QUANTUM PHASE CORRELATION IN SMALL SUPERCONDUCTORS

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We have observed for very narrow tin films a depression of the superconducting transition temperature below that of wider portions of the same films. We propose a model which accounts for this depression and also suggests a condition on the maximum microwave enhancement of critical current in narrow thin-film bridges as observed by Wyatt et al.

In the present experiments we have found that in very narrow tin films the superconducting transition temperature is depressed below the value characteristic of wider portions of the same films. The magnitude of this depression, ΔT_c, depends on both the thickness and width of the film. We propose that this effect is due to the presence of an energy of order kT^* which is associated with the "degree of freedom" corresponding to uncorrelated differences in the quantum mechanical phase. T^* is an effective average temperature for the sample and its associated measuring circuitry.

The transition to the superconducting state occurs at a temperature for which the Gibbs potentials of the normal and the condensed state are equal, and the condensed state is favored when $\Delta G_{n-s} > 0$. Between two electrically connected superconductors whose quantum mechanical phases are uncoupled there is a voltage proportional to the time derivative of the phase difference.[1,2] Thus the system has, in a certain sense, a degree of freedom corresponding to this phase difference.[3] If the connecting link also becomes superconducting the phases are correlated, this "degree of freedom" is lost, and an energy of order kT^* is released. Thus, the connecting link will remain "normal," separating the two superconductors, at temperatures below the usual transition temperature until this gain in thermal energy (kT^*) can be absorbed by the free-energy loss, ΔG_{n-s}, at the transition of the link. The transition of the connecting link can thus occur only when

$$(\Delta G_{n-s}) \sim kT^*. \tag{1}$$

If we assume that the link is "normal" in the usual sense then $(\Delta G_{n-s}) = \mu_0 H_c^2/2 \times [\text{volume}]$. The volume will necessarily be as small as possible, consistent with the requirement that it separate the phases, since this allows the maximum amount of sample material to become superconducting. The shortest link in which the electrons can lose their phase memory is a link one mean free path long. We therefore choose to identify the volume as the product of the cross-sectional area and the electronic mean free path. Assuming that $H_c = H_0(1 - T^2/T_c^2)$ one finds that the depression of the transition temperature $\Delta T_c \equiv T_c - T_c'$ has the form

$$(\Delta T_c)^2 \sim kT^* \left[\frac{T_c^2}{2\mu_0 H_0^2 w\delta l}\right], \tag{2}$$

where w is the film width, δ the thickness, l the effective mean free path due to surface and impurity scattering, and T_c' the depressed transition temperature.

We have measured the superconducting transition temperatures of a variety of narrow tin films and in each case have compared the transition temperature T_c' of the narrow strip with that of the much wider sections of the same film which served as leads. Using a simple four-terminal method, curves of resistance versus temperature were recorded during pumpdown using He^4 vapor-pressure thermometry and during warmup using a calibrated carbon resistance thermometer. No hysteresis in T_c' or T_c was observed. Small test currents, of the order of 10^{-8} A, were used in order to avoid any significant suppression of the transition temperature in the narrow strip where the current density is largest. The critical currents in the narrow strips far exceed the test current except for temperatures within 10^{-3} K° of the transition temperature. The test current was varied about these small values with no apparent change in the observed ΔT_c. The films were produced by vacuum evaporation of 99.999% pure Sn at a pressure of about 5×10^{-7} Torr onto microscope-slide substrates cooled with liquid nitrogen. The technique for the preparation of masks for the 1- to 5-μ-wide thin-film strips has been discussed

Table I. Experimental values of the transition temperature depression ΔT_c for a variety of tin films.

Thickness δ (Å)	Width w (μ)	Length L (μ)	Effective mean free path l (calculated) (Å)	ΔT_c (K°)	T_c (°K)
495	1.0	300	396	0.047	3.90
495	3.0	300	396	0.018	3.86
495	0.9	300	396	0.050	3.87
1500	1.7	440	858	0.018	3.85
525	~5.0	8.5	415	0.005	3.99
170	1.7	280	112	0.105	3.99
267	40	800	174	0.019	3.92
200	160	10 000	135	0.011	3.97
170	2.9	350	112	0.085	4.02
2000	3.5	~1	1000	0.005[a]	3.85

[a]Data for this film were taken from the paper by Wyatt et al. (Ref. 2).

elsewhere.[4] No tapering of the film edges could be observed at a resolution of about 0.1 μ.

In Table I we have given for several samples the dimensions and experimental values for the depression of the transition temperature ΔT_c. For these films the width of the transition region was usually of order 10^{-2} K° in both wide and narrow sections. The values given for the mean free path were calculated using the usual relation, $l^{-1} = \delta^{-1} + l_{imp}^{-1}$, where l_{imp} is the impurity-limited mean free path. The effective impurity-limited mean free path is shorter in the case of the thinnest films due to the effects of film agglomeration, but this correction is not very important for the samples used here.

In Fig. 1 the experimental values of ΔT_c are plotted as a function of $(w\delta l)^{-1/2}$ to provide a test of the linear dependence on this quantity predicted by Eq. (2). The fit to a straight line is within the limits of experimental uncertainty. The temperature T^* is found from Fig. 1 to be 26.5°K, not an unreasonable average temperature for the superconductor and measuring wires.

