POLARIZATION OF COSMIC OH 18-cm RADIATION

P. L. Bender
Joint Institute for Laboratory Astrophysics,* Boulder, Colorado
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In a recent Letter,[1] Heer suggested that the anomalously high degree of circular polarization observed in cosmic OH 18-cm radiation[2] may be related to a saturation effect predicted earlier for maser amplifiers by Heer and Graft.[3] This effect involves the gain competition between right and left circularly polarized waves passing through an amplifying medium. For a single incident frequency, zero magnetic field, steady-state conditions, unidirectional propagation, and $F=2\rightarrow F=2$ or $F=1\rightarrow F=1$ transitions, Heer and Graft showed that whichever circular polarization was more intense would reduce the gain for the other circular polarization more than for itself. In a strongly amplifying and saturating medium, this would lead to one polarization becoming much stronger than the other if the source intensities were even slightly different. In contrast, for ordinary saturation behavior with incoherent waves, any fractional difference in source intensities for two polarizations with the same low-intensity gain would be reduced by amplification under saturation conditions.

The purpose of this Letter is to present a calculation indicating that the mechanism suggested by Heer is not applicable to the OH microwave emission problem. The reason is that the incident radiation is not monochromatic but is instead amplified spontaneous emission with a spectral width which is very large compared with the non-Doppler part of the linewidth for the amplifying medium. This leads to the terms responsible for the effect of Heer and Graft being small compared with the normal saturation terms. For the calculation we assume that the pumping rates for the two polarizations are essentially the same, as is the case for the appealing differential absorption pumping mechanism which was suggested recently by Litvak et al.[4]

We will consider the saturation behavior in a medium which is amplifying unidirectional spontaneous emission of both circular polarizations. For the ionization front between HII and HI regions where OH inversion is believed to occur, the collisional width is unlikely to be much larger than the 10^{-6}-sec^{-1} estimate of Litvak et al.[4] based on 10^4 hydrogen atoms/cm^3 and is probably less. The radiation width due to ultraviolet absorption is estimated to be[4,5] roughly 10^{-7} sec^{-1}, and the natural width is about 10^{-10} sec^{-1}. The incident spectral width at any point in the medium, even after gain narrowing, is unlikely to be a great deal less than the 10^4-sec^{-1} Doppler width characteristic of OH at about 30°K. It is therefore much larger than the inverse lifetime of the levels.

For simplicity we consider only a transition $J=1\rightarrow J=0$ and take all decay constants to be Γ. The states are labeled as follows: $(1,1)=1$, $(1,0)=2$, $(1,-1)=3$, and $(0,0)=4$. The microwave field is taken to be of the form

$$\sum_n\{U_n[(\hat{i}+i\hat{j})/\sqrt{2}]\exp i\omega_n t+V_n[(\hat{i}-i\hat{j})/\sqrt{2}]\exp i\nu_n t\},$$

where U_n and V_n are complex. Letting λ_1, λ_0 be the rates of populating the upper and lower levels, and β be the dipole moment, we find that the time derivatives for the necessary density-matrix elements are given by

$$(\dot{\rho}_{11}-\dot{\rho}_{44})=(\lambda_1-\lambda_0)-\Gamma(\rho_{11}-\rho_{44})-2R_+-R_-,$$

$$(\dot{\rho}_{33}-\dot{\rho}_{44})=(\lambda_1-\lambda_0)-\Gamma(\rho_{33}-\rho_{44})-R_+-2R_-,$$

$$\dot{\rho}_{41}=(i\omega_0-\Gamma)\rho_{41}-i\beta(\rho_{11}-\rho_{44})\sum_m U_m\exp i\omega_m t-i\beta\rho_{31}\sum_m V_m\exp i\nu_m t,$$

$$\dot{\rho}_{43}=(i\omega_0-\Gamma)\rho_{43}-i\beta(\rho_{33}-\rho_{44})\sum_m V_m\exp i\nu_m t-i\beta\rho_{31}{}^*\sum_m U_m\exp i\omega_m t,$$

$$\dot{\rho}_{31}=[i(\omega_0-\nu_0)-\Gamma]\rho_{31}-i\beta\rho_{41}\sum_n V_n{}^*\exp(-i\nu_n t)+i\beta\rho_{43}{}^*\sum_n U_n\exp i\omega_n t.$$

