles are 11.2 BeV/c and the closed circles at 5.9 BeV/c. The solid curves are the best fits at 5.9 and 11.2 BeV/c while the dashed curve is the prediction at 18.2 BeV/c. (b) Charge-exchange differential cross section $d\sigma/dt$ μb $(\mathrm{BeV}/c)^2$ versus momentum transfer variable $-t$ $(\mathrm{BeV}/c)^2$. Experimental points are from Ref. 5. Data at 5.9, 9.8, 13.3, and 18.2 BeV/c were used but only the 5.9- and 18.2-BeV/c data are shown along with their corresponding best fit curves. The dashed curve is the direct-channel contribution at 18.2 BeV/c.

$$Y_i(W)=\frac{A_i(W-W_1)+B_i(W-W_1)^2}{1+C_iW}, \qquad (8)$$

where W_1 is the inelastic threshold energy. The Y_i's are plotted in Fig. 1(b). The X_i for Δ_δ is the largest among the three and is the least rapidly varying. It plays an important role in explaining the magnitude and the energy dependence of the polarization. The ρ parameters are given in BeV units: $\alpha_\rho(0)=0.58$, $\alpha_\rho'(0)=1.09$, $C_0=1.50$, $C_1=0.26$, $C_2=11.4$, $D_0=25.8$, $D_1=0.01$. Most of the parameters remain unchanged from those of Chiu, Phillips, and Rarita[11] and, therefore, the total cross-section fits are unaffected.

There are two important points to be emphasized in connection with our model. Firstly, baryon resonances should exist also at higher energies.[6] The contribution to the direct channel at a given energy comes from a resonance at that energy and its nearby neighbors, unlike the earlier models[2] where it was the tail end of a lower energy resonance that contributed (predominantly to the real part). Thus our model avoids the questionable assumption made in earlier models of using a Breit-Wigner form far away from a resonance.[2] Secondly, even though the X_i's are small, the direct-channel contribution is non-negligible.[13,14] It is entirely possible that the X_i's fall off as a power at high energies.[12] If this power is compatible with that given by the ρ contribution, then the polarization may, in fact, remain essentially unchanged above a certain energy. Our prediction at 18 BeV/c on the basis of (7) and (8) is given in Fig. 2(a). Experiments at this energy should shed further light on this subject. Meanwhile, it would be most interesting to investigate theoretically the asymptotic energy dependence of the elasticity factors and the question of a proper Regge representation and, generally, to explore the avenues of the kind proposed here.

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[6]The possibility that the trajectories may, in fact, rise indefinitely has been pointed out by S. Mandelstam (unpublished). This possibility is also consistent with certain versions of the quark model.

[7]The information regarding the resonance parameters

is taken from V. Barger and D. Cline, Phys. Rev. Letters 16, 913 (1966); and to be published. The first four resonances are chosen for N_γ and Δ_δ, and for N_α the $N(938)$, $N^*(1688)$, and a possible $\frac{5}{2}^-$ resonance mentioned in Ref. 8 are taken.

[8]C. B. Chiu and J. D. Stack, Phys. Rev. 153, 1575 (1967). Our results reproduce their $W=0$ intercept of the N_α trajectory.

[9]The N_α and N_γ trajectories do not turn around, while Δ_δ turns around at $W\approx 35$ BeV and $\alpha\approx 125$. Whether or not the trajectories turn around depends on the detailed asymptotic behavior of $\mathrm{Im}\alpha_i$. Our results for the polarization in question (where $W \lesssim 5$ BeV) are insensitive to such details.

[10]Since $\mathrm{Im}\alpha_i$ is small compared with $\mathrm{Re}\alpha_i$, the imaginary part of X_i is ignored.

[11]C. B. Chiu, R. J. N. Phillips, and W. Rarita, Phys. Rev. 153, 1485 (1967).

[12]One can use more complicated expressions for $X_i(W)$ which would, among other things, take account of its threshold properties. However, experimental information on these quantities at low energies is not very good and, consequently, no useful purpose is served in increasing the number of parameters. We have implicitly assumed that, asymptotically, X_i falls off as an exponential. This need not be true in practice.