Results recently reported by Wyatt et al.[5] and by Dayem and Wiegand[6] show that when two large thin-film sections are connected by a narrow and short bridge, it is possible under certain conditions to increase the dc critical current in the bridge by imposing a microwave field upon the sample. This apparent enhancement may simply indicate a restoration of the critical current to the thermally undisturbed value. Without specifying any particular dynamic mechanism for this process, it is possible to arrive at some general consequences. If the microwave-induced currents independently impose a constraint upon the quantum phase difference between the two superconductors, the phase is no longer a degree of freedom in the thermodynamic sense.[1] Thus, under optimum microwave stimulation, the connecting bridge is no longer required to constrain the relative phases in order to become superconducting, and its transition tempera-

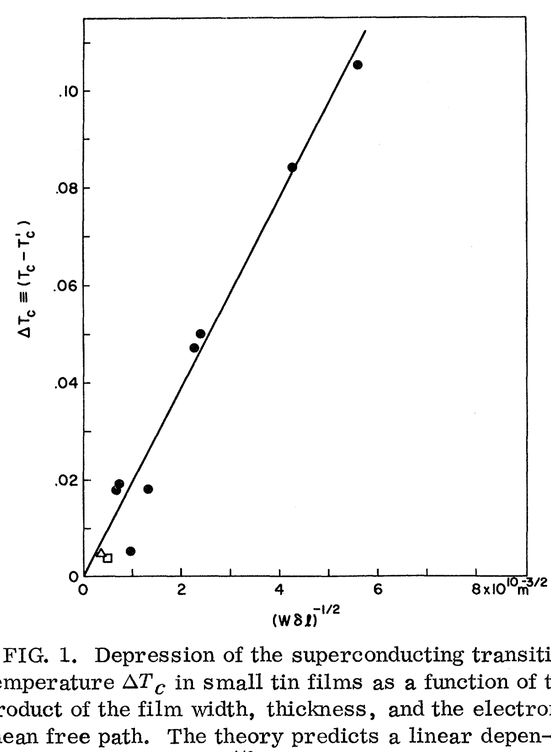

FIG. 1. Depression of the superconducting transition temperature ΔT_c in small tin films as a function of the product of the film width, thickness, and the electronic mean free path. The theory predicts a linear dependence of ΔT_c on $(w\delta l)^{-1/2}$.

ture will increase to the value characteristic of the bulk film. On this hypothesis one may readily calculate the maximum ratio $j_c(T_c-T)/j_c(T_c'-T)$ of the current with optimum microwave phase synchronization to that in the absence of microwaves. For $T_c'-T \ll T_c'$ one can describe the critical current density adequately[4] by the relation $j_c=[4j_0/T_c'^{3/2}](T_c'-T)^{3/2}$. In Fig. 2 calculated maximum-to-minimum critical-current ratios have been plotted as a function of temperature for various values of ΔT_c. The results of Wyatt et al. are indicated by the triangles and they are in rather good agreement with the curve for $\Delta T_c = 0.005$ K°. This is consistent with their observation of an apparent partial transition some 5 mdeg above their measured T_c'. Using this value for ΔT_c and calculating $(w\delta l)^{-1/2}$ from their reported average sample geometry, we have obtained the point indicated by the triangle in Fig. 1. The point indicated by the square was taken from data supplied in the paper by Dayem and Wiegand. These values for ΔT_c are quite consistent with those which we are reporting here. The model presented here also predicts that in the temperature range $T_c' < T < T_c$ it should be possible to observe supercurrents in the presence of microwaves when none occur without the microwaves. This feature has also been observed by Dayem and Wiegand.

The superconducting transition temperature of small thin-film strips has been observed to be depressed below that of larger sections of the same films. We have sought an explanation in the concept that for two coherent quantum systems each consisting of a thermodynamically macroscopic number of particles, there is an average energy of order kT^* associated with uncorrelated differences in their quantum phase.

The proposed model implies further that for a link connecting two superconductors there is a minimum cross-sectional area which can exhibit zero resistance at a finite temperature. The quantum phases of two superconductors connected by a link of smaller area should remain uncorrelated at all temperatures. For tin this critical area is about 10^{-16} m^2. Quantum interference experiments utilizing point-contact geometries[7] have shown that for estimated contact areas in this range (10^{-16} m^2) such contacts exhibit electrodynamic behavior very similar to that of Josephson junctions. Experiments are being conducted to investigate further the effect of small contact areas on quantum phase correlations between superconductors.

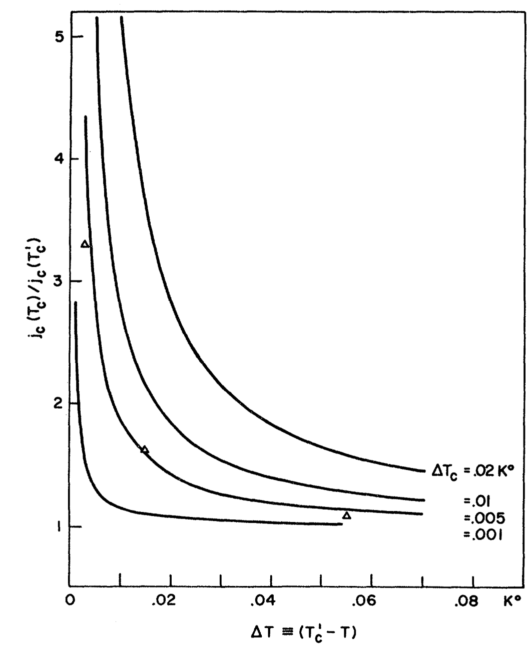

FIG. 2. The calculated temperature dependence of the ratio of the maximum to minimum critical current density for tin films with different values of the transition temperature depression $\Delta T_c \equiv T_c - T_c'$.

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