thermally induced stresses exist suggest that stress and strain may be instrumental in producing the observed interface charge distribution.

The authors are indebted to A. Schoeler, V. Klints, and V. Sapione for their valuable assistance in construction of the experimental apparatus and in carrying out the measurements. Helpful discussions with Dr. F. du Pré and Dr. E. S. Rittner are gratefully acknowledged.

---

[1]Y. Miura, J. Appl. Phys. (Japan) 4, 958 (1965).

[2]J. F. Delord et al., Bull. Am. Phys. Soc. 10, 546 (1965).

[3]P. Balk, P. J. Burkhardt, and L. V. Gregor, Proc. IEEE 53, 2133 (1965).

[4]E. Kooi, Philips Res. Repts. 21, 477 (1966).

[5]N. Kawamura and H. Iwasaki, to be published.

[6]J. Ladell, Trans. Am. Cryst. Assoc. 1, 86 (1965).

[7]G. Abowitz and E. Arnold, Rev. Sci. Instr. 38, 564 (1967).

[8]A. Many, Y. Goldstein, and N. B. Grover, Semiconductor Surfaces (John Wiley & Sons, Inc., New York, 1965), Chap. 4.

[9]C. N. Berglund, IEEE Trans. Electron Devices ED-13, 701 (1966).

[10]C. S. Barrett, Structure of Metals (McGraw-Hill Book Company Inc., New York, 1952), p. 26.

[11]The symbol ⟨111⟩ is used generically to include [111], [$\bar{1}$11], [1$\bar{1}$1], [11$\bar{1}$], etc., where [111] denotes the direction normal to the (111) planes.

[12]One possibility is that the slight shifts result from the fact that the experiment was performed on a hemisphere rather than a full sphere.

[13]J. A. Dillon, Jr., and H. E. Farnsworth, J. Appl. Phys. 29, 1195 (1958). These authors determined the work function for ion-bombarded silicon in high vacuum. Although these are the best available values, it is not certain that the same values of the work function are applicable to thermally oxidized silicon surfaces.

[14]J. F. Nye, Physical Properties of Crystals (Oxford University Press, London, 1957), p. 143.

[15]American Institute of Physics Handbook (McGraw Hill Book Company, Inc., New York 1957), p. 3-81.

[16]A. G. Revesz, K. H. Zaininger, and R. J. Evans, J. Phys. Chem. Solids 28, 197 (1967).

[17]W. A. Pliskin, IBM J. Res. Develop. 10, 198 (1966).

POLARIZABILITY OF A TWO-DIMENSIONAL ELECTRON GAS

Frank Stern
IBM Watson Research Center, Yorktown Heights, New York
(Received 13 February 1967)

The response of a two-dimensional electron gas to a longitudinal electric field of arbitrary wave vector and frequency is calculated in the self-consistent-field approximation. The results are used to find the asymptotic screened Coulomb potential and the plasmon dispersion for a plane of electrons imbedded in a three-dimensional dielectric.

There has been increased interest in the theory of two-dimensional systems recently, partly because of the relevance of such theories to the properties of thin films and surfaces. A particularly interesting example is the n-type inversion layer of a Si-SiO_2-metal structure, whose density of states has been shown to have the behavior expected of a two-dimensional electron gas,[1] and whose carrier concentration can be varied by at least two orders of magnitude simply by changing the voltage across the oxide layer.

We present results for the response of a two-dimensional electron gas to longitudinal electric fields of arbitrary wave vector $\vec{q}$ and frequency ω, the two-dimensional analog of the longitudinal Lindhard[2] dielectric constant. From this the screening behavior of the system, the plasmon frequency as a function of wave vector, and the energy loss of moving charged particles can be calculated. We give as examples the asymptotic expression for the potential due to an external charge, and the approximate solution of the plasmon dispersion equation.

A longitudinal electric field $\vec{E}(\vec{q},\omega)=\vec{E}_0\exp(i\vec{q}\cdot\vec{r}-i\omega t)$ acting on a two-dimensional electron gas will induce a polarization

$$\vec{P}(\vec{q},\omega)=\chi(\vec{q},\omega)\vec{E}(\vec{q},\omega)\delta(z), \qquad (1)$$

where $\vec{q}$ has only x and y components, $\vec{q}\times\vec{E}_0=0$, and the electrons are in the plane $z=0$.

The self-consistent-field treatment of the response of the electron gas gives the same expression for the polarizability χ as in the three-dimensional case[3]:

$$\chi(\vec{q},\omega)=\frac{e^2}{q^2\Omega}\lim_{\alpha\to 0}\sum\frac{f_0(E_{\vec{k}})-f_0(E_{\vec{k}+\vec{q}})}{E_{\vec{k}+\vec{q}}-E_{\vec{k}}-\hbar\omega-i\hbar\alpha},\quad(2)$$

where the sum is taken over all one-electron states; $E_{\vec{k}}$ is their energy, f_0 is the Fermi-Dirac occupation probability, Ω is the normalization area, and $q=|\vec{q}|$. We use Gaussian units.

If we evaluate (2) at absolute zero for a two-dimensional electron gas with energy levels $E_{\vec{k}}=\hbar^2k^2/2m^*$,[4] Fermi wave vector k_F, and Fermi velocity $v_F=\hbar k_F/m^*$, and introduce the usual[2] dimensionless quantities $z=q/2k_F$ (not to be confused with the coordinate z perpendicular to the electron plane) and $u=\omega/qv_F$, we find that the real and imaginary parts of χ are

$$\chi_1=G\{2z-C_-[(z-u)^2-1]^{1/2}-C_+[(z+u)^2-1]^{1/2}\},\quad(3a)$$

$$\chi_2=\sigma/\omega=G\{D_-[1-(z-u)^2]^{1/2}-D_+[1-(z+u)^2]^{1/2}\},\quad(3b)$$

$$G=Ne^2/m^*zq^2v_F^{\ 2},\quad(3c)$$

$$C_\pm=(z\pm u)/|z\pm u|\text{ and }D_\pm=0\text{ for }|z\pm u|>1,\quad(3d)$$

$$C_\pm=0\text{ and }D_\pm=1\text{ for }|z\pm u|<1,\quad(3e)$$

where N is the number of electrons per unit area and $\sigma(\vec{q},\omega)$ is the conductivity.

The polarizability (3) satisfies the dispersion relations, and at high frequencies its real part is $\chi_1\sim -Ne^2/m^*\omega^2$. The two-dimensional conductivity, therefore,[5] obeys the sum rule[5]

$$\int_0^\infty\sigma(\vec{q},\omega)d\omega=\pi Ne^2/2m^*.\quad(4)$$

What we have found so far is the polarization of the electron-gas plane in the presence of a total field $\vec{E}$ in the plane. We still need a constitutive relation between the induced field and the total field to calculate a dielectric constant. This relation, and, therefore, the dielectric constant itself, depends not only on the properties of the electron gas, but also on its surroundings. We assume here that the electron plane is surrounded by a medium with dielectric constant κ_b. Then the induced field associated with the polarization (1) is proportional to $\exp(i\vec{q}\cdot\vec{r}-i\omega t-\beta|z|)$, where $\beta=(q^2-\kappa_b\times\omega^2/c^2)^{1/2}$. (If β is complex, the root with negative imaginary part is to be taken.) The induced field in the plane $z=0$ is found to be

$$\vec{E}_{\text{ind}}(\vec{q},\omega)=-2\pi\beta\chi(\vec{q},\omega)\vec{E}(\vec{q},\omega)/\kappa_b.\quad(5)$$

We define the dielectric constant for this geometry by the relation $\kappa/\kappa_b=(\vec{E}-\vec{E}_{\text{ind}})/\vec{E}$, and find

$$\kappa(\vec{q},\omega)=\kappa_b+2\pi\beta\chi(\vec{q},\omega).\quad(6)$$

The dependence of the plasmon frequency ω_p on wave vector is found from (6) by solving the equation $\kappa(\vec{q},\omega_p)=0$. If κ_b is dispersionless, and $u\gg 1$, we find

$$q^2=\kappa_b\omega_p^{\ 2}/c^2+(\omega_p^{\ 2}/a)^2,\quad(7)$$

where $a=2\pi Ne^2/m^*\kappa_b$. For small values of q, the first term on the right dominates, and the phase velocity ω_p/q approaches the phase velocity of a propagating wave in the dielectric, but is always smaller. Thus the plasmons cannot radiate.[6] For larger values of q, the first term on the right becomes negligible, and the plasma frequency rises like $q^{1/2}$. For still larger q, the condition $u\gg 1$ used to derive (7) becomes invalid. The leading terms in the expansion of $\omega_p^{\ 2}$ in powers of q are then found to be

$$\omega_p^{\ 2}\sim aq+3q^2v_F^{\ 2}/4.\quad(8)$$

Another result which can be obtained easily from the dielectric constant (6) is the screening of a static point charge. If a charge Ze is located at $x=y=0$, $|z|=d$, the potential in the electron plane is

$$\varphi(r)=Ze\int_0^\infty\kappa^{-1}(q,0)\exp(-qd)J_0(qr)dq,\quad(9)$$

where J_0 is the Bessel function of order zero, and $r=(x^2+y^2)^{1/2}$. The static dielectric constant is

$$\kappa(q,0)=\kappa_b(1+s/q),\quad q\le 2k_F,\quad(10a)$$

$$=\kappa_b[1+(s/q)\{1-[1-(2k_F/q)^2]^{1/2}\}],\quad q>2k_F;\quad(10b)$$

$$s=4\pi Ne^2/m^*\kappa_bv_F^{\ 2}=2n_v/a^*,\quad(10c)$$

where n_v is the degeneracy factor defined in Ref. 4, and $a^* = \kappa_b \hbar^2/m^* e^2$ is the effective Bohr radius.