[13]This is partly due to the fact that higher partial-wave amplitudes carry larger statistical weight.

[14]The direct-channel contribution is comparable with ρ only around the dip region, where the ρ amplitude is $\sim\frac{1}{5}$ its forward value. However, ρ is a rather weak secondary trajectory because at $t=0$ it is already $\sim\frac{1}{10}$ the Pomeranchuk (P) amplitude. Therefore, compared with P the direct channel is quite small and the basic Regge hypothesis of the asymptotic dominance of the leading poles in unhampered.

EVIDENCE FOR ΛNN REPULSIVE FORCES FROM p-SHELL HYPERNUCLEI

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A strongly repulsive three-body ΛNN force is proposed for the explanation of some exceptionally large binding-energy differences in the p-shell hypernuclei. It is argued that heavy hypernuclei maintain their stability.

Λ binding energies of hypernuclei in the p shell follow a linear trend when plotted against baryon number A (Fig. 1). The over-all linear increase with A is attributed, in various approaches,[1,2] to the strong spin-independent central part of the ΛN interaction. For each p-shell nucleon, an average of about 1- to 1.5-MeV binding has been obtained.[2,3] The spin-dependent components of the ΛN interaction are expected to produce deviations of about 0.5 MeV[3,4] from the straight line shown in Fig. 1.

There are some exceptions to this general description of p-shell hypernuclei, notably at A numbers 9 and 13. Both ${}_\Lambda\mathrm{Be}^9$ and ${}_\Lambda\mathrm{C}^{13}$ lie considerably below the straight line of Fig. 1. For these two A values surprisingly large binding-energy differences are found[5-7]:

$$\Delta B_\Lambda({}_\Lambda\mathrm{Li}^9 - {}_\Lambda\mathrm{Be}^9) = 1.62 \pm 0.19\ \mathrm{MeV},$$
$$\Delta B_\Lambda({}_\Lambda\mathrm{B}^{13} - {}_\Lambda\mathrm{C}^{13}) = 1.6 \pm 0.8\ \mathrm{MeV}. \qquad (1)$$

It is to be noticed that $B_\Lambda({}_\Lambda\mathrm{C}^{13}) = 10.9 \pm 0.3$ MeV[7] is even lower than $B_\Lambda({}_\Lambda\mathrm{B}^{12}) = 11.06 \pm 0.14$ MeV,[5] in spite of the smaller A value of ${}_\Lambda\mathrm{B}^{12}$. The third example we wish to consider is that of the $A=8$ hypernuclei. The "exotic" ${}_\Lambda\mathrm{He}^8$[8] lies considerably above the straight line of Fig. 1. Here we have[5,8]

$$\Delta B_\Lambda({}_\Lambda\mathrm{He}^8 - {}_\Lambda\mathrm{Li}^8) = 1.0 \pm 0.8\ \mathrm{MeV}, \qquad (2)$$

which might appear less problematic than in the previous examples (1). However, by coupling the particularly low first excited state of Li^7 ($J_N = \frac{1}{2}^-$, 0.478 MeV) to the Li^7 ground state ($J_N = \frac{3}{2}^-$), the spin-dependent effects in ${}_\Lambda\mathrm{Li}^8$ may reduce the "bare" ΔB_Λ.

Some α-α-Λ models[9] for ${}_\Lambda\mathrm{Be}^9$ give a rearrangement energy of $\lesssim 1$ MeV due to the fact that Be^8 is unstable against α-α formation. This provides a possible explanation of the large $\Delta B_\Lambda({}_\Lambda\mathrm{Li}^9 - {}_\Lambda\mathrm{Be}^9)$. However, the large rearrangement energy is not reproduced by a shell model calculation.[10] Clearly, the rearrangement arguments, when applied to the $A=8$ hypernuclei, affect $\Delta B_\Lambda({}_\Lambda\mathrm{He}^8 - {}_\Lambda\mathrm{Li}^8)$ in the opposite direction, since He^7 is particle unstable. The $A = 12$-13 hypernuclei are not affected by these considerations.

Here we propose that three-body ΛNN forc-