Here R_+, R_- are the stimulated emission rates for right and left circularly polarized radiation and are given by

$$R_+=2\,\mathrm{Re}\{i\beta\rho_{41}\sum_n U_n{}^*\exp(-i\omega_n t)\},$$

$$R_-=2\,\mathrm{Re}\{i\beta\rho_{43}\sum_n V_n{}^*\exp(-i\nu_n t)\}.$$

For large t, setting $\Delta_n \equiv \omega_n - \omega_0$ and $\delta_n \equiv \nu_n - \nu_0$, we find to zero and second order in β times the electric field:

$$(\rho_{11}-\rho_{44})^{(0)} = (\rho_{33}-\rho_{44})^{(0)} = (\lambda_1-\lambda_0)/\Gamma,$$

$$\Gamma(\rho_{11}-\rho_{44})^{(2)} = -2R_+^{(2)} - R_-^{(2)},$$

$$\Gamma(\rho_{33}-\rho_{44})^{(2)} = -R_+^{(2)} - 2R_-^{(2)},$$

$$\Gamma\rho_{31}^{(2)} = -\beta^2(\lambda_1-\lambda_0)\sum_{mn} U_m V_n^* \left[\frac{1}{\Gamma+i(\Delta_m-\delta_n)}\right]\left[\frac{1}{\Gamma+i\Delta_m}+\frac{1}{\Gamma-i\delta_n}\right]\exp[i(w_m-\nu_n)t],$$

where

$$\Gamma R_+^{(2)} = 2\beta^2(\lambda_1-\lambda_0)\sum_{mn}\mathrm{Re}\{U_m U_n^* \exp[i(\omega_m-\omega_n)t]/[\Gamma+i\Delta_m]\},$$

$$\Gamma R_-^{(2)} = 2\beta^2(\lambda_1-\lambda_0)\sum_{mn}\mathrm{Re}\{V_m V_n^* \exp[i(\nu_m-\nu_n)t]/[\Gamma+i\delta_m]\}.$$

This gives to fourth order

$$R_+^{(4)} = -(2\beta^4/\Gamma^2)(\lambda_1-\lambda_0)\sum_{klmn}\mathrm{Re}\Big\{U_k U_l^* \exp[i(\omega_k-\omega_l)t]$$

$$\times\Big[2U_m U_n^* \exp[i(\omega_m-\omega_n)t]\left(\frac{1}{\Gamma+i\Delta_m}+\frac{1}{\Gamma-i\Delta_n}\right)\left(\frac{1}{\Gamma+i(\Delta_k+\Delta_m-\Delta_n)}\right)$$

$$+V_m V_n^* \exp[i(\nu_m-\nu_n)t]\left(\frac{1}{\Gamma+i\delta_m}+\frac{1}{\Gamma-i\delta_n}\right)\left(\frac{1}{\Gamma+i(\Delta_k+\delta_m-\delta_n)}\right)$$

$$+V_m V_n^* \exp[i(\nu_m-\nu_n)t]\left(\frac{1}{\Gamma+i\Delta_k}+\frac{1}{\Gamma-i\delta_n}\right)\left(\frac{\Gamma}{\Gamma+i(\Delta_k-\delta_n)}\right)\left(\frac{1}{\Gamma+i(\Delta_k+\delta_m-\delta_n)}\right)\Big]\Big\}.$$

Calculating the expectation value of $R_+^{(4)}$ by summing over k and m and using the fact that the phases of U_i and V_j are random, we have

$$\langle R_+^{(4)}\rangle = -\left(\frac{2\beta^4}{\Gamma^2}\right)(\lambda_1-\lambda_0)\,\mathrm{Re}\Big\{2\sum_{ln}|U_l|^2|U_n|^2\left[\frac{1}{\Gamma+i\Delta_l}\right]\left[\frac{2}{\Gamma-i\Delta_n}+\frac{1}{\Gamma+i\Delta_l}+\frac{1}{\Gamma+i\Delta_n}\right]$$

$$-2\sum_n |U_n|^4\left[\frac{1}{\Gamma+i\Delta_n}\right]\left[\frac{1}{\Gamma+i\Delta_n}+\frac{1}{\Gamma-i\Delta_n}\right]+\sum_{ln}|U_l|^2|V_n|^2\left[\frac{1}{\Gamma+i\Delta_l}\right]\left[\frac{2\Gamma}{\Gamma^2+\delta_n^2}\right]$$

$$+\sum_{ln}|U_l|^2|V_n|^2\left[\frac{1}{\Gamma+i\Delta_l}\right]\left[\frac{\Gamma}{\Gamma+i(\Delta_l-\delta_n)}\right]\left[\frac{1}{\Gamma+i\Delta_l}+\frac{1}{\Gamma-i\delta_n}\right]\Big\}.$$