The long-wavelength approximation (10a) to the static dielectric constant leads to the same result as does the Thomas-Fermi approach to the screening,[7] with a potential whose asymptotic form at large r is $\varphi \sim Zes(1+sd)/\kappa_b(sr)^3$.[7] Because of the discontinuity in $d\kappa(q,0)/dq$ at $q = 2k_F$, we must add to this an oscillatory term which can be evaluated from (9) and (10) using a theorem of Lighthill.[8] The oscillatory term dominates at large r, and has the asymptotic form

$$\varphi(r) \sim -\frac{Zes}{\kappa_b} \frac{4k_F^2 \exp(-2k_F d)}{(2k_F+s)^2} \times \left[\frac{\sin\xi}{\xi^2} + \frac{8^{1/2}s\cos(\xi-\pi/4)}{\pi^{1/2}(2k_F+s)\xi^{5/2}} + \cdots\right], \quad (11)$$

where $\xi = 2k_F r$. The leading term in (11) has the same dependence on r as does the result of Roth, Zeiger, and Kaplan[9] for a three-dimensional semiconductor with cylindrical energy bands.

The screened Coulomb potential due to external charges is discussed more fully in a forthcoming paper,[7] where it is used to calculate bound states and ionized impurity scattering of the electrons confined to the plane.

I am indebted to R. A. Ferrell, W. E. Howard, G. J. Lasher, M. I. Nathan, and T. D. Schultz for discussions and comments.

---

[1]A. B. Fowler, F. F. Fang, W. E. Howard, and P. J. Stiles, Phys. Rev. Letters 16, 901 (1966).

[2]J. Lindhard, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 28, No. 8 (1954).

[3]H. Ehrenreich and M. H. Cohen, Phys. Rev. 115, 786 (1959). These authors use a time dependence $\sim\exp(i\omega t)$.

[4]We assume that the electrons occupy a single band, and ignore transitions to higher bands. If the occupied band has N electrons/cm^2, a spin degeneracy of 2, and an orbital degeneracy of n_v ($n_v = 1$ for a free-electron gas, and $n_v = 2$ for the results of Ref. 1), then the Fermi wave vector is $k_F = (2\pi N/n_v)^{1/2}$.

[5]The sum rule (4) applies to the model that led to (3), but does not have the generality of the corresponding three-dimensional result [see, for example, F. Stern, in Solid State Physics, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1963), Vol. 15, p. 299, esp. Sec. 23]. In a physical system such as a thin metallic film or the inversion layer of a semiconductor, our results are valid only for excitation energies $\hbar\omega$ smaller than the separation between the two lowest electric sub-bands.

[6]R. A. Ferrell, Phys. Rev. 111, 1214 (1958). See also K. L. Kliewer and R. Fuchs, Phys. Rev. 153, 498 (1967), and references cited therein.

[7]F. Stern and W. E. Howard, to be published.

[8]M. J. Lighthill, Introduction to Fourier Analysis and Generalised Functions (Cambridge University Press, Cambridge, England, 1958), p. 52.

[9]L. M. Roth, H. J. Zeiger, and T. A. Kaplan, Phys. Rev. 149, 519 (1966), Eq. (37).

PHONON-DISPERSION MEASUREMENTS ON A KRYPTON SINGLE CRYSTAL*

W. B. Daniels
Princeton University, Princeton, New Jersey, and Brookhaven National Laboratory, Upton, New York

and

G. Shirane, B. C. Frazer, H. Umebayashi, and J. A. Leake†
Brookhaven National Laboratory, Upton, New York

(Received 10 February 1967)

Phonon-dispersion relations for the symmetric [100], [110], and [111] branches in fcc krypton have been measured by triple-axis neutron spectrometry. Measurements were carried out at 79°K on a single-crystal sample grown from the melt at a pressure of 2.31 kbar.

The phonon-dispersion relations in fcc krypton have been measured on the triple-axis spectrometer at the Brookhaven high-flux-beam reactor. The single crystal used for the experiment was grown from the melt at a pressure of 2.31 kbar in an aluminum-alloy pressure cell incorporating a nucleation tip at the bottom. The cell was cylindrically shaped with an inside diameter of 12 mm and an outside diameter of 47 mm. The growth process was carried out in a temperature-controlled Dewar with heaters appropriately placed so as to pre-