Here the first two terms are from $R_+^{(2)}$, the third from $R_-^{(2)}$, and the fourth from $\rho_{31}^{(2)}$. For U and V monochromatic and $\Delta_1 = \delta_1$, we note that

$$\langle R_+^{(4)}\rangle = -(2\beta^4/\Gamma^2)(\lambda_1-\lambda_0)[2\Gamma/(\Gamma^2+\Delta_1^2)]^2\{|U|^4+|U|^2|V|^2\}.$$

This agrees with the result of Heer and Graft that for transitions $J=1\to J=0$ the coefficients of $|U|^4$ and of $|U|^2|V|^2$ become equal, leading to a saddle point,[3] if $|\Delta_1-\delta_1| \ll \Gamma$.

To demonstrate the effect of the incident spectrum being broad compared with Γ, we take U_n, V_n to be uniform in amplitude but random in phase from $\omega_0-\Delta$ to $\omega_0+\Delta$ and from $\nu_0-\Delta$ to $\nu_0+\Delta$. Changing to integrals, and noting that $\beta^2|U|^2$, $\beta^2|V|^2$ now have units of frequency instead of frequency squared, we have

$$\langle R_+^{(4)}\rangle = -(2\beta^4/\Gamma^2)(\lambda_1-\lambda_0)\{I_1|U|^4+(I_2+I_3)|U|^2|V|^2\},$$

$$I_1 = 2\,\mathrm{Re}\int_{-\Delta}^{\Delta}\int_{-\Delta}^{\Delta}\left[\frac{1}{\Gamma+ix}\right]\left[\frac{1}{\Gamma+ix}+\frac{1}{\Gamma+iy}+\frac{2}{\Gamma-iy}\right]dxdy,$$

$$I_2 = 2\Gamma^2\int_{-\Delta}^{\Delta}\left[\frac{dx}{\Gamma^2+x^2}\right]\int_{-\Delta}^{\Delta}\left[\frac{dy}{\Gamma^2+y^2}\right],$$

$$I_3 = \Gamma\,\mathrm{Re}\int_{-\Delta}^{\Delta}\int_{-\Delta}^{\Delta}\left[\frac{1}{\Gamma+ix}\right]\left[\frac{1}{\Gamma+i(x-y)}\right]\left[\frac{1}{\Gamma+ix}+\frac{1}{\Gamma-iy}\right]dxdy.$$

It is easily shown that $I_1 \to 6\pi^2+8$, $I_2\to 2\pi^2$, and $I_3\to 0$ as $(\Gamma/\Delta)\to 0$. The coefficient of $|U|^4$ will thus be larger than that of $|U|^2|V|^2$ and normal saturation behavior depending only on changes in the level populations will occur.[3] For a more realistic line shape and after integrating over the Doppler distribution of absorber frequencies, the ratio of the coefficients will change somewhat, but the contribution from $\rho_{31}^{(2)}$ to the saturation will still be small. Therefore, for a $J=1\to J=0$ transition, the coherence effect discussed by Herr and Graft is unimportant if the incident spectral width is large compared with the non-Doppler part of the linewidth. The calculation has not been done for transitions $F=2,1\to F=2,1$ or for $\beta^2|U|^2/\Gamma$ large, but the coherence effect would be expected to vanish here also. It thus seems unlikely that the mechanism suggested by Heer can be responsible without some modification for the anomalously high degree of circular polarization observed in the case of cosmic OH 18-cm radiation.

It is of interest to note that the assumption of a finite spectral width for the incident radiation will reduce the relative importance of the coherence effect even though the width 2Δ is much less than Γ. If both U and V are near resonance, $I_1\cong 32\Delta^2/\Gamma^2$, $I_2\cong 8\Delta^2/\Gamma^2$, and $I_3\cong 8\Delta^2/\Gamma^2$. The doubling of the $|U|^4$ term compared with the $|U|^2|V|^2$ one is due to the amplitude fluctuations of the two polarizations being uncorrelated. If the incident light comes from a laser or maser oscillator, however, the monochromatic analysis is correct since feedback in the source makes the amplitude fluctuations small.

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*Of the National Bureau of Standards and the University of Colorado.